Chapter

Literacy for Mathematicians

It is not enough to have a good mind. The main thing is to use it well.

—René Descartes, *Discourse on Method*

Problems of the Day

- Why us?
- Why is literacy important?
- What is the relationship between literacy and disciplinary literacy?
- How do we develop literate mathematicians?

A colleague of mine attempted to go to the public library on March 31—Cesar Chavez Day—only to find it closed in Chavez's memory. She remarked on how appalled the civil rights activist would be by this "honor." Chavez himself understood the key role that literacy plays in promoting a just society; as he stated to the Commonwealth Club in San Francisco on Nov. 9, 1984, "Once social change begins, it cannot be reversed. You cannot un-educate the person who has learned to read. You cannot humiliate the person who feels pride. You cannot oppress the people who are not afraid anymore." As a teacher, you know that education in general—and literacy specifically—has the power to change lives.

Educators and policymakers, parents and teachers, want everyone to be literate. Literacy is the key to individual opportunity, a pillar of our democracy, and a responsibility of a civilized nation. But how exactly we define literacy, whose job it is to produce literate learners, and how we successfully coax a child from here to there can be more controversial. Simply put, literacy is the ability to make and communicate meaning. As teachers of any content area, it is indeed our role to support learners in meeting the literacy demands of our discipline. This book will offer you insight about how you can be an effective instructor of literacy in general and, more important, how you can support all learners in developing proficiency with the disciplinary literacy of mathematics.

Why Us?

Given our changing standards, math teachers are no longer able to focus exclusively on teaching mathematical concepts and are now expected to offer all learners explicit, embedded instruction in literacy. Astute mathematicians recognize that students' literacy is inherently linked to their mathematical success. Historian of mathematics Glen Van Brummelen of Quest University Canada describes the relationship between literacy and mathematics:

> I consider mathematics learning and literacy to be so closely aligned that they are inseparable. There are two main purposes to learning mathematics. The first is to learn how to think clearly and logically. It goes almost without saying that someone who cannot express ideas and reasoning clearly is very unlikely to be thinking clearly in the first place. In fact, one might even define mathematics to be the art of clear communication in the world of numbers and geometry. The second purpose of learning mathematics is to be effective in quantitative and spatial relations when they arise in the physical world. In these cases, being able to solve a practical problem is entirely useless unless one is able to communicate it clearly to the target audience. (personal communication with the author)

Van Brummelen assures us of the intimate link between our content and literacy.

If only to multiply learners' mathematical prowess, if not to change the trajectory of their futures, math teachers are now called upon by the newest standards to integrate explicit literacy instruction into their regular math teaching routines—the specialized skills that students need to read, think, talk, and write as mathematicians.

In this chapter, we will articulate the nature and importance of literacy in general and then delve into the meaning of disciplinary literacy for mathematicians.

Why "General" Literacy Matters

As a mathematician, let the following statistics propel you to take up the charge of literacy instruction with gusto. Evidence clearly indicates that literacy has the power to save lives, transform the social order, and uplift a nation.

Literacy facilitates health

Literacy is a stepping-stone to school success and a prerequisite to high school graduation, which paves the way to healthy living. On average, high school graduates live longer, are less likely to become teen parents, and are more likely to raise healthier, better-educated children (Alliance for Excellent Education 2012).

By contrast, students who do not complete high school, often because of low literacy skills, encounter greater health challenges and incur significant costs. In a 2002 research review, medical professionals Andrus and Roth found that "forty-nine percent of patients with hypertension and 44% of patients with diabetes had inadequate functional health literacy" (Andrus and Roth 2002, p. 292). Colorado alone, according to statisticians' estimates, would save more than $280 million in health-care costs over the course of the lifetimes of each class of high school dropouts, had they earned their diplomas (Montelores Early Childhood Council 2013).

Illiterate individuals are at risk of poor health, may require expensive medical care, and might shorten their own lives because of their inability to understand doctors' instructions or follow written prescriptions.

Literacy promotes social justice

Literacy is a gateway to student achievement and high school graduation, yet literacy rates differ along racial lines. Longitudinal data produced by the National Assessment of Educational Progress (NAEP) demonstrates that, in reading, the achievement gap between white and black students and white and Hispanic students has narrowed since 1973. Still, in 2012, thirteen-year-old black students trailed their white peers by an average of 23 points, while Hispanic students scored 21 points below the average of their white counterparts (National Center for Education Statistics 2015).

Literacy is not the only predictor of high school graduation, but it is a significant factor. Nationwide, 70 percent of all students graduate from high school on time, yet the racial disparity in graduation rates is striking:

- 80 percent of Asian American students graduate from high school
- 76 percent of white students
- 58 percent of Latino students
- 53 percent of African American students
- 49 percent of Native American students (Alliance for Excellent Education 2012)

To close this achievement gap is to facilitate greater equality of opportunity for all children. A critical step in closing this gap is increasing the literacy rates of traditionally underperforming students. Literacy, therefore, is a social justice issue.

Speed Bumps on the Road to Literacy

How can it be possible that in a great modern nation such as ours, only 86 percent of our adult population is literate? There are a number of impediments that learners can encounter en route to full literacy, including—

- lack of access to books;
- parents' low literacy levels;
- dearth of literate role models;
- different language backgrounds;
- ineffective teachers or teaching methods;
- intellectual, physical, or emotional disabilities; and
- mismatch between learning styles and modes of instruction.

These are not excuses, just speed bumps some youngsters have to overcome. As teachers of all stripes, we can each work to decrease the possibility that hundreds of thousands of youth will continue to enter adulthood, year after year, ill-equipped to read the instructions on a bottle of medicine prescribed to save their mother's life, or their own.

Literacy creates individual opportunity

Literacy opens doors to individual economic opportunity. In 2011, the average annual income for a high school dropout was $25,100, as compared with $35,400 for a high school graduate, and $56,500 for those with a college degree (Baum, Ma, and Payea 2013). On average, in her lifetime, a college graduate today will earn $1 million more than a high school dropout.

Almost 85 percent of those tried in the juvenile court system are functionally illiterate; more than 60 percent of inmates of all ages are also functionally illiterate (Blankenship 2013). There is a positive correlation between illiteracy and crime, and a similar positive correlation between literacy and economic opportunity.

The nature of jobs is changing. While our grandparents' generation could earn good wages in stable manufacturing jobs with a high school diploma or less, the fastest-growing professions today have far greater than average literacy demands. Meanwhile, the fastest-declining professions have lower than average literacy demands. To be prepared for the jobs of the future, students need literacy skills.

Literacy promotes economic development

According to *The Economist* ("Counting Heads" 2004), a 1 percent increase in literacy scores leads to a 2.5 percent increase in labor productivity. Businesses in America currently spend upward of $60 billion annually on employee training, the greatest portion of which is devoted to remedial reading, writing, mathematics, and computer skills (ProLiteracy 2014). Supporting all learners in developing proficient literacy skills while in school will create far-reaching economic benefits: High school graduates are less likely to commit crimes, to depend on government health care, or to use public services such as housing assistance or food stamps. Economists estimate that cutting the nation's high school dropout rate in half would save the federal government $45 billion each year (Levin et al. 2006). Literacy is good for the economy.

Literacy learning, in general, is highly prized. Some students enter our math classrooms each year with basic literacy skills already in their backpacks, while others somehow missed picking them up along the way. As mathematics teachers attending to the literacy needs of learners, we not only enhance their mathematical understanding but also prepare them for lifelong success.

Literacy by the Numbers

Percent of U.S. adults who can't read	14%
Number of U.S. adults who can't read	32 million
Percent of U.S. adults who read below a fifth-grade level	21%
Percent of prison inmates who can't read	63%
Percent of high school graduates who can't read	19%

Source: National Center for Education Statistics 2013

Reflect

- What mathematical and literacy skills did you engage while reading these statistics about the importance of literacy?

What Is Literacy?

How do you define literacy for your students? What language skills do you believe they need to succeed in mathematics learning—and in life? As you review literacy standards authored by the National Council of Teachers of English as well as those by developed by the authors of the Common Core State Standards, I invite you to take this question personally, with a pen in hand: Underscore what you value and agree are key skills, and wrap parentheses around those you might leave to another teacher in another content area or year.

Literacy standards

The federal Workforce Investment Act of 1998 defined literacy as "an individual's ability to read, write, speak in English, compute and solve problems at levels of proficiency necessary to function on the job, in the family of the individual and in society" (Workforce Investment Act of 1998).

The National Council of Teachers of English (2013) defined literacy more broadly, asserting that twenty-first-century readers and writers need to—

- develop proficiency with the tools of technology;
- build relationships with others to pose and solve problems collaboratively and cross-culturally;
- design and share information for global communities to meet a variety of purposes;
- manage, analyze, and synthesize multiple streams of simultaneous information; and
- create, critique, analyze, and evaluate multimedia texts.

The Common Core State Standards Initiative, in the in the Common Core State Standards (CCSSI 2010), defines literacy even more globally and specifically. The Standards for English Language Arts state that students who are college and career ready will—

- demonstrate independence;
- build strong content knowledge;
- respond to the varying demands of audience, task, purpose, and discipline;
- comprehend as well as critique;
- value evidence;
- use technology and digital media strategically and capably; and
- come to understand other perspectives and cultures.

To this end, the Common Core's authors made an explicit effort to delineate how literacy instruction ought to be integrated across all content areas by articulating a separate set of literacy standards for history/social studies, science, and technical subject teachers, as described briefly in figure 1.1. Holding the broad goals on the preceding page as a backdrop, the majority of this book focuses on the specific components of literacy as defined by the Common Core: reading, writing, speaking, listening, and language.

Common Core literacy shift	In math, students . . .
Regular practice with **complex texts** and their academic language	• read nonfiction from textbooks, peers' work, or other resources • learn and use mathematical vocabulary
Reading, writing, and speaking **grounded in evidence from texts**, both literary and informational	• read, write, and discuss ideas and solutions • use evidence and argument to evaluate and respond to the thinking of others
Building knowledge through content-rich nonfiction	• read complex nonfiction chock-full of abstract concepts and challenging vocabulary

Fig. 1.1. Math learning aligned with Common Core (CCSSI 2014) literacy shifts

Disciplinary Literacy

With accountability to so many standards and responsibility to all our students, math teachers are wise to home in on what literacy instruction really ought to look and sound like in our classrooms. So, let us zero in now on the traditional domains of literacy—reading, writing, and speaking—which allow us access to all the higher-order opportunities required for college and career readiness as described by the Common Core.

Researchers Timothy and Cynthia Shanahan (2008) describe literacy learning as including three layers:

- Basic literacy: literacy skills such as decoding and knowledge of high-frequency words that underlie virtually all reading tasks
- Intermediate literacy: literacy skills common to many tasks, including generic comprehension strategies, common word meanings, and basic fluency
- Disciplinary literacy: literacy skills specialized to history, science, mathematics, literature, or other subject matter

Elementary school teachers devote a great deal of instructional time to ensuring that learners achieve basic and intermediate literacy skills. Yet explicit content-area literacy instruction often drops off abruptly in middle and high school, just as students are expected to comprehend and compose more technical texts written with complex academic language in content-specific genres. Accessing and responding to these specialized texts often requires specialized literacy skills unique to the content area, yet few content-area teachers invest time and energy in developing students' disciplinary literacy.

"Because students do not usually enter content-area classrooms knowing how to read and write the specialized print and non-print texts of the various disciplines, teachers must provide literacy instruction in content-area classrooms," Tom Bean explains in the foreword to (*Re)Imagining Content-Area Literacy Instruction* (Draper 2005, p. 2).

Timothy and Cynthia Shanahan (2008) go on to describe the nature of high-quality content-area literacy instruction: It is authentic to the discipline, and it cultivates learners' true literacy in that content area.

Disciplinary literacy includes an understanding of the ways knowledge is created and shared within a discipline, as well as an understanding of the texts and discourse used by that discipline. For mathematics teachers, disciplinary literacy instruction begins with understanding what it means to read, write, speak, and listen as a mathematician. Shanahan and Shanahan (2008, p. 52) provided this example of how mathematicians read:

> When one of the mathematicians was thinking aloud during the reading of a journal article, he explained that one of the first things he did when reading was to memorize the variables that were to be used in the rest of the article. Even though the article began as mostly prose, he would soon be reading only symbols, and he did not want to interrupt his flow of thought by having to return to the definitions.

This example, unique to the content area of mathematics, illustrates a specialized reading skill unlikely to be introduced in any other course. So as content-area teachers, we can both continue to develop students' basic and intermediate literacy skills and build on those by offering direct instruction in disciplinary literacy.

Essentially, the work of the Shanahans and their colleagues illuminates the reality that literacy is learned in layers, that the literacy of each subject area is unique, and that as content teachers our role is to support learners by intentionally introducing them to the complexity and nuances of literacy as it applies to our chosen discipline. A study of the writing of professionals in our content area will help us gain insight into the genres of disciplinary writing; eavesdropping on the discussions of professional engineers, mathematicians, statisticians, accountants, and economists at work will assist us in identifying key features of mathematical discourse, as we will do later in this book.

The disciplinary literacy of mathematics

Disciplinary literacy is based on the premise that students can develop deep conceptual knowledge in a discipline *only* by using the habits of reading, writing, talking, and thinking which that discipline values and uses.

—McConachie et al., "Tasks, Text, and Talk: Literacy for All Subjects"

Knowing that literacy is a cornerstone of lifetime achievement, that literacy involves many modes of communication, and that literacy must be taught in each discipline throughout a student's career in school, let us look closely at the disciplinary literacy of mathematics.

Disciplinary literacy can be considered as the means by which students engage with and learn new material, as well as the ways in which they demonstrate their understanding of it. Reflecting on your own experience as a math learner in school, you probably recollect numerous ways that you were expected to rely on your literacy skills both to learn content and to present your learning: reading the text, explaining a solution, writing a proof, and so

forth. Can you recall ever being explicitly taught how to do so? Disciplinary literacy requires more than assigning literacy tasks in the context of math teaching. Rather, it needs to evolve to incorporate direct instruction about what these literacy skills are and how we can harness their power to generate mathematical understanding.

Mathematics and Literacy through History

Mathematics learning is a journey of understanding. To understand is to make knowledge one's own, readily available for application in new circumstances. Mathematics and literacy are and have always been symbiotic. The earliest mathematical understandings were shared in spoken words. The first geometric discoveries were documented in written treatises.

The early history of mathematics is known by its texts: notched animal bones, carved clay tablets dried in the sun. Greek, Chinese, and Indian mathematicians left us centuries-old evidence of their thinking in books, on cloth or parchment. We know that the early centers of learning, such as Plato's fourth-century academy, engaged mathematicians in great debates. Vast libraries could be filled with the mathematical texts of earlier civilizations, and Euclid's textbook, *The Elements,* at 2300 years old, still forms the basis for introductory geometry courses today. Text and conversation have always been the means to mathematical understanding.

Presently, professional mathematicians, as well as the vast array of professionals who use mathematics in their work, rely on their literacy skills to learn, to share, and to teach. Just take one look at the seven Millennium Prize Problems (posed by the Clay Mathematics Institute in 2000 with a $1 million reward for the solution to each) for a glimpse into what professional-level literacy entails for mathematicians. Mathematics learning is difficult. Yet without literacy, it is nigh impossible.

Our students, as mathematicians, require instruction in the disciplinary literacy of mathematics to succeed. As math teachers, we must therefore allow time and space for direct instruction and rehearsal of literacy skills in service of content learning targets.

Disciplinary literacy in this book

Let us now consider broadly what literacy in mathematics includes (subsequent chapters will address each of the following strands in greater depth):

- Reading: Making sense of a variety of genres for multiple purposes—prose, problems, diagrams, graphs, equations, proofs, models, solutions, or justifications (chapter 3)
- Vocabulary: Learning and using, understanding and remembering mathematical vocabulary; being able to use root words to make sense of new, unknown terms (chapter 4)

- Discourse: Engaging in purposeful conversations about mathematical ideas, speaking and listening about concepts, solutions, errors, strategies (chapter 5)
- Writing: Composing mathematical texts in a variety of formats to explain solutions, analyze errors, and reflect on learning experiences (chapter 6)

All these aspects of literacy can naturally be encompassed within learners' mathematical experience, and you may already be asking students to engage with most on this list. Although for the purpose of this text the four areas of literacy described above have been separated, we know that they are, in fact, inextricably linked, interconnected as part of each learner's web of meaning making. Simply for the purpose of explanation, we artificially tease them apart.

The purpose of this book is to offer you strategies that invite students not only to have literacy experiences while learning math but also to learn and master the literacy skills needed by successful mathematicians.

Literacy in Any Language

In my home state of Colorado, the number of students whose first language is not English has grown by nearly 300 percent in the past decade (Klaus-Quinlan and Nathenson-Mejía 2010), and our state is certainly not unique. Many students are challenged to master mathematics content while also developing second language skills. In these cases, students can benefit from exercising their literacy in their native language and using it as a tool for mathematics learning. More critical than that they discuss their mathematical ideas in English is that they discuss them at all.

Research describes the value to learners of code-switching, changing back and forth from one language to another:

> It is important for analyses of bilingual mathematical conversations to avoid interpreting code switching as a deficiency and instead explore how code switching can be a resource for mathematical communication. Sociolinguistics research suggests that we should not expect bilingual students to switch into their first language only to provide a missing English vocabulary term. While some students may sometimes use their first language in this way, other students will use their first language to explain a concept, justify an answer, describe mathematical situations or elaborate, expand and provide additional information. . . . In general, code witching has been documented as a resource for elaborating on a point that is repeated, without repeating the initial utterances word for word. In particular, code switching can provide resources such as phrases from the mathematics register in two languages and multiple ways to participate in mathematical discourse practices. (Moschkovich 2007, p. 18)

Mathematical talk, therefore, need not be limited by English proficiency. To discuss ideas in any language is to develop both literacy and understanding.

Developing literate mathematicians is best done in the context of workshop-model instruction, where the bulk of students' learning time is devoted to reading, problem solving, discussing, and writing as mathematicians. A workshop allows time for students to build their agency through tasks that engage them in productive struggle in a community of learners with whom they collaborate in the development of rich mathematical understanding. In chapter 2, we will explore this instructional approach in greater detail.

Disciplinary literacy in the NCTM Mathematics Teaching Practices

Principles to Actions, published by the National Council of Teachers of Mathematics (NCTM) in 2014, articulates a short set of descriptors of effective instruction—the Mathematics Teaching Practices—known to enhance math learning at all grade levels. Figure 1.2 helps us examine how each of those teaching practices is linked with literacy. Chapter references are included in the right-hand column; further detail on how to implement each strategy is included later in the text.

NCTM Mathematics Teaching Practice	Opportunity to integrate disciplinary literacy instruction
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	Along with content learning goals, establish process goals for daily math workshops that include reading, writing, speaking, and listening about mathematics (chapter 2).
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allows multiple entry points and varied solution strategies.	Explicitly teach learners the **reading** strategies they need to make sense of word problems and complex mathematical texts and tasks (chapter 3). Offer instruction and support for the development of **discourse** skills required to engage in mathematical discussions (chapter 5).
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and in using tools for problem solving.	Support learners in making sense of multiple representations by teaching the **reading** skills needed to understand surface and deep structures of varied text (chapter 3).
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.	Scaffold learners' ability to engage in respectful, meaningful mathematical **discourse** and in collaborating with peers (chapter 5).

Fig. 1.2. Integrating literacy instruction into NCTM's (2014) Mathematics Teaching Practices

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.	Invite learners to respond effectively both **orally** (chapter 5) and in **writing** (chapter 6) as a means of synthesizing their thinking.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.	Support learners in articulating problem solving strategies in **written** form (chapter 6). Develop conceptual understanding with rich **vocabulary** instruction (chapter 4).
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.	Provide specific scaffolds for literacy challenges with explicit mini-lessons and opportunities to reflect during math workshop (chapters 2, 3, 4, 5, 6).
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.	Through **workshop-model** instruction (chapter 2), offer learners the means to communicate their thinking, progress, and growth, **orally** (chapter 5) and in **writing** (chapter 6).

Fig. 1.2. *Continued*

Responsibility for Literacy

If this all feels a bit overwhelming at this stage, take heart. As a content-area teacher, you are a member of a cross-curricular team of teachers, each of whom is chipping away at the challenges and working toward the literacy standards from her own angle. So rather than feel burdened by the plethora of literacy learning targets you are now explicitly expected to ladle into your lessons, try looking on that list as a buffet, not a fixed menu. The next chapter will explore how workshop-model instruction can support you and your students in digesting your literacy selections efficiently, without sacrificing content learning.

So, ask yourself: Which aspects of literacy instruction would best serve my students? What could I authentically tackle in the context of our math learning this year? Take up your answer to that question as your charge in moving forward, and feel confident that by doing this, you are doing enough. Students have many content teachers and more years in school to master the craft of disciplinary literacy, and, with your help, they will.

"Yeah, but . . . "

- ***"It's not my job to teach literacy—just math."***

Math learning and literacy learning have always been interdependent. Historically, math teachers have instructed students only on the former, yet what we know now is that when we

also add direct instruction in the latter, students learn more math. When we include disciplinary literacy as an integral part of our math instruction, we are better math teachers.

- ***"If I teach literacy, I will run out of time to cover my content."***

Literacy instruction does take time but also enhances learners' understanding. Our role as math teachers is to support students in developing that understanding, rather than hurrying them through the chapters.

- ***"My students will get mad if I make them read and write in math class."***

Unfortunately, modern schooling has perpetuated the myth that the subjects are all separate, that there is to be no singing in science and no muckraking in math. To help learners rethink this misconception, we can describe for them the value of integration between content areas: We use timelines to help us understand history, graphs to make sense of science, fractions to interpret musical notation, so why not writing to support us as mathematicians?