



CHAPTER

1

Discovering rules for odds and evens

CASE 1

Number of the day

Dolores

Grade 3, October

CASE 2

Reds are even; blacks are odd

Nadine

Grade 1, October

CASE 3

Why can't you count to 5
by 2s?

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CASE 4

Adding evens

Lucy

Grade 3, June

CASE 5

Defining even numbers

Carl

Grades 7-9, September

One day, a second-grade student asked, “Why is it that you can count to 10 by 5s, and you can count to 10 by 2s, but you can’t count to 5 by 2s?” Although he asked his question in terms of specific numbers, he might be wondering about a more general issue. A third grader in a different school was wondering about a similar question. Staring at the two rows of five cubes he had created by separating ten cubes, he said, “It’s like what I’ve been wondering since like kindergarten. How does two odd numbers equal an even number?”

The behavior of odd and even numbers often piques the curiosity of students, young and old. Though their questions may arise from considering specific numbers, they are soon drawn to make generalizations about whole classes of numbers—for example, that any two odd numbers added together make an even number.

In the cases featured in this chapter, students in grades 1 through 9 are discussing odd and even numbers. As you read the cases, consider the following questions: Are students making generalizations? If so, what are they, and what makes them general? What do students do to convince themselves or their classmates as to the validity of the generalizations?

case 1

Number of the day

Dolores
GRADE 3, OCTOBER

I decided to open an exploration of odd and even numbers by asking my new class of third graders to share what they already knew. When I asked what they had learned earlier, one boy’s hand shot up immediately. Then he struggled for a minute, searching for the wording he wanted, before saying, “I could say it to myself in my head, but I can’t explain it now.” 5

Other students contributed comments on what they had learned:

- CORY: Odd numbers are like one, that’s the first one in the odd family.
- EVA: Even numbers are 2, 4, 6, 8, and so on.
- KIM: Even numbers in the ones place are usually the even numbers, so 2, 4, 6, 8, 0. You can test 32. Just pay attention to the 2. It’s even. 10
- DOUG: Nine is on my soccer shirt. It’s odd. So is 1, 3, 5, 7, 9.
- AMY: We can start with 1 and go maybe to 100; it goes odd, even, odd, even, odd. I think 0 is even, too.
- JACK: If 0 is at the end of the number, it is even. So in 100, you look at the last 0 for even. 100 is 50 and 50. 30 is 15 and 15. 50 is 25 and 25. 40 is 20 and 20. 15
- MARGE: If 0 is in a number, it doesn’t make it even, does it? Like 101, the end matters right?

As the discussion continued, it seemed most children, like Jack, were using the definition that an even number can be divided into two equal parts using only whole numbers. I discussed that definition with students before we went on. Then the class got busy trying to divide numbers that end with zero into two equal parts. This lasted for about 10 minutes and, after offering help to each other, the students determined that all of the numbers were “splittable.” 20

Next, I asked students to try to find out what kind of a number we would get as an answer if we added an even number to an even number. They set off to try all sorts of possibilities. As we came back together at the end of the period, all twenty children reported that their sums were even, after a few addition corrections. 25

The next day, I reminded the class of the work we had completed earlier, emphasizing the definitions of even and odd numbers. Then I gave each child a sheet of paper with the question, How can we be sure an even number added to an even number will give an answer that is even? Students supplied a variety of responses: 30

- RICKY:** We will never know because there's too many numbers.
- D. C.:** We could never tell because the numbers don't stop.
- LAURIE:** We can never know for sure because the numbers do not stop.
- EVA:** We really can't! Because we might not know about an even number and if we add it with 2 it might equal an odd number! 35
- ZOE:** I know it will add to an even number because $4 + 4 = 8$ and $8 + 8 = 16$.
- CLAUDIA:** We don't know because numbers don't end— $1,000,000 + 100$; you can always add another hundred.
- MIM:** Your answer will be even because you are using even numbers.

Building on this background of thinking about odd and even numbers, I decided to try something new based on the number of the day. As we make our way through 182 school days, I begin each day with “Happy Day #...” A usual activity is to think of interesting arithmetic expressions that equal the number of the day. We also figure out ways to determine the number of days ahead of us in the year. 40

I decided to ask the children to create problems that would have answers of 24, but this time they had to work with some restrictions. They could only use the operation of addition and only even numbers to get the sum. 45

My third graders threw themselves into this task. In just a few minutes, everyone had a supply of math sentences to share. For the most part, they were all done correctly. There were just a few arithmetic errors. Sometimes the difficulty was in keeping track of how many 2s were in the problem. 50

The next day, we worked on making problems with sums of 25 using the same restrictions. Again, the children busied themselves with calculations. After a few minutes of working alone, we talked about what they had found. A few kids raised their hands to offer ideas. In each case, an odd number had “snuck” into the equation. No one actually said they could not do it. The comments revealed more of a mood that they had not found a way to do it yet. Thinking back on this, I wish I had asked if 24 had been easier to do, and if so, why. I wish I had pushed a little on their thinking. 55

I knew we would come back to this on another “odd” day. I am often wondering about how and when children notice patterns. I knew it would take a few times at this same kind of work for third graders to expect particular results. This first time I think they were pretty focused on actually getting even numbers and adding correctly. 60

On day 28, we tried using only odd numbers to add up to 28. There were lots of $1 + 1 + 1 + 1 + \dots$ For the most part the class had little difficulty with this. They just poured themselves into the job. The next day they breezed through finding odd numbers that totaled 29. No one noticed an odd number of odd numbers was needed to get an odd sum. They were happily and busily adding. (There will still be time to notice those patterns.) 65

As days passed I continued to work on making math time more than just doing calculations. It is always a lot of work at the beginning of a year to get a new batch of children to appreciate the value of sharing ideas with each other. There is such a tendency to just take turns talking without recognizing that they also need to consider the ideas and strategies of others.

I was also wondering if students were noticing trends, rules, and patterns with numbers by doing lots of examples. Could they help each other make generalizations?

The next time we did the “number of the day” work was on day 33.

I gave the kids a few minutes to try to make the number 33 with the restrictions of using only addition and only even numbers. The results were interesting.

Mim immediately wrote in giant-sized letters on the back of her spelling pretest, “YOU CAN’T DO IT!! evens $10 + 10 + 10 + 2 + 2$ can’t do it!! Sorry, you can’t do it.”

Kim wrote, “ $16 + 16 = 32$ There is no posiple way to make 33 with all even numbers. You can only make evens with all evens.”

Claudia said, “Even numbers plus even numbers always adds up to even, and 33 is odd.”

Carry insisted, “You can’t do it. You would at least use one odd. You can’t make 33 with only even numbers.”

Jack tested it out by adding a string of 2s. He wrote, “ $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 - 1 = 33$ I tried and I can’t come up with even numbers to equal 33.” 85

Noah had lots of calculations on his page, which were crossed out. Below all of that he wrote, “You can’t do it.”

Beth used a different strategy. She put together a series of even numbers and ended up with sums of 32 or 34:

$$30 + 2 + 2 = 34 \qquad 90$$

$$20 + 10 + 2 + 2 = 34$$

$$10 + 12 + 10 = 32$$

$$10 + 10 + 10 + 4 = 34$$

She didn't make any generalizations.

Anna did some exploring and made an observation: 95

$1 + 4 + 4 + 2 + 10 + 10 + 2$ 1 is odd. I think it is not possible.

$$10 + 10 + 10 + 2 + 1 = 33 \qquad 1 \text{ IS ODD.}$$

$10 + 10 + 10 + 2 = 32$ You need one more but one is odd.

Eva tried something similar:

$$20 + 12 = 32 \text{ NO} \qquad 30 + 2 + 1 \text{ ODD} \qquad 100$$

$$30 + 2 + 1 \text{ ODD}$$

I think this problem cannot be done with the even numbers we know.

Others took time to try but made no “discoveries” or generalizations.

As we came together to talk about it, everyone was relieved to find it could not work. I think it is still a “number-specific” idea for about half of the children. I expect they will jump into trying many combinations of even numbers to make numbers 39 and 41, and maybe even 55. It may take time for them to get it.

case 2

Reds are even; blacks are odd

Nadine

GRADES 1, OCTOBER

One of our regular morning activities involves the number of days we have been in school. Instead of using a number line, we keep track of numbers in a 10 x 10 grid. (I will add a second grid on the 100th day of school.) Our chart starts with 0 and a new number is added every day, so on the first day 0 and 1 were filled in. The last number will be 180 for the last day of school. We alternate using red and black pens, so the even numbers are written in red and odds are written in black.

The discussion I am writing about was initiated by my students on day 19, so the chart looked like this:

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19

The children wanted to tell one another what they noticed about the chart. I wrote down their observations.

- This is a pattern: red, black, red, black.
- There's a column of red—0, 10—and then a column of black—1, 11—and then a column of red—2, 12—and then black—3, 13—and it keeps going. (Anita had said “rows,” but I pointed out that because they were going up and down, we call them “columns.”)
- There is a 0 in the first box, and then there's a 1 with a 0 for the ten in the box under it.
- The same thing happens with the other numbers. The number on the top row shows up right under it with a one in front of it.
- The red numbers are even; the black ones are odd.

We had never talked about odd and even numbers before, so I asked, “What is even? What is odd?”

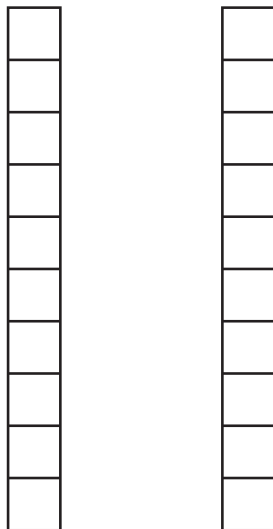
Some students called out numbers: “4 is even.” “3 is odd.” I decided that today, I would just listen, and based on what I heard, would work to design a lesson later.

Terry noticed that half the numbers were red—0, 2, 4, 6, 8, 10, 12, 14, 16, 18 (10 numbers)—and half were black—1, 3, 5, 7, 9, 11, 13, 15, 17, 19 (10 numbers).

Jack said that the next number, 20, would be even. He knew that because of two things: 1) it would be red, and 2) $10 + 10 = 20$. He continued his explanation: “If you take two numbers that

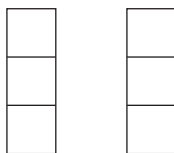
are even and put them together, you have an even number.” I was extremely impressed. He then added, “You have to have two even numbers to make another even number.”

The students now had cubes out so Jack could show everyone else what he meant. He showed us two stacks of 10; then he put them together to make 20. 140



Shay, listening very carefully to Jack, added, “What about $3 + 3$?” Shay held up two stacks of 3 cubes. “The 3s are odd, but when you put them together, it makes 6, and 6 is even.”

145



Jack looked back at his recorded statement that you have to have two even numbers to make another even number and said, “I can fix that. If you take two numbers that are the same and put them together, it makes an even number.”

At this point, the discussion was very interesting for several of my students, but I was definitely losing the attention of the rest of the class. So we stopped there. I have plenty to think about. What made my day was Jack’s comment just before lunch: “Math discussions are really fun.” 150

case 3

Why can't you count to 5 by 2s?

Ingrid

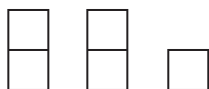
GRADE 2, FEBRUARY

One day, at the end of a recent math activity, Sam said he had a question. “Why is it that you can count to 10 by 5s, and you can count to 10 by 2s, but you can’t count to 5 by 2s?” I told Sam that his question was too important to try to think about in the time we had left, but promised we would return to it the next day. I could not have come up with a better question to get at what my students might be thinking about odd and even numbers and how they relate to each other.

The following day, I asked my students to be prepared to use cubes to explore Sam’s question and then demonstrate their thinking to the class.

CALVIN: Me and Craig were thinking of taking 5 cubes. . . . If you have 5 cubes and you break them up into 2s, you have 1 missing. So, it’s 2, 4, and not 6, it’s 5 because there’s two 2s that makes 4 and there’s 1 more after the 2s to make 5. 155

Calvin points to this model.



TEACHER: Could somebody say in his or her own words what Calvin and Craig were noticing? 160

JEREMY: I don’t get it, because if you put another 1 onto the 5, it will equal 6, not 5.

CALVIN: If I put 1 to that 1, it would equal 6, and I’m not doing that. I’m counting to 5 on 2s and I was doing 4 here with two even numbers. (*Calvin again points to his model.*) 2 and 2 are even numbers. 1 is an odd number and if we have three 165 even numbers, that would make 6 to make another even number, but we’re trying to make an odd number and it’s 5 and it’s very difficult to understand, because... I just don’t know what to say.

It appears that Calvin is interjecting some thoughts about odd and even numbers that he has accumulated on his way to second grade. I am concerned that the conversation will head off 170 into recalling notions about “odds” and “evens” and stray away from Sam’s question, so I decide to set aside the general ideas of odds and evens for now. I make an attempt at refocusing the conversation to stick specifically to the numbers Sam asked about.

TEACHER: I’m hearing words like odd and even. We don’t have to use those words. What I think I hear Calvin saying is, “If you put another cube down, you would 175

have 6, but we can only have 5 cubes. So you can't get there by 2s. You can only get to 4, and the next number would be 6. When you have 5 cubes, you have two groups of two and one cube left over."

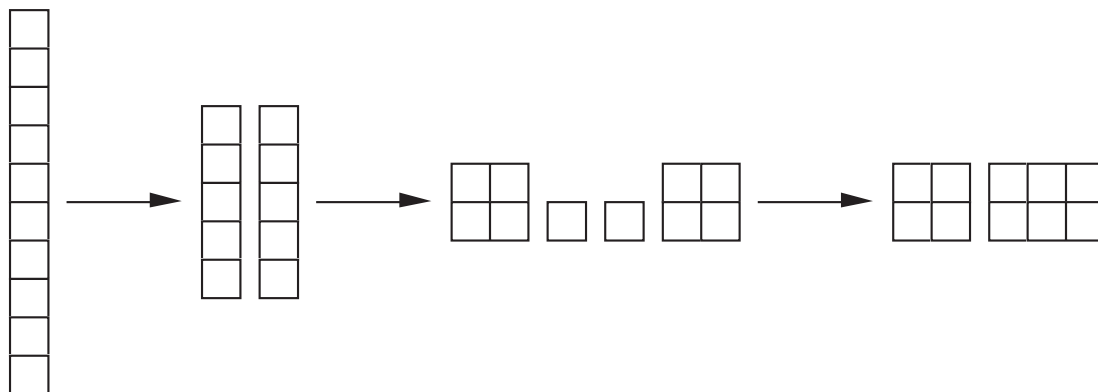
CALVIN: Yes, to make 5.

TEACHER: The next part of Sam's question is, "You can't count to 5 by 2s, but you can count to 10 by 2s and 5s. What's happening there?" 180

CALVIN: Craig got this idea, so I think that he should explain it.

Craig has been a silent partner up to this point. He does not like to be the spokesperson, but he uses the following model to help his classmates understand:

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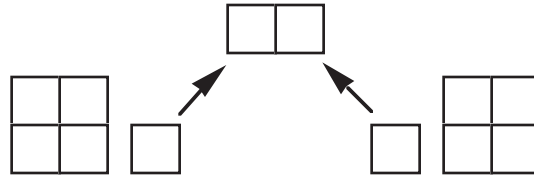
Sam says that Craig has not answered his question. I'm not sure that most of the class understands what Craig has done, so I ask for a paraphrase. Janelle is happy to oblige.

JANELLE: He took 10 cubes, and he took 5 off of them, and then he put 2 in a block of cubes and another 2 in a block of cubes, and then 1 was left. And then he did the same with the other 5. And then the two 1s that were left he put together, and that made another 2 and that made it equal 10. 190

Calvin feels the need to point out a difference between Janelle's explanation and Craig's model. In Craig's model, the leftover from one group of 5 is added to the other group to make a group of 6.

CALVIN: Craig is saying that, if you have the 4 and the 1, the 2 and the 2 and the 1; he made a copy of it over here. Then he saw these two loose ones that were not together, and he took this one and put it with the other group, and then he made 6 and 4. And that is how you make 10 by 2s, but it's actually by 5s a little, too. 195

I wonder if Craig's model might obscure the idea that each group of five has one left over, which comes through in Janelle's explanation. So I decide to model what she has said. 200



TEACHER: So Craig had two groups of 5 and each of those groups of 5 had one left over. He moved the extra from one group and put it with the other group. What if I took the extra 1 from one group of 5 and put it with the extra 1 from the other group of 5? Does that show it in a different way? 205

CALVIN: It would be the same thing as Craig did, but these are separated from the two 4s. If you add those 2 to one of these 4s, then it would be 6. It would be $6 + 4$ and that would be easier to think about.

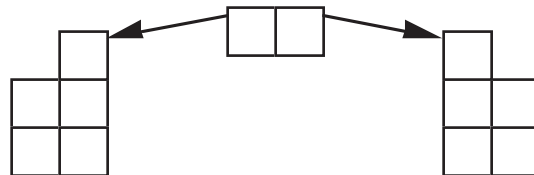
TEACHER: Which would be easier to think about? 210

CALVIN: Now that you took the two up there, now no one will get confused by the way Craig does it.

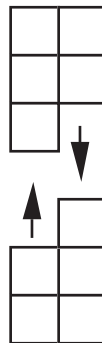
TEACHER: So that feels clearer to you, to take the extra 1s and put them together.

SAM: I have another idea. What Craig and Calvin did, that gave me an idea.

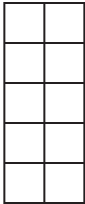
Sam splits the two cubes apart and returns one cube to each of the sets of 4 cubes. 215



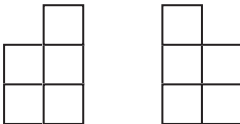
Then he picks up the two sets of 5 and joins them together to make columns of 5 cubes.



SAM: That equals 10. If you take them apart, they equal 5, and that makes 5 an odd number. This equals $10 - 2, 4, 6, 8, 10$. 220



SAM: And this equals 5.



TEACHER: So, it looks like you're showing the extra 1s on each 5 coming together to make a pair.

case 4**Adding evens****Lucy****GRADE 3, JUNE**

I set out to investigate odd and even numbers with my class. As third graders, they had worked on odd and even numbers before, but to make sure we were all on the same page, we started our session with a discussion about how to define these terms. We agreed that even numbers are those that can be made into pairs with none left over, or that they can be divided into two equal groups with none left over. (We did clarify that those groups needed to be whole numbers—not fractions.) Some of the children pointed out that even numbers are numbers you get to when you count by 2, starting with 2. We also agreed that if you have a number that fits one of those conditions—it can be made into pairs, it can be made into two equal groups, or it can be reached by counting by 2—it also fits the other conditions.

The class defined odd numbers as those that have one left over when you make pairs or when you try to make two equal groups.

Next, I gave the class a worksheet I had created. I intended to have students practice applying these definitions. I didn't want them just to say whether a number is odd or even but to show how they used the definition to make this determination. Then, I wanted them to see if they could predict whether sums of given pairs of numbers are even or odd. The questions on the worksheet were:

1. Are these numbers odd or even? How do you know? What rules or pictures can you write to show your thinking about whether a number is odd or even?

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2. Are the answers to these problems odd or even? Do you know without solving them? Why is that? Show your thinking with pictures or explain with words.

 $4 + 8 =$ $5 + 3 =$ $4 + 5 =$ $9 + 2 =$

255

3. Does your thinking work for these problems, too? Why? Do you know if the answer is odd or even without solving the problem? How do you know?

$35 + 49 =$

$66 + 105 =$

$96 + 244 =$

260

4. Do your rules work for any size numbers? Why is that?
5. How could you tell whether an answer was even or odd if you had more than two numbers?

When it was time to bring the class together for a whole-group discussion about what they had figured out from this activity, many students suggested generalizations—the sum of two evens is even; the sum of two odds is even—but none offered a proof. That is, nobody offered a proof until Amanda spoke up: 265

AMANDA: Two evens, no matter what they are, have to equal an even.

TEACHER: Why?

AMANDA: Um. I just figured out something.... If you counted something by 2s, and 2s always work on an even number, they can't work on an odd number, and every even number you count by 2s with it and if you added the 2s of both even numbers on top of each other, they both count by 2s, so they would have to equal an even. 270

At this point, I wasn't at all sure what Amanda was saying, but I wanted to give her classmates a chance to think together with her. 275

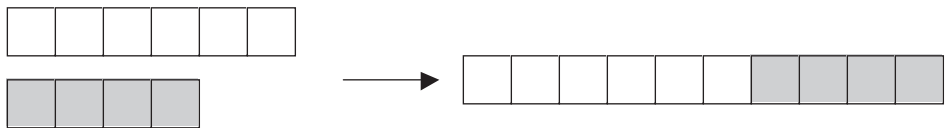
TEACHER: Does somebody know what Amanda is talking about? Ellen?

ELLEN: She's kind of talking about—No, I'm confused.

TEACHER: Does somebody want to hear it again? Ellen wants to hear it again, Amanda.

AMANDA: Well, if you have two even numbers, 2s work on both of them, so if you put them on top of each other. Umm. Can I have some cubes? 280

As Amanda built two sticks of cubes representing even numbers, she explained her idea again, showing the two sticks of cubes that could be counted by 2s, and then joining them to make one stick. 285



- AMANDA:** I have this, both of them (the two sticks) count by 2s, so if I put them on top of each other, you keep counting by 2s, and then you get to an even number.
- Once I saw Amanda's demonstration with cubes, I understood what she was talking about.
- TEACHER:** Ohhhh. That's not what I thought you meant when you said you put them on top of each other. Oh. What do people think about what Amanda just said? 290
Elizabeth?
- ELIZABETH:** I think she said that if you have two even numbers, and they're counting by 2s, then you put them on top of each other.
- TEACHER:** You stick them together; I think that's what she meant.
- ELIZABETH:** Yeah. But then I don't know what she said after that. 295
- TEACHER:** (*Checking with Amanda to make sure she's correctly paraphrasing*) So, you've got an even number over here, and an even number over here, and you stick them together. Doing that has to give you an even number.
- AMANDA:** Because you can count by 2s up to 6 and if I add the 4 on, you can just keep counting up by 2s, and that would have to equal an even number because 2s 300
only get you to even numbers.
- At this point, Elizabeth was working hard to follow Amanda's proof. At first, she seemed to agree that the generalization felt right. Then she reconsidered.
- ELIZABETH:** I think she's just like adding the even numbers that are counting by 2s, so if she had two even numbers on both sides and then put them together, then she 305
would get an even number, so it's kind of like you're adding the even numbers.
- TEACHER:** But why would it be an even number, if she had an even number and another even and she put them together. Why do you...does it feel like she's right, that it has to be an even?
- ELIZABETH:** Well, yes... Well, I don't know, because in some cases, well, um, I can't really 310
think of it now but like, if you had one that was an even plus an even, if like, I haven't figured this out, but sometime maybe it could equal an odd.
- TEACHER:** And Amanda's saying it couldn't ever equal odd. Is that what you're saying, Amanda? That an even plus an even could never equal odd? And Elizabeth 315
is wondering if it (the sum) sometimes could be an odd, but you're saying it could never be an odd? Do you want to say more about why that is?
- AMANDA:** Because 2s don't get to odds. And if they're two even numbers, they're both counting by 2s, and if you put them on top of each other you keep counting by 2s and that always equals an even number.

ELIZABETH: So she's saying she already knows that it always equals it. 320

TEACHER: Amanda thinks she knows that it's always going to be even.

That is as far as we got today, but it leaves me with much to think about. Amanda has come up with what seems to be a very convincing proof, based on counting by 2s, that the sum of two even numbers must always be even. Elizabeth seemed quite close to understanding Amanda's argument, but she just doesn't seem able to make a claim about all even numbers. "I can't really think of it now, but...sometime maybe it could equal an odd," she said. Certainly, these two girls are in two different places with regard to thinking about and proving generalizations. 325

But should I think of Elizabeth's position as weak for a third grader? No. Actually grasping the notion that numbers go on forever is new for children at this age. It is important that Elizabeth realizes it is not sufficient to argue that a generalization is always true simply by looking at particular examples. After all, the examples chosen might be special cases, and something might change with other numbers. Elizabeth's insight is key toward understanding the need for mathematical proof based on reasoning, rather than by example. 330

Elizabeth and Amanda's classmates were quiet during their exchange. I don't know where they are with these ideas, and this is something for me to look into in coming days. 335

case 5

Defining even numbers

Carl

GRADE 7-9, SEPTEMBER

As I begin the school year, I see a focus on odd and even numbers as an opportunity to convey something about the heart of mathematical endeavor: that it is about ideas and reasoning; that it is about looking for patterns, making claims, and trying to show why those claims are true. I also want to illustrate the power of using different representations when considering mathematical ideas and situations. 340

Representations organize information and enhance our perception of features and relationships that might not be obvious upon first consideration. It is often through contrasts between different representations that these features and relationships become apparent. These new perceptions are powerful tools for reasoning about the situation. I want students to value and develop skill using representations so that they see the use of diagrams, physical models, graphs, stories, and verbal arguments is as essential to doing mathematics as the ability to compute or solve equations. Finally, I hope to encourage the development of a learning community in which students view themselves and classmates as partners in making sense of mathematics and their world. I want them to learn to listen to their own ideas and the ideas of their classmates. I want them to feel comfortable sharing their ideas with others and using the ideas of others to help them make mathematical progress. 345 350

It was with these goals in mind that I started the session about odd and even numbers. I put the following prompt on the board: “What is an even number? How do you know?” I instructed students to write their ideas in their journal. After a couple of minutes, I had students share what they had written with their “Think Team” (groups of three). After a couple more minutes, we had the following class discussion: 355

TEACHER: So, let’s hear some of your ideas.

ALEXANDER: It is any number that ends in 0, 2, 4, 6, or 8.

I expect that most of my students came in with this characterization of even numbers, but I wanted to go beyond that, to think about the importance of 2 in the structure of even numbers. This would be necessary for the goals that I had for the next session in which we would be making arguments about sums of odd and even numbers. So, I pushed the class on Alexander’s idea. 360

TEACHER: How do you know that? How do you know that is true for all numbers, even really big ones? 365

DAVID: It is just the way even numbers are.

TEACHER: So what about 90? It ends in 0 but 9 is odd, so I think 90 is odd.

The class buzzed with discussion, and after a few seconds I posed my question again.

TEACHER: How are you going to convince me that 90 is even?

KRISTA: The 9 is not a “9,” but it means nine 10s. 370

TEACHER: So how is that supposed to convince me?

KRISTA: We know 10 is even. Every one of the nine 10s is even, so 90 has to be even.

TEACHER: This takes us back to the original question: “How do you know 10 is even?”

KIMBERLY: An even number is a number that you can divide by 2.

CHRIS: But I can divide 5 by 2 and 5 is not an even number. 375

KRISTA: I meant that it divides without leaving a remainder.

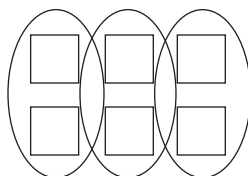
VINOD: You get a whole number when you divide by 2.

Now we were getting to a definition we could work with. I wrote on the board, “An even number is a number that can be divided by 2 with the result being a whole number.”

WILLIAM: I have a different way. I was drawing cubes. I drew 6 using cubes and I saw that an even number is made up of pairs. 380

TEACHER: Let’s have you come up here and draw it. Explain what you are seeing. I am going to ask someone else to restate what William is saying.

William drew the following:



385

WILLIAM: I tried this a couple of times, and it always works.

TEACHER: So what do the rest of you think of this?

CAROLINA: It’s like what I was thinking. An even number is a multiple of 2.

TEACHER: What do you mean by a multiple of 2?

CAROLINA: It is something that is made up of 2s. 390

BRIANNA: Two times 4. It’s multiplication. Eight is 2 times 4.

TEACHER: What does that have to do with William’s idea of pairs?

BRIANNA: There are pairs of 2 in 8...4 of them.

TEACHER: Interesting. So, it seems like we have two different ideas going on here.

The board now looked like this:

395

1. An even number is a number that can be divided by 2 with the result being a whole number.
2. An even number is a number that can be made up of pairs or multiples of 2.

TEACHER: How is it that we can have two different definitions for the same thing? Or do they define the same thing? I want you to talk in your Think Teams about these two ideas and decide if they are both true and why? 400

I watched students in their teams putting cubes together and breaking them in half or into pairs. After a few minutes, I called them back together.

TEACHER: Alex, I heard your group talking about something interesting. What were the three of you focusing on? 405

ALEX: We think both are true because they are kind of saying the same thing.

TEACHER: Explain what you mean.

ALEX: Any number that is made up of 2s has to be able to be divided by 2.

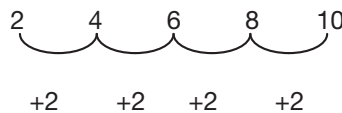
ALANA: It's like division. Multiplication is division backwards. Two times 4 is 8. Eight divided by 2 is 4. 410

KEVIN: I think I know why what we have been talking about is true for all even numbers. It's kind of like a chain of 2s.

TEACHER: (*At first, I was not sure what he meant.*) Kevin, before you explain your chain idea, help us get clear about what you are referring to. What are you thinking is true for even numbers? 415

KEVIN: Well, all of it. . . . that all evens are made up of 2s and that all evens end in 0, 2, 4, 6, or 8. It's like a chain. Between zero and 2 is 1. Between 2 and 4 is 3. To get from 2 to 4 you have to add 2. There is no way to get from the number 2 to an odd number going by 2s. So if you keep adding 2s, the number has to be made up of 2s. 420

I wrote on the board:



TEACHER: So, does this show what you are thinking?

KEVIN: Yes.

I asked the class to discuss Kevin's idea in their Think Teams, and then brought them back together again. 425

TEACHER: Zach, a few minutes ago you said something to your team. Before we see what everyone thinks about Kevin's "chain" argument, I would like you to say it to the whole group.

ZACH: You can't know something is true for all even numbers because there are too many. 430

BRIANNA: We talked about that, too. You can't ever convince someone because they can always think of other numbers we haven't tried.

TEACHER: Exactly. I wanted Zach to state this because it highlights something very important about mathematics. By looking at many examples, we can never be totally confident that something is true because there are more numbers than we can try. But with mathematics we can think about the structure of the situation; we can think about what is happening with the numbers and come up with an argument based on that structure to explain why. This is why mathematics is so powerful and why I said that this class was going to be about developing your mathematical power to explain things that would be difficult to figure out otherwise. So, Kevin is noticing something about even numbers that he thinks explains why they always are made up of 2s and why they always end in 0, 2, 4, 6, or 8. Does anyone think they know what he is saying? 435
440

ELLA: Because numbers go odd-even-odd-even, you can never get to the number in between using 2s. You can get to a 1, 3, 5 by 2 or 2, 4, 6, but you can't get between these. 445

TEACHER: So, because evens have this "every other" quality, the idea of 2s is built into them.

TAYLOR: Mr. Fromm, but what about four point four? Isn't that even? 450

This statement surprised me. I needed to know what she was thinking. I didn't just want to say no because she may have a very powerful rationale for why she thinks that 4.4 is even. I do not want to undermine her sense of being able to make sense of the math. I am also aware that one other group was talking about 5.3 as being odd.

TEACHER: What leads you to believe that 4.4 is even? 455

TAYLOR: Four is even and 4.4 can be divided by 2.

TEACHER: I like the way you are thinking about this and trying to make sense of the ideas. This is exactly how a mathematician might think: Are there any cases that seem to contradict what we are thinking? First of all, 4 is, of course, even but what does "point 4" mean? 460

TAYLOR: It means 4 tenths.

TEACHER: Yes. So this is like what you were saying about the number 90 earlier. The 9 is odd, but the 9 in 90 means 9 tens.

TAYLOR: Oh, yeah. The point 4 is not really a 4 but 4 tenths.

Taylor's question now became an opportunity for the class. I asked students to create an argument to explain why they think 4.4 is or is not even. After a few minutes, I brought them back together. 465

ALEX: We say it is not because when you divide by 2 you get 2.2. This is not a whole number.

MARGOT: We don't think it is even because you can't make 4.4 with 2s. Two plus 2 is 4, and 2 plus 2 plus 2 is 6. You can't get to 4.4. 470

TEACHER: Wow, you have been doing some powerful thinking. You can see that our Think Teams are working. We are helping each other to make sense of stuff, and we are using each other's ideas to come up with new insights into evens. Taylor's question forced us to really think harder and to apply what we have been talking about. Way to go. I can't wait for tomorrow. Before we end this session, I have one more thing for you to think about. 475

I wrote on the board:

Based on the work we have been doing with even numbers, write a definition for odd numbers. 480

I feel very pleased about this discussion because I think that, as a class, we were able to make some progress toward my goals. At the same time, I am wondering where each student is in his or her thinking about these ideas. I am going to need to look closely at each student's homework. I am also concerned about Danny. I noticed that Danny had written on his paper that an odd number is a strange or weird number. Is he thinking about the everyday sense of the term odd? Does he notice the "one left over" feature of odd numbers? I will need to pay attention to his ideas about adding odds tomorrow. 485

