Part 3

Expressions and Equations

Expressions and Equations

Domain Overview

GRADE 6

At this level, the study of expressions and equations centers on the use of variables in mathematical expressions. Students write and evaluate numerical expressions and use expressions and formulas to solve problems. Students also solve simple one-step equations and use equations such as $3x = y$ to describe relationships between quantities. The sixth-grade study of expressions and equations is foundational in the transition to algebraic representation and problem solving, which is extended and formalized in Grade 7.

GRADE 7

Seventh graders use properties of operations to generate equivalent expressions. They use the arithmetic of rational numbers to formulate expressions and equations in one variable and use these equations to solve problems.

The seventh-grade focus of solving real-world and mathematical problems using numerical and algebraic expressions and equations provides the foundation for equation work in Grade 8 and assists in building the foundation work for writing equivalent nonlinear expressions in later grades.

GRADE 8

Eighth graders focus on more complex equations by learning about and applying the properties of integer exponents, square and cube roots, and scientific notation. They also connect previous understandings about proportional relationships to linear equations. Systems of two linear equations in two variables are introduced, and three methods for finding solutions are learned.

SUGGESTED MATERIALS FOR THIS DOMAIN

6	7	8	
		✓	Books: *My Full Moon is Square* by Elinor Pinczes and *Sea Squares* by Joy Hulme
✓		✓	Cubes such as linking cubes, Unifix cubes™, wooden cubes
✓			Number line
✓	✓	✓	Square tiles (paper or commercially produced)

KEY VOCABULARY

6	7	8	
✓	✓		**base** in 4^2, 4 is the base; the number multiplied by itself
	✓	✓	**bivariate data** data in two variables, one to be graphed on the *x*-axis and the other on the *y*-axis
✓	✓		**coefficient** a number or variable used to multiply a variable; in $3x + 7$, 3 is the numerical coefficient; in $y = mx + b$, *x* is the variable and *m* is the variable coefficient
✓	✓		**constant** a fixed value; in $3c + 5 = 11$, 5 and 11 are constants
✓	✓		**dependent variable** the output variable in a function; the variable whose value depends on the input
✓	✓	✓	**distributive property** property that states that multiplying a sum by a number is the same as multiplying each addend by the number and then adding the products. The distributive property states that if *a, b,* and *c* are real numbers, then $a \times (b + c) = (a \times b) + (a \times c)$.
✓	✓	✓	**equation** statement using an equal sign (=) showing that two expressions have the same value
✓		✓	**equivalent** the same as; equal to
✓		✓	**evaluate** solve
✓		✓	**exponents** in 4^2, 2 is the exponent; the number that dictates how many times the base multiplies by itself; $4^2 = 4 \times 4$
✓			**exponential notation** written in the form of B^x
✓	✓	✓	**expression** a value expressed as numbers and/or variables, and operation symbols (such as +, −, ×) grouped together; $9y + 7$ is an expression; one side of an equation
✓	✓		**factor** as a verb, to break down into the terms that multiply to make the quantity to be factored

(Continued)

(Continued)

KEY VOCABULARY

6	7	8	
✓			**independent variable** the input value in a function; the variable whose value determines the value of the dependent variable
✓	✓		**inequality** statement that two values are not equal; the inequality symbol is ≠; inequalities can also use the symbols > and <
		✓	**irrational number** any real number that cannot be expressed as a ratio $\frac{a}{b}$, where *a* and *b* are integers, with *b* non-zero, and is therefore not a rational number
		✓	**linear equation** an equation whose graph is a straight line
✓		✓	**numerical expressions** expressions using all numbers such as 34 × 82
✓		✓	**rational numbers** any number that can be expressed as the quotient $\frac{a}{b}$ of two integers, with the denominator *b* not equal to zero
		✓	**scientific notation** a number written in the form of a number between 1 and 10 (including 1) times a power of 10; 4.2×10^6 is 4,200,000 written in scientific notation
✓			**simplify** to change an expression or equation into its lowest terms; combining like terms is one method to simplify an equation or expression
		✓	**simultaneous equations** a set of equations whose solution(s) are all the points that make all the equations in the set true; when graphed, the solutions are shown as the point(s) of intersection; also known as a system of equations
✓			**substitution** use of a numerical value to replace a variable
✓	✓	✓	**variable** a symbol that stands for an unknown number or any number in a specified set

Notes

Expressions and Equations
6.EE.A*

Cluster A

Apply and extend previous understandings of arithmetic to algebraic expressions.

STANDARD 1 **6.EE.A.1:** Write and evaluate numerical expressions involving whole-number exponents.

STANDARD 2 **6.EE.A.2:** Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract* y *from 5" as* $5 - y$.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = \frac{1}{2}$.

STANDARD 3 **6.EE.A.3:** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$; *apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$; *apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$.

STANDARD 4 **6.EE.A.4:** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* $y + y + y$ *and* $3y$ *are equivalent because they name the same number regardless of which number* y *stands for.*

*Major cluster

Expressions and Equations 6.EE.A

Cluster A: Apply and extend previous understandings of arithmetic to algebraic expressions.
Grade 6 Overview

The focus for this cluster is writing and evaluating numerical expressions involving whole number exponents, finding the value of an expression using exponential notation such as $3^3 = 27$, and using the appropriate terminology to explain how to evaluate an expression. Students are applying the properties of operations to generate equivalent expressions including the distributive property to produce equivalent representation.

Standards for Mathematical Practice
SFMP 2. Reason abstractly and quantitatively.

Sixth graders decontextualize to manipulate symbolic representations by applying properties of operations.

SFMP 4. Model with mathematics.

Students model real-world scenarios with equations and expressions.

SFMP 6. Attend to precision.

Students communicate precisely with others and use clear mathematical language when describing expressions.

Related Content Standards

5.OA.A.1 5.OA.A.2 7.EE.A.1 7.EE.B.3 7.EE.B.4

Notes

STANDARD 1 (6.EE.A.1)

Write and evaluate numerical expressions involving whole-number exponents.

This standard concentrates on whole-number exponents with a focus on understanding the meaning of exponents and exponential notation such as $3^2 = 3 \times 3$. Students find the value of an expression using exponential notation such as $4^3 = 64$. Students write and evaluate numerical expressions such as: $5 + 2^4 \cdot 6$.

What the TEACHER does:

- Plan experiences for students to investigate that an exponent is notation representing repeated multiplication such as $10 \cdot 10 \cdot 10$ as 10^3. The 10 in 10^3 is called the base and the 3 in 10^3 is known as the exponent. The exponent is the number that tells how many factors of 10 there are. The expression 10^3 is called the exponential expression. 10^3 is read as "10 to the third power" or "10 cubed." It means $10 \cdot 10 \cdot 10$, or 1,000. Exponential notation was developed to write repeated multiplication more efficiently. It represents multiplication where the base *always* remains the same, such as $6^2 = 6 \times 6$ or 36. 6^2 is read as "6 to the second power" or "6 squared" and means $6 \cdot 6$, or 36. 5^4 is read as "5 to the fourth power." It means $5 \cdot 5 \cdot 5 \cdot 5$, or 625.
- Present a few sets of exponential expressions in the form $2^1 = 2$, $2^2 = 4$, $2^3 = 8 \ldots 2^8 = 256$. Repeat with bases 2, 3, 4, and 5. Facilitate a class discussion about what patterns students notice in the sets. Once students begin to notice that if you divide the value of the exponential expression by the base ($2^3 = 8$, $8 \div 2 = 4$), you get the value of the previous exponential expression in the list, pose a question that leads students to discover why any number to the 0 power is 1. A sample question is, "*Using the patterns you just discovered, what should* 2^0 be *equal to?*"
- Allow students to use manipulatives to show $3^2 = 9$ square units. Ask students to model the meaning of $3^3 = 27$ cubic units with manipulatives.
- Have students write exponential notation for examples such as $4 \times 4 \times 4 \times 4$. Have students evaluate a variety of examples such as 5^3, $5 + 2^4 \cdot 6$, and $7 \cdot 4^3$.
- Focus on the following vocabulary terms: *exponents*, *base*, *numerical expressions*, *evaluate*, *sum*, *term*, *product*, *factor*, *quantity*, and *quotient*.
- Provide cyclical, distributed practice over time to continually review writing and evaluating numerical expressions.

What the STUDENTS do:

- Understand the meaning of exponents and exponential notation such as $3^2 = 3 \times 3$.
- Use appropriate terminology to explain how to evaluate an expression.
- Evaluate numerical expressions containing exponents.
- Discover that any base to the zero power is 1.

Addressing Student Misconceptions and Common Errors

Some students interpret 3^2 as $3 \times 2 = 6$. This is a common error. Use a number line representation to model the expression. Also, writing the expanded notation of $3^2 = 3 \times 3$ helps students.

STANDARD 2 (6.EE.A.2)

Write, read, and evaluate expressions in which letters stand for numbers.

a. *Write expressions that record operations with numbers and with letters standing for numbers.* For example, express the calculation "Subtract y from 5" as $5 - y$.

b. *Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.* For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.

c. *Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).* For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

Parts a–c of Standard 2 emphasize translating expressions from verbal expressions to numerical ones and from numerical expressions to verbal expressions. Students evaluate expressions given values for the variables such as in the example in part c of this Standard using the order of operations when appropriate. Students identify parts of an algebraic expression including sum, term, product, factor, quotient, coefficients, and constants.

What the TEACHER does:

- Explore with students that letters called variables represent unknown numbers and the same rules that apply in operations with numbers also apply in operations with variables.
- Help students translate verbal expressions into numerical expressions by providing a verbal expression such as "the sum of 6 and 4" and asking them to explore ways to write it with numbers. Encourage students to explore synonyms for operations such as: *and*, *plus*, and *sum*, which can all signify addition.
- Model the notation $6n$ for $6 \cdot n$ because a number and variable written together means to multiply.
- Provide varied practice translating numerical expressions into word form and from a word form into variable expressions such as "8 less than 2 times a number is $2x - 8$," "4 times the sum of a number and 2 is $4(x + 2)$," or "$2(8 + 7)$ is read as the product of 2 times the quantity or sum of $8 + 7$."
- Help students define the parts of an algebraic expression, including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient), as this helps students understand the structure of expressions and explain their process for simplifying expressions. Explain the following:
  - **Terms** are values in an expression separated by addition and subtraction, such as: $x + 3$ contains two terms and $2x - 5$ contains two terms.
  - Multiplication and division in an expression represent a **single term,** such as $5y$ is a term and $\frac{1}{2}x$ is a term.
  - A **coefficient** is a number that multiplies a variable, such as 4 is the coefficient of $4y$.
- Provide practice identifying parts of an expression, such as in the expression $2x - 5$: $2x$ is the first term where 2 is the coefficient of the variable x, and 5 is the second term, a constant.
- Help students concentrate on evaluating algebraic expressions for a given value of a variable, using the order of operations such as the following: Evaluate the expression $2(x + 3^2)$ when $x = 5$.
- Focus attention on the following vocabulary terms: *algebraic expressions*, *evaluate*, *sum*, *term*, *product*, *factor*, *quantity*, *quotient*, *coefficient*, *constant*, *like terms*, *equivalent expressions*, and *variable*.
- Provide cyclical, distributed practice over time to continually review translating and evaluating expressions.

What the STUDENTS do:

- Recognize that variables represent unknown quantities.
- Translate verbal expressions into numerical expressions and numerical expressions into verbal expressions.
- Communicate orally and/or in writing about translating and evaluating variable expressions using precise mathematical language, including the vocabulary variables, coefficients, constant, and term.
- Evaluate expressions for given values of variables using the order of operations when appropriate.

Addressing Student Misconceptions and Common Errors

Some students misunderstand or incorrectly read expressions. Students often confuse x^3 and $3x$. To address this, ask students to create a chart with the meaning of x^3 and $3x$ such as:

x^3 means	$3x$ means
✓ x times x times x ✓ x to the third power	✓ 3 times x ✓ $x + x + x$

Notes

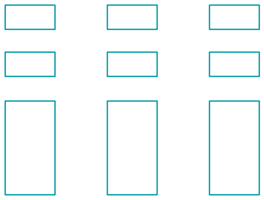
STANDARD 3 (6.EE.A.3)

Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.

Standard 3 spotlights applying the properties (distributive property, the multiplicative identity property of 1, and the commutative property for multiplication of operations) with expressions involving variables to generate equivalent expressions.

What the TEACHER does:

- Provide learning opportunities for students to use multiplication to interpret $3(2 + x)$ as three groups of $(2 + x)$. Have students create an array with three columns and $x + 2$ in each column to show the meaning of $3(2 + x)$. Ask students to discuss and explain why $3(2 + x)$ is equal to $6 + 3x$. (Note that the bars below represent x and the small boxes represent units.)



- Use manipulatives to interpret y as referring to one y, and y plus y plus y is $3y$. Discuss the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$.
- Encourage students to generate equivalent expressions for $5(x - 2)$ and $4x + 3y$.
- Ensure students have opportunities to talk with the teacher and each other to make sense of equivalent expressions.
- Focus on the following vocabulary terms: *equivalent expressions*, *properties*, *multiplicative identity*, *distributive*, and the *commutative property for multiplication*.
- Provide cyclical, distributed practice over time to review applying the properties of operations to generate equivalent expressions.

What the STUDENTS do:

- Understand that the properties used with numbers also apply to expressions with variables.
- Apply the properties of operations with expressions involving variables to generate equivalent expressions.

Addressing Student Misconceptions and Common Errors

When using the distributive property, some students may multiply the first term in the parentheses but forget to do the same to the second term. To address this error, give students a plastic ziplock bag of approximately 25 counters in two different colors mixed in each bag. Direct students to empty the bag and count the number of each color counter such as there are 14 yellows and 11 reds. Ask students to use the distributive property to write an expression to show how many of each color would be in 4 bags. Students write the expression $4(14y + 11r)$. Using the distributive property, the expression is $56y + 44r$. Interpret this as 56 yellows and 44 reds in 4 bags. Provide other examples.

Give students error analysis problems such as the following: "*Fred said* $3(2 + x)$ *and* $6 + x$ *are equivalent expressions. He was incorrect. Tell Fred what he did incorrectly*." One solution is to remind Fred that 3 must be distributed through both terms in the parentheses.

Using manipulatives such as Algeblocks™ or Algebra Tiles™ is also helpful in modeling the distributive property.

Notes

STANDARD 4 (6.EE.A.4)

Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

This standard focuses on combining like terms in expressions. Students substitute values into expressions to prove equivalence. For example, *Are $3(x + 4)$ and $3x + 12$ equivalent expressions?* Substitute a numerical value for x such as 2. Then, $3(2 + 4) = 18$ and $(3 \times 2) + 12 = 18$ so the expressions are equivalent.

What the TEACHER does:

- Have students explore adding or subtracting like terms as quantities that contain the same variables and exponents. For example, $5x + 4x$ are like terms and can be combined as $9x$; however, $5x + 4x^2$ are not like terms since x and x^2 are not the same. Manipulatives such as Algeblocks™ or Algebra Tiles™ can be used to explore this concept.
- Provide practice for students to prove equivalence with substitution. They will use substitution to verify that both expressions are equivalent such as $3(2 + x) = 6 + x$. They should substitute any number for x. If the expressions have different values, they are not equivalent.
- Ensure students have adequate practice with equivalent expressions using the associative, commutative, and distributive properties.
- Focus on the following vocabulary terms: *equation*, *equivalent*, *expression*, *variable*, and *substitution*.
- Provide cyclical, distributed practice over time to review how to identify when two expressions are equivalent.

What the STUDENTS do:

- Explore the concept of like terms and apply combining like terms in expressions accurately.
- Reason that two expressions are equivalent through the use of substitution.
- Explain reasoning to other classmates and the teacher using precise mathematical vocabulary.

Addressing Student Misconceptions and Common Errors

Some sixth graders do not recognize when letters are used to represent variables and when letters are used to represent units of measure such as, *4m* and 4 m as in meters or 3*h* and 3 h as in hours. Use contextual examples to distinguish between the two.

Some students may continue to combine $4x$ and $4x^2$. Use a manipulative such as square tiles to demonstrate the difference between the two terms.

Notes

Expressions and Equations
6.EE.B*

Cluster B

GRADE 6

Reason about and solve one-variable equations and inequalities.

STANDARD 5 **6.EE.B.5:** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

STANDARD 6 **6.EE.B.6:** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

STANDARD 7 **6.EE.B.7:** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

STANDARD 8 **6.EE.B.8:** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

*Major cluster

Expressions and Equations 6.EE.B

Cluster B: Reason about and solve one-variable equations and inequalities.

Grade 6 Overview

Students focus on the meaning of an equation and use reasoning and prior knowledge to solve it. They use variables to represent numbers and write expressions when solving problems. Students learn to write inequalities of the form $x > c$ or $x < c$ and use number line representation to show the solutions of the inequalities.

Standards for Mathematical Practice

SFMP 1. Make sense of problems and persevere in solving them.

Sixth graders solve real-world and mathematical problems through the application of algebraic concepts. They look for meaning of a problem and find efficient ways to represent and solve it.

SFMP 2. Reason abstractly and quantitatively.

Grade 6 students use properties of operations to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers.

SFMP 4. Model with mathematics.

Students write expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. They use number lines to compare numbers and represent inequalities.

SFMP 6. Attend to precision.

Students communicate precisely with others and use clear mathematical language when describing equations and inequalities.

SFMP 7. Look for and make use of structure.

Sixth graders apply properties to generate equivalent expressions and solve equations by the subtraction property of equality.

Related Content Standards

5.OA.A.1 7.EE.A.2 7.EE.B.3 7.EE.B.4 7.EE.B.4.a 7.EE.B.4.b

STANDARD 5 (6.EE.B.5)

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

The center of attention for this standard is solving an equation or inequality as a process of answering the following question: *Which values from a specified set make the equation or inequality true?* Students simplify numerical expressions by substituting values for given variables and use substitution to determine whether a given number in a specified set makes an equation true or which set of numbers makes an inequality true. Limit solving inequalities to selecting values from a given set that would make the inequality true. For example, find the value(s) of y that will make $7.2 + y \geq 9$. Select your value(s) from the set = {1, 1.3, 1.8, 2, 3}.

What the TEACHER does:

- Provide experiences for students to focus on understanding the meaning of solving an equation before developing the procedural knowledge. Start with a balance scale model to represent and solve equations. Say, *"There are 42 centimeter cubes on the left side of the scale and 200 cubes on the right side of the scale. All the blocks are the same size. How many cubes must be added to the left side of the scale to make the scale balance?"*
- Pose questions for variable equations such as the following: *What numbers could possibly be the solution for* $x + 17 = 27$? Note that equations should not require using the rules for operations with negative numbers.
- Supply scenarios where the solution is a single answer or multiple answers for the students to explore. This helps students establish the difference between equations and inequalities.
- Ask questions for variable inequalities such as the following: *What numbers could possibly be the solution for* $x + 17 > 27$? Present a set of possible solutions for students to select from. Include rational numbers in the set.
- Emphasize simple equations for students to solve using reasoning and prior knowledge such as *"Maria has 42 dollars in her bank. For her birthday she received some more dollars and now has $200. How many dollars did she receive for her birthday?"* Provide a set of possible solutions so that students may use substitution to find the solution. One possible set is {78, 58, 158, 258}.
- Focus on the following vocabulary terms: *inequalities* and *equations.*
- Provide cyclical, distributed reviews over time to practice solving an equation or inequality as a process of answering a question.

What the STUDENTS do:

- Use precise mathematical vocabulary to explain the differences between equations and inequalities.
- Discover that solutions to inequalities represent a range of possible values rather than a single solution.
- Reason the value(s) that make an equation or inequality true and select from a given set of values.
- Simplify numerical expressions by substituting values for given variables.

Addressing Student Misconceptions and Common Errors

Many students have difficulty understanding that an inequality can have more than one solution. The best way to work on this concept is to use real-world examples that are familiar to students. For example, I have $25 and want to buy some bracelets. The bracelets cost $8 each. How many could I buy? This results in the inequality $8b \leq 25$ where b is the number of bracelets I can buy. Since students are not solving inequalities in this standard, if you include a negative number in the set of possible solutions, have a discussion about how the negative value only works for the equation and not the real-world scenario.

STANDARD 6 (6.EE.B.6)

Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

This standard concentrates on writing expressions using variables that represent real-world or mathematical problems. Students learn that a variable represents an unknown number or any number in a specified set.

What the TEACHER does:

- Provide a variety of experiences for students to write expressions for solving real-world word problems. Focus on reading algebraic expressions to make the connection that a variable represents a number, such as the following: "*Sean has five more than twice as many pencils as John. Write an algebraic expression to represent the number of pencils Sean has.*" $2j + 5$ where j represents the number of pencils John has.
- Encourage students to identify what the variable represents in each real-world scenario when they write an expression.
- Help students describe problem situations solved using an equation such as $4c + 5 = 25$, where c represents the cost of an item. Provide other problems where a variable represents any number in a specified set.
- Emphasize the following vocabulary terms: *expressions*, *equations*, and *variables*.
- Provide cyclical, distributed reviews over time to practice using variables to represent numbers and to read/write expressions when solving real-world or mathematical problems.

What the STUDENTS do:

- Understand that a variable represents a number or a specified set of numbers.
- Represent real-world scenarios with variable expressions, identifying what the variable represents.
- Use precise mathematical vocabulary when discussing expressions and variables.

Addressing Student Misconceptions and Common Errors

Some students continually misrepresent real-world scenarios with expressions. They each make different errors. Do an error analysis on the work of the students who repeatedly make errors. Are they mistaking what the variable is? Do they have trouble translating verbal expressions to variable expressions? Are they seeking to write equations instead of expressions? Write error analysis questions for the students to solve that use each of the common student errors being made in class that you have identified. An example of such a problem is as follows: Fred wrote $x + 6$ when asked to find an expression for *Sam has 6 times as many fish as Paul*. Fred was incorrect. Write Fred a note explaining his error. Note: Do not use real student names. This is meant to clarify misconceptions generally, not embarrass students who made the errors initially.

Notes

STANDARD 7 (6.EE.B.7)

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ *and* $px = q$ *for cases in which* p, q *and* x *are all nonnegative rational numbers.*

Attention for Standard 7 is placed with solving equations for real-world and mathematical problems that involve positive rational numbers and zero. To solve the equation, students can draw pictures such as this example: *"Juan spent $48.99 on three T-shirts. If each shirt is the same amount, write an algebraic equation that represents this situation and solve to determine how much one T-shirt costs. The picture created is a bar model chart."* Each bar is labeled S for T-shirt, so each pair of jeans costs the same amount of money. The bar model represents the equation 3S = $48.99. To solve the problem, students divide the total cost of $48.99 by 3.

$48.99		
S	S	S

What the TEACHER does:

- Pose problems for students to explore solving equations based on real-world scenarios such as *"Corry bought 6 CDs that each cost the same amount. Without tax, he spent $89.94. How much did he spend on each CD? Write and solve an equation to solve the problem."* Note that problems should only use positive rational numbers (including 0), fractions, and decimals. Encourage students to illustrate the equation in problem situations by drawing a picture or using reasoning and prior knowledge. Solving equations using reasoning, pictures, diagrams, and prior knowledge allows students to develop effective strategies on their own.
- Ask students to generate equations based on situations from their daily lives such as texting friends. Have students explain the meaning of the variables used.
- Ensure students have opportunities to talk with the teacher and each other to make sense of equations in the form of $x + p = q$ and $px = q$.
- Focus on the following vocabulary terms: *equations* and *nonnegative rational numbers.*
- Provide cyclical, distributed review over time to continually practice solving real-world and mathematical problems by writing and solving equations.

What the STUDENTS do:

- Solve equations that represent real-world mathematical problems that involve positive rational numbers and zero.
- Model real-world situations with equations and use a variety of strategies to solve them.
- Use precise mathematical vocabulary to communicate with the teacher and classmates.

Addressing Student Misconceptions and Common Errors

Some students may need additional, on-going practice with writing and solving equations. Use advertisements in newspapers to generate real-world scenarios that may be used to write and solve the equations.

Notes

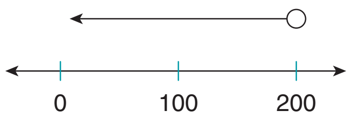
STANDARD 8 (6.EE.B.8)

Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

The essence of Standard 8 is graphing inequalities on a number line and writing inequalities to solve real-world mathematical problems. Students check by substitution to determine if the graph of an inequality is correct.

What the TEACHER does:

- Provide opportunities for students to represent inequalities on a number line. Present problems such as "*Less than $200.00 was spent by the Mrs. Smith for the class party. Write an inequality to represent this amount and graph this inequality on a number line.*" Explain that the open circle above the 200 means that 200 is not included in the solution boundary set. The ray represents all numbers in the solution set. Check the solution by having students each select a number represented on the number line as part of the solution set and determine if it makes the inequality $x < 200$ true. Ask students if –200, which makes the statement $x < 200$ true, is a realistic answer to the word problem. Then, facilitate a class discussion.



- Focus on the vocabulary term *inequality*.
- Provide cyclical, distributed reviews over time to practice writing an inequality of the form $x > c$ or $x < c$.

What the STUDENTS do:

- Discover that a variable can stand for an infinite number of solutions when used in inequalities.
- Graph inequalities on a number line.
- Write inequalities to solve real-world mathematical problems.
- Check by substitution to determine if the graph of an inequality is correct.

Addressing Student Misconceptions and Common Errors

Some students may need additional on-going practice with writing inequalities to represent a real-world mathematics situation. Use advertisements in newspapers to generate ideas of real-world scenarios that can be used to write an inequality to represent an amount. Ask students to talk about the problems and the number lines they created to show the inequalities.

Notes

Expressions and Equations
6.EE.C

Cluster C

Represent and analyze quantitative relationships between dependent and independent variables.

STANDARD 9

6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.*

Expressions and Equations 6.EE.C

Cluster C: Represent and analyze quantitative relationships between dependent and independent variables.

Grade 6 Overview

The focus for this cluster is using variables to represent two quantities in a real-world problem that change in relationship to one another. Students write an equation and analyze the relationship between the dependent and independent variables using graphs and tables.

Standards for Mathematical Practice

SFMP 1. Make sense of problems and persevere in solving them.

Sixth graders solve real-world problems through the application of algebraic concepts.

SFMP 4. Model with mathematics.

Students model real-life situations with mathematics and use variables to represent two quantities in real-world problems. Problem situations are modeled symbolically, graphically, tabularly, and contextually.

SFMP 6. Attend to precision.

Students communicate precisely with others and use clear mathematical language when describing dependent and independent variables.

SFMP 7. Look for and make use of structure.

Sixth graders represent mathematics to describe a situation with either an equation or a diagram and interpret the results.

Related Content Standards

5.OA.A.1 7.EE.B.3 7.EE.B.4

Notes

STANDARD 9 (6.EE.C.9)

Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

This standard accents using variables to represent two quantities in real-world scenarios. Students recognize that a change in the independent variable creates a change in the dependent variable, such as the following: As x changes, y also changes. Emphasis is placed on writing an equation to express the quantity in terms of the dependent and independent variables. Students also identify relationships between tables, graphs, and equations and relate these back to the equation.

What the TEACHER does:

- Provide experiences for students to understand multiple representations such as tables, equations, and graphs that can be used to analyze relationships between quantities. Students should describe the relationships using language. Ensure students understand that each representation shows the same relationship.
- Include numerous situations for students to analyze and determine the unknown that is dependent on the other components such as how far someone travels is dependent on the time and rate.
- Focus on the following vocabulary terms: *dependent variable* and *independent variable*.
- Provide cyclical, distributed practice over time to continually review using variables to represent two quantities in a real-world problem that change in relationship to one another.

What the STUDENTS do:

- Use variables to represent two quantities.
- Identify relationships between tables, graphs, and equations.
- Recognize that a change in the independent variable creates a change in the dependent variable such as the following: As x changes, y also changes.
- Write an equation to express the quantity in terms of the dependent and independent variables.

Addressing Student Misconceptions and Common Errors

Some students may confuse what a graph represents. To help, have students explain in their own words what the graph means.

Notes

Sample PLANNING PAGE

Standard: 6.EE.C.9. *Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.* For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Mathematical Practice or Process Standards:

SFMP 1. Make sense of problems and persevere in solving them.

Sixth graders solve real-world problems through the application of algebraic concepts.

SFMP 4. Model with mathematics.

Students model real-life situations with mathematics and use variables to represent two quantities in real-world problems. Problem situations are modeled symbolically, graphically, tabularly, and contextually.

SFMP 6. Attend to precision.

Students communicate precisely with others and use clear mathematical language when describing dependent and independent variables.

SFMP 7. Look for and make use of structure.

Sixth graders represent mathematics to describe a situation with either an equation or a diagram and interpret the results.

Goal:

To focus on independent and dependent variables using real-world problems, sixth graders make connections between different representations of the problem using a table, an equation, and a graph.

Planning:

Materials: pencil and paper for each partner pair of students

Sample Activity:

- Provide the following real-world problem and allow students to work as partners.

> Our sixth-grade class is selling wrapping paper to fund our class party. Each roll of wrapping paper sells for $2.50 and is packaged 12 rolls to a box. Our class sold a total of 15 boxes of wrapping paper.

- Give students the following directions:
  - Complete the table to show the money collected for 15 boxes.
  - Write an equation to show the amount of money collected and the boxes of wrapping paper sold.
  - Graph the equation using ordered pairs from the table.

Sample PLANNING PAGE (Continued)

# of Boxes Sold	Money Collected
1	$30.00
2	$60.00
3	
4	$120.00
5	
6	
7	
8	
9	$270.00
10	
11	
12	
13	
14	
15	

- Look at the equation you wrote to show the amount of m money collected if b boxes of wrapping paper were sold. Explain which is the independent variable and which is the dependent variable.

Questions/Prompts:

- Ask, *"What is the relationship between the variables? Write an expression that illustrates the relationship."*
- Ask, *"How many boxes of wrapping paper do the students in the class need to sell if they want to make approximately $2,000?"*

Differentiating Instruction:

Struggling Students: Some students may need more experience with completing tables, writing equations, and constructing graphs with ordered pairs. With these students, try the problem again, making it a simpler problem and an easier table for students to complete.

Extension: Extend this work by having students generate their own scenarios to create a table, write an equation, and construct the graph with ordered pairs.

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Expressions and Equations
7.EE.A*

Cluster A

Use properties of operations to generate equivalent expressions.

STANDARD 1 **7.EE.A.1:** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

STANDARD 2 **7.EE.A.2:** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example*, $a + 0.05a = 1.05a$ *means that "increase by 5%" is the same as "multiply by 1.05."*

*Major cluster

GRADE 7

Expressions and Equations 7.EE.A

Cluster A: Use properties of operations to generate equivalent expressions.
Grade 7 Overview

In this cluster students apply properties of operations previously learned as strategies to add, subtract, factor, and expand linear equations that have rational coefficients. This skill leads to students being able to rewrite expressions in different forms so they can solve contextual problems and understand how the quantities in the problem are related.

Standards for Mathematical Practice
SFMP 2. Reason abstractly and quantitatively.

Students use expressions in different forms to understand how quantities in an equation are related.

SFMP 4. Model with mathematics.

Students write expressions and equations to model contextual problems.

SFMP 6. Attend to precision.

Students communicate their reasoning using precise mathematical vocabulary.

Related Content Standards

6.EE.A.3 6.EE.A.4 7.NS.A.1.d 7.NS.A.2.c 8.EE.C.7.b

Notes

STANDARD 1 (7.EE.A.1)

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Apply previously learned properties of operations (distributive, commutative, associative, identity, and inverse properties of addition and multiplication, as well as the zero property of multiplication) as strategies for adding, subtracting, factoring, and expanding linear expressions. Coefficients are limited to rational numbers that include integers, positive/negative fractions, and decimals. Use the properties to write equivalent expressions; for example, $3(4a + 2) = 12a + 6$ uses the distributive property.

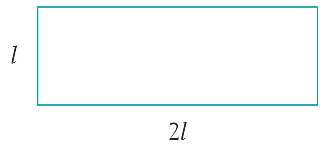
Substituting a numerical value for the variable and then evaluating the expressions to find the same solution is a tool to determine whether two expressions are equivalent. For example, $3(4a + 2)$ is equal to $12a + 6$. Let $a = 5$ and substitute 5 for a in both expressions.

$3(4a + 2)$	$12a + 6$
$3((4 \cdot 5) + 2)$	$(12 \cdot 5) + 6$
$3(20 + 2)$	$60 + 6$
$3(22)$	66
66	

What the TEACHER does:

- Present sets of expressions and ask which are equivalent. Allow time for students to reason using properties. For example, "*Maria thinks the two expressions* $2(3a - 2) + 4a$ *and* $10a - 2$ *are equivalent. Is she correct?*" Explain your reasoning.
- Provide students with opportunities to explain their reasoning in writing about how they are creating an equivalent expression using precise mathematical vocabulary. For example, vocabulary includes the terms *distributive property*, *identity*, and so on.
- Use substitution as a method to determine if two expressions are equivalent.
- Use equivalent expressions for real-world problems. For example: "*A rectangle is twice as wide as long. One expression to find the area is* $l \cdot 2l$. *Write the expression another way.*"

  Solution: $2l^2$



What the STUDENTS do:

- Reason to identify sets of equivalent expressions.
- Discover that there can be more than one expression equivalent to a given expression.
- Change an expression into an equivalent expression using properties of operations and combining like terms.
- Represent real-world problems with equivalent expressions using properties of operations, combining like terms and substitution, and solve them.
- Communicate orally and/or in writing using precise mathematical vocabulary how an equivalent expression is created.
- Defend why two expressions are or are not equivalent.

Addressing Student Misconceptions and Common Errors

When students work with several steps in an expression, sometimes they forget about the order of operations such as in the following example: $7 + 2(3x - 5) + 2x$. Students may want to add the $7 + 2$ first or only multiply the 2 by the $3x$ and not the -5. A review of the order of operations can help. For students who need more assistance, have them create their own order of operations card with steps outlined to reference when needed to check their work. Students can also create their own pneumonic device to help them recall the steps.

STANDARD 2 (7.EE.A.2)

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

Using equivalent expressions from the previous standard, focus on how writing an equivalent statement can better show the relationship among the terms in the expressions. For example, $6x + 15 = 3(2x + 5)$ means that three groups of $2x + 5$ is the same as one group of $6x$ and 15.

GRADE 7

What the TEACHER does:

- Present students with real-world problems that can be modeled with more than one expression. For example: *"An item that is on sale for 20% off costs 80% of the original price. Write an expression using* x *as the original price."* Allow students to explain their expressions, decide if one another's expressions are equivalent, and explain how a particular expression relates the quantities in the problem. This can be done individually, in groups or as projects.

What the STUDENTS do:

- Model contextual problems with multiple variable expressions.
- Explain orally and/or in writing, using precise mathematical vocabulary, how two equivalent expressions relate the quantities.

Addressing Student Misconceptions and Common Errors

Many students have difficulty seeing that expressions are equivalent when the expressions are out of context. Use simple contexts so that students can reason with a context to explain why two expressions are equivalent. For example: *"Write two equivalent expressions for the following situation—All music downloads are 99 cents today. Maria wants to download 2 R&B hits, 1 rap hit, and 3 hits by her favorite artist. Two equivalent expressions are 6 × 0.99 and (2 × 0.99) + (1 × 0.99) + (3 × 0.99)."* Focus student attention on how 6 hits for 0.99 each is the same as 2 hits and 1 hit and 3 hits for 0.99 each.

Notes

Expressions and Equations
7.EE.B*

Cluster B

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

STANDARD 3

7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional* $\frac{1}{10}$ *of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar* $9\frac{3}{4}$ *inches long in the center of a door that is* $27\frac{1}{2}$ *inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

STANDARD 4

7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

*Major cluster

Expressions and Equations 7.EE.B

Cluster B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Grade 7 Overview

Students focus on solving real-world problems and learn to use equations and inequalities to solve the problems by reasoning about the quantities. Students learn to solve equations in the forms $px + q = r$ and $p(x + q) = r$ fluently through practice. They compare algebraic solutions to arithmetic ones to demonstrate that they understand the sequence of operations in each approach and how they are the same and different. For inequalities, students graph solutions and then describe the solutions in terms of the context of the problem.

Standards for Mathematical Practice

SFMP 1. Make sense of problems and persevere in solving them.

Students solve multi-step real-world mathematical problems. Students use equations and inequalities to solve problems.

SFMP 2. Reason abstractly and quantitatively.

Students solve problems by reasoning about quantities.

SFMP 4. Model with mathematics.

Students write equations to model contextual problems.

SFMP 6. Attend to precision.

Students estimate answers to problems as a check to accurate solutions.

Related Content Standards

7.NS.A.3 6.EE.B.6 6.EE.B.7 8.EE.C.7 8.F.A.3

GRADE 7

Notes

STANDARD 3 (7.EE.B.3)

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

Students solve multi-step real-world and mathematical problems. The problems should contain a combination of whole numbers, positive and negative integers, fractions, and decimals. Students will apply what they learned in previous standards about converting fractions, decimals, and percents and use properties of operations to find equivalent forms of expressions as needed. Students will be expected to check their work for reasonableness using estimation strategies, which may include but are not limited to the following:

- rounding the values in the problem up or down and then adjusting the estimate to make up for the closeness of the rounded values to the originals,
- using friendly or compatible numbers for the values in the problem that allow for common factors for multiplication or easy addition such as grouping hundreds or thousands, and
- using benchmark numbers that are easy to work with such as using 2 for $1\frac{7}{8}$ to make an estimate.

What the TEACHER does:

- Pose a variety of multistep real-world and mathematical problems to solve, including integers, fractions, decimals, and percents. Students should convert fractions, decimals, and percents as in the example in the Standard where a 10% raise was interpreted as $\frac{1}{10}$ of the base salary.
- Encourage the use of rounding, compatible numbers, and benchmark numbers to check for reasonableness of results.
- Expect students to use a check for reasonableness on every problem. Have them explain orally and/or in writing their estimation strategies for some of the problems using journals or on exit slips.

What the STUDENTS do:

- Solve multi-step real-world and mathematical problems with precision.
- Select an appropriate estimation strategy and apply it to a problem. Values in problems lend themselves to different strategies.
- Justify the estimation process used by explaining, orally and/or in writing, how it proved their answer to be reasonable. If the estimate did not show an answer to be reasonable, explain how it helped lead to an accurate answer.

Addressing Student Misconceptions and Common Errors

It is common for students to have difficulty with multi-step problems. Scaffold the problems by adding a question mid-way. Display the first step of the problem, allow students to find the answer, and then present the next part that relies on the first step. Gradually remove the middle question as students get used to finding a middle question and identifying it themselves. For example: "*Fred goes out to eat and buys a pizza that costs \$12.75, including \$.50 tax. He wants to leave a tip based on the cost of the food. What must Fred do?*"

First, present the following: "*Fred goes out to eat and buys a pizza that costs \$12.75, including \$.50 tax. How much did the pizza cost?*" Solve this part of the problem. Then, using the answer from Part 1, introduce the second part of the problem: "*He wants to leave a tip based on the cost of the food. What must Fred do?*"

Some students' work may indicate a weakness representing numbers in different forms such as 10% as $\frac{1}{10}$. These students need additional practice. Use number lines, visuals such as bars, and hands-on materials instead of memorizing rules.

STANDARD 4 (7.EE.B.4)

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. *Solve word problems leading to equations of the form* $px + q = r$ *and* $p(x + q) = r$, *where* p, q, *and* r *are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach*. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

GRADE 7

Students will become fluent in solving equations. Students use the arithmetic from the problem to generalize an algebraic solution.

Use word problems that lend themselves to equations in the forms of $px + q = r$ and $p\ (x + q) = r$. Two examples are as follows:

1. Three consecutive even numbers add up to 48. What is the lowest number of the three?
   $x + x + 2 + x + 4 = 3x + 6 = 48\ (px + q = r)$
2. Ms. Thomas had \$25 to spend on party favors. She had \$10.40 left after buying 10 balloons. How much did she spend on each balloon? $0.1(25 - 10.40) = r\ (p(x + q) = r)$

Students should develop fluency solving word problems that can be modeled by linear equations in the form $px + q = r$. Integers, fractions, and decimals should be included as values in the word problems.

What the TEACHER does:

- Select word problems for students that lend themselves to algebraic equations in the forms $px + q = r$ and $p(x + q) = r$, such as:
  - Diane had \$30 to spend on party favors. She had \$17.50 left after buying 10 balloons. How much did she spend on each balloon? $0.1(30 - 17.50) = r$.
  - Three consecutive even numbers add up to 48. What is the lowest number of the three?

$$x + (x + 2) + (x + 4) = 48$$

$$x + x + 2 + x + 4 = 48$$

$$3x + 6 = 48$$

- Facilitate a classroom discussion about the importance of using the order of the operations. Demonstrate with an incorrect sequence of operations to emphasize the point.
- Provide students with problems that can be solved arithmetically but also have an algebraic solution such as problems that apply formulas for area or perimeter as in the example in the Standard. Ensure students can relate an arithmetic solution to an algebraic one using the example from the Standard.

What the STUDENTS do:

- Model word problems with equations in the forms $px + q = r$ and $p(x + q) = r$.
- Fluently solve equations of the forms $px + q = r$ and $p(x + q) = r$.
- Compare algebraic equations with arithmetic solutions for the same problem using precise mathematical vocabulary.

Addressing Student Misconceptions and Common Errors

Students who have difficulty becoming fluent in solving equations may need a hands-on approach. Manipulatives such as Algeblocks™, Hands-On Equations,™ and Algebra Tiles™ can be useful.

b. *Solve word problems leading to inequalities of the form* $px + q > r$ *or* $px + q < r$, *where* p, q, *and* r *are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.* For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

In this standard, students move from solving word problems with equations to word problems with inequalities. Inequalities follow a similar form to those of the equations, $px + q > r$ and $px + q < r$. Students graph the solution set of the inequality on a number line and describe what it means in terms of the context of the word problem. Be aware that sometimes the solution set to the inequality contains values that do not make sense as solutions for the word problem. For example, in the word problem, "*Donna has at most $60 to spend on a shopping spree. She wants to buy a dress for $22 dollars and spend the rest on bracelets. Each bracelet costs $8. How many bracelets can she purchase?*" we see a solution of

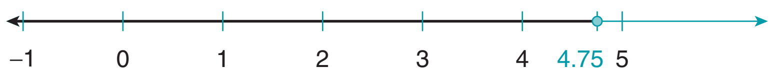
$$\$60 - \$22 = \$38$$

$$8x \le 38$$

$$\frac{8x}{8} \le \frac{38}{8}$$

$$x \le 4.75$$

The number of bracelets is less than or equal to 4.75. However, Donna cannot buy .75 of a bracelet, so when we graph the inequality as below:



we see that the only viable solutions to the word problem are 4, 3, 2, 1, or no bracelets.

What the TEACHER does:

- Compare word problems that can be modeled with equations to those where an inequality is needed to find a solution set. Inequalities may have negative coefficients. Ask the students to compare how they are the same and how they are different.
- Model solving an inequality while facilitating a classroom discussion about how the procedure for solving inequalities is similar to that of equations.
- Present students with many examples of word problems that can be modeled by and solved with inequalities such as the following:
    - Erin has at most $73 to spend on jewelry. She wants to buy a watch for $25 and spend the rest on necklaces. Each necklace costs $8. Write an inequality for the number of necklaces she can purchase and solve it.
- Provide examples of inequalities with negatives so that students learn to reverse the direction of the inequality sign when multiplying or dividing by a negative.
- Encourage students to substitute the answer in the inequality to see if it makes the inequality true.
- Have students graph the solution sets of the inequalities on a number line and make sense of the solution set in context as opposed to whether it is a correct solution set to the mathematical inequality. Ask students to identify the maximum and minimum numbers that make sense within the context of the problem.

What the STUDENTS do:

- Recognize whether a word problem can be represented with an equation or an inequality.
- Create inequalities of the forms $px + q > r$ and $px + q < r$.
- Solve inequalities that contain the symbols $<, >, \le, \ge$.
- Check answers with substitution.
- Graph solutions to inequalities on number lines and discuss whether all of the answers in the solution set make sense in the context of the problem.

Addressing Student Misconceptions and Common Errors

Students may forget to switch the inequality sign when multiplying or dividing by a negative. Help students by asking them to check answers in their solution sets in the original inequality to see if they satisfy the inequality. For other students who consistently make errors, check their number line graphs. Some seventh graders may have difficulty drawing the graphs accurately. For example, some students will reverse the location of negative and positive integers. For these students, supply them with graph paper or simply a sheet of pre-drawn number lines for them to fill out.

Notes

Sample PLANNING PAGE

Standard: 7.EE.A.2. *Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.* For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

Mathematical Practice or Process Standards:

SFMP 2. Reason abstractly and quantitatively.

Students use expressions in different forms to understand how quantities in an equation are related.

SFMP 4. Model with mathematics.

Students write expressions and equations to model contextual problems.

SFMP 6. Attend to precision.

Students communicate their reasoning to one another using precise mathematical vocabulary.

Goal:

Students understand that equivalent expressions can help show the relationship between the quantities in the problem more clearly and, thus, make it easier to explain the relationship.

Planning:

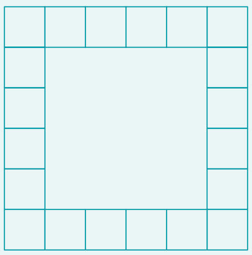
Materials: copies of the problem, paper and pencil, 1 sheet of chart paper per group

Sample Activity:

- Students work in groups to solve the following problem:

Pablo is making a square picture frame of square tiles as shown:

On a sheet of poster paper, write three different expressions Pablo can use to find the total number of tiles in the frame.



1. Explain how each expression relates to the diagram.
2. Demonstrate that the expressions are equivalent.
3. Which expression does your group think is the most useful? Explain your thinking to the class.

Sample PLANNING PAGE (Continued)

- Have groups make class presentations. Ask questions during the presentations to facilitate class discussion such as, *"That is an interesting expression. Did any other group come up with that one? What did you need to understand to create that expression?"*

Questions/Prompts:

- For groups having trouble getting started, ask them to describe the problem in their own words.
- When students need to look at the problem from a different perspective, ask, *"How is this side related to this side?"* (as you point).
- If a group needs more practice, say, *"Talk to me about the thinking you have used so far."*

Differentiating Instruction:

Struggling Students: Struggling students may better understand the problem if they model it with square tiles and manipulate them to make sense of the problem.

Extension: Ask students to find a fourth equivalent expression for this problem.

Notes

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Expressions and Equations
8.EE.A*

Cluster A

Work with radicals and integer exponents.

STANDARD 1 **8.EE.A.1:** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

STANDARD 2 **8.EE.A.2:** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

STANDARD 3 **8.EE.A.3:** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.*

STANDARD 4 **8.EE.A.4:** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

*Major cluster

Expressions and Equations 8.EE.A

Cluster A: Work with radicals and integer exponents.
Grade 8 Overview

In this cluster students learn how to compute with integer exponents. Students build on what they have learned about square roots to solve equations in the form of $x^2 = p$ and $x^3 = p$, where p is a positive rational number, evaluating perfect square and perfect cube roots. Students learn how to express very large and very small numbers in scientific notation and express how many times larger or smaller one number written in scientific notation is than another. Students use the properties of integer exponents to perform operations with numbers written in scientific notation. Students interpret numbers written in scientific notation using technology.

Standards for Mathematical Practice
SFMP 2. Reason abstractly and quantitatively.

Students use reasoning to express how many times larger (or smaller) one number is than another when both are expressed in scientific notation.

SFMP 5. Use appropriate tools strategically.

Students learn to read scientific notation as expressed by technology.

SFMP 6. Attend to precision.

Students compute with integer exponents and numbers in scientific notation accurately.

SFMP 7. Look for and make use of structure.

Students will see and make use of patterns in scientific notation and in square and cube numbers.

Related Content Standards

7.NS.A.D.1 8.NS.A.1 8.NS.A.2

Notes

STANDARD 1 (8.EE.A.1)

Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

Students learn how to compute using integer exponents building on their earlier experiences with adding and subtracting integers. For any non-zero real numbers *a* and *b* and integers *n* and *m*, the properties of integer exponents are as follows:

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = \frac{1}{a^n}$
6. $\frac{a^n}{a^m} = a^{n-m}$

What the TEACHER does:

- Introduce the laws of integer exponents one at time. Use a conceptual approach as opposed to asking students to memorize the rules.
- Provide examples of the processes that lead to the rules for each law, such as $4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = 4 \times 4 \times 4 \times 4 \times 4 = 4^5$. Allow students to try a few similar expressions to see if they can find the solution and posit a rule or property they may discover.
- Provide examples: $6^2 7^2 = (6 \times 6)\ (7 \times 7) = (6 \times 7)\ (6 \times 7) = (6 \times 7)^2$. Ask students to try several of their own to see if they can discover a rule or property.
- Provide examples: $(5^3)^4 = (5 \cdot 5 \cdot 5) \times (5 \cdot 5 \cdot 5) \times (5 \cdot 5 \cdot 5) \times (5 \cdot 5 \cdot 5) = 5^{3 \times 4} = 5^{12}$ and $(3 \times 7)^4 = (3 \cdot 7) \times (3 \cdot 7) \times (3 \cdot 7) \times (3 \cdot 7) = (3 \cdot 3 \cdot 3 \cdot 3) \times (7 \cdot 7 \cdot 7 \cdot 7) = 3^4 \times 7^4$. Ask students to try several of their own to see if they can discover a rule or property.
- Have students practice the properties by generating equivalent expressions and writing them in simplest form such as $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.
- Provide examples to lead students to discover how a negative exponent translates to a positive exponent in the denominator of a fraction, such as how $3^{-3} = \frac{1}{3^3}$.
- Assign a project for students to design and create posters summarizing the rules they discovered to hang in the classroom of the properties of exponents with integers.

What the STUDENTS do:

- Discover the properties of integer exponents by making sense of the examples presented. For example, $(3 \times 7)^4 = (3 \cdot 7) \times (3 \cdot 7) \times (3 \cdot 7) \times (3 \cdot 7) = (3 \cdot 3 \cdot 3 \cdot 3) \times (7 \cdot 7 \cdot 7 \cdot 7) = 3^4 \times 7^4$ with the rule discovered as $(ab)^n = a^n b^n$.
- Generate equivalent expressions in simplest form for products and quotients of numbers with integer exponents having the same bases.

Addressing Student Misconceptions and Common Errors

Students often confuse the rules. This occurs primarily when students are taught to memorize the rules rather than understand what is happening in the properties by working with numerical expressions as in the suggestions above. It is important to present examples and let students discover what the rules are. Then students should be encouraged to write their reasoning so they can clarify the explanations for themselves.

STANDARD 2 (8.EE.A.2)

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Students learn that squaring and cubing numbers are the inverse operations to finding square and cube roots. This standard works with perfect squares and perfect cubes, and students will begin to recognize those numbers. Equations should include rational numbers such as $x^2 = \frac{1}{4}$ and $x^3 = \frac{1}{64}$ and fractions where both the numerator and denominator are perfect squares or cubes:

$$x^2 = \frac{1}{4}$$

$$\sqrt{x^2} = \pm\frac{\sqrt{1}}{\sqrt{4}}$$

$$x = \pm\frac{1}{2}$$

Square roots can be positive or negative because $2 \times 2 = 4$ and $-2 \times -2 = 4$.

What the TEACHER does:

- Introduce squaring a number and taking the square root as inverse operations, providing students opportunities to practice squaring and taking roots.
- Repeat the previous instruction for cubes and cube roots, also including fractions where the numerator and denominator are both perfect cubes.
- Relate perfect square numbers and perfect cubes to geometric squares and cubes using square tiles and square cubes to build the numbers. A square root is the length of the side of a square, and a cube root is the length of the side of a cube.
- Encourage students to find patterns within the list of square numbers and then with cube numbers.
- Facilitate a class discussion around the question, "*In the equation* $x^2 = p$, *when can* p *be a negative number?*" Students should come to the conclusion that it is not possible.
- Discuss non-perfect squares and non-perfect cubes as irrational numbers such as $\sqrt{2}$.

What the STUDENTS do:

- Recognize perfect squares and perfect cubes.
- Solve equations containing cube and square roots.
- Discover and explain the relationship between square and cube roots and the sides of a square and the edges of a cube, respectively, by using hands-on materials.
- Reason that non-perfect squares and non-perfect cubes are irrational, including the square root of 2.

GRADE 8

Addressing Student Misconceptions and Common Errors

It is important for students to have multiple opportunities and exposures with perfect cubes. This is a new concept in the curriculum and many students struggle with finding cube roots. A common misconception for cube roots is that any number times 3 is a perfect cube. Building larger cubes from smaller ones gives students a visual that they can rely on.

Notes

STANDARD 3 (8.EE.A.3)

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.

This standard emphasizes scientific notation. Students write very large and very small numbers in scientific notation using positive and negative exponents. For example, 123,000 written in scientific notation is 1.23×10^5, and 0.0008 written in scientific notation is 8×10^{-4}. When mastered, students use the skill to determine how many times larger (or smaller) one number written in scientific notation is than another. To compare, if the exponent increases by 1, the value increases 10 times. In the example of the U.S. and world populations, the exponent increased by 1, and the 7 is a little more than 2 times 3. So 2×10 makes for 20 times larger.

What the TEACHER does:

- Introduce examples of very large and very small numbers in contexts. Contexts can be found in sources such as government statistics websites, population sizes, land mass in area, and science. Ask students why writing very large and very small numbers in scientific notation would be beneficial. Who would use it?
- Provide students with the opportunity to research very large and very small numbers and present them written in scientific notation, along with the contexts, to the class. Discuss why these numbers are considered estimates. Keep a bank of these numbers and their contexts for students to use at a later time to create real-world problems.
- Provide contextual problems for students to compare numbers written in scientific notation.
- Use the bank of numbers created by the students to ask about how some of the numbers are related to one another by comparing similar contexts.

What the STUDENTS do:

- Understand the benefits of using scientific notation.
- Research to find examples of very large and very small numbers.
- Write very large and very small numbers in scientific notation.
- Understand that some numbers written in scientific notation are estimates. Explain why that is true.
- Compare numbers written in scientific notation to determine how many times larger (or smaller) one number written in scientific notation is than another.

Addressing Student Misconceptions and Common Errors

Students often confuse a very large number for a small number when written in scientific notation such as 4,000,000 for 4×10^{-6}. This usually is a result of students trying to memorize a rule about moving a decimal point to the left or the right. Instead of teaching a rule, rely on students' background knowledge of negative exponents. Before rewriting a number in standard form, look to the exponent to determine whether it is a small or large number. This can also be used as a check.

Students who do not understand the properties of exponents also make errors in computation with scientific notation. Teachers may need to review these properties.

Notes

STANDARD 4 (8.EE.A.4)

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

This standard builds on previous standards as now students use what they know about scientific notation and properties of integer exponents to solve problems. Quantities in the problems can be expressed in scientific notation and decimal form. Students focus on the size of the measurement to determine which units are appropriate for the context such as millimeters for very small quantities. This standard also calls for students to use technology and be able to interpret the scientific notation used. The teacher needs to check the class calculators to be familiar with the notation used by those particular calculators as the notation used by calculators to express scientific notation is not standard.

What the TEACHER does:

- Pose problems that require students to perform operations with numbers written in scientific notation.
- Present problem-solving opportunities for students to choose correct units of measurement when working with very large and very small numbers including making conversions between units such as in the following problem: *"An average ant is 10^{-1} centimeters long. If you laid ants end to end, how many would it take to make a line from New York City to Disney World?"* The distance to Disney World from NYC is 1,513 kilometers.
- Provide students with calculators. Give them a calculation to perform that results in a number displayed in scientific notation. Facilitate a large group discussion about what the notation in the display means and how it is a form of scientific notation. Provide other opportunities for students to interpret scientific notation expressed with technology. Ask, *"Do all calculators use the same display for scientific notation?"*

What the STUDENTS do:

- Perform operations with numbers written in scientific notation. Solve both mathematical and real-world problems.
- Choose correct units for very large and very small numbers when solving problems.
- Discover and interpret the rules for scientific notation displayed on a given calculator.

GRADE 8

Addressing Student Misconceptions and Common Errors

When performing operations with numbers in scientific notation, such as $(7 \times 10^5) \times (18 \times 10^9)$, some students will be overwhelmed with keeping track of what they should do. Encourage these students to color code the numbers such as highlighting the numbers in exponential form in the given example so students remember to work them together.

Notes

Expressions and Equations
8.EE.B*

Cluster B

Understand the connections between proportional relationships, lines, and linear equations.

STANDARD 5

8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

STANDARD 6

8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

*Major cluster

Expressions and Equations 8.EE.B

Cluster B: Understand the connections between proportional relationships, lines, and linear equations.

Grade 8 Overview

In this cluster students connect proportional relationships, lines, and linear equations. First, students compare proportional relationships represented in different ways such as graphs, tables, and linear equations. Unit rate is interpreted as the slope of a line, and students learn that the slope is the same between any two points on a line by using similar triangles. Then the general equations for a line ($y = mx + b$ and $y = mx$) are derived.

Standards for Mathematical Practice

SFMP 2. Reason abstractly and quantitatively.

Students compare two proportional relationships represented in different forms.

SFMP 6. Attend to precision.

Students give explanations that are precise and use appropriate vocabulary.

SFMP 7. Look for and make use of structure.

Students see a pattern that results in the general form of a linear equation.

Related Content Standards

6.RP.A.3.b 7.EE.B.4.a 8.G.A.4 8.F.A.2 8.F.A.3

Notes

STANDARD 5 (8.EE.B.5)

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

Students build on their work from Grade 6 with unit rates and their work with proportional relationships in Grade 7 to compare graphs, tables, and equations of linear (proportional) relationships. Students identify the unit rate as slope in graphs, tables, and equations to compare proportional relationships presented using different representations. For example, compare the unit rate in a problem about a phone bill presented in graphic form on a Cartesian plane to a phone bill from a different company where the unit rate can be found represented in an equation or table.

What the TEACHER does:

- Present a single, graphed, proportional relationship to the class. Facilitate a class discussion about the unit rate, using students' background knowledge, and interpret the unit rate as the slope of the line.
- Present a second, related, proportional relationship, written in a different form such as a table or equation. Facilitate a class discussion about how to compare the two situations. Use questions such as, *"What is happening in each situation? How are they the same? Different? How can we tell? What about the slopes? What do they tell us? What can we do to help us compare the slopes (graph the second relationship)?"*
- Provide opportunities for students to compare proportional relationships and write their conclusions using precise mathematical vocabulary.

What the STUDENTS do:

- Make use of the structure of a representation to compare different representations. For example, students will find that if a problem asks them to compare the unit rates for information presented in a table and information presented in equation form, graphing both of them will make the comparison easier.
- Compare proportional relationships presented in different forms (graphs, tables, equations, verbal descriptions) and explain comparisons in writing using clear and precise mathematical language. Comparisons will include slope interpreted in context of the relationships.

Addressing Student Misconceptions and Common Errors

Errors occur when students are overwhelmed by being presented with too much information at a time. Encourage students having difficulty making the comparisons to work with one relationship at a time. Graphing may be a difficult skill for some students. Use graph paper larger than 1 cm for these students so they can see the unit rate easier.

Students who are overwhelmed can also be helped by using graphs of experiences that are familiar to them. This makes the information more accessible so students can better understand and interpret proportional relationships.

Notes

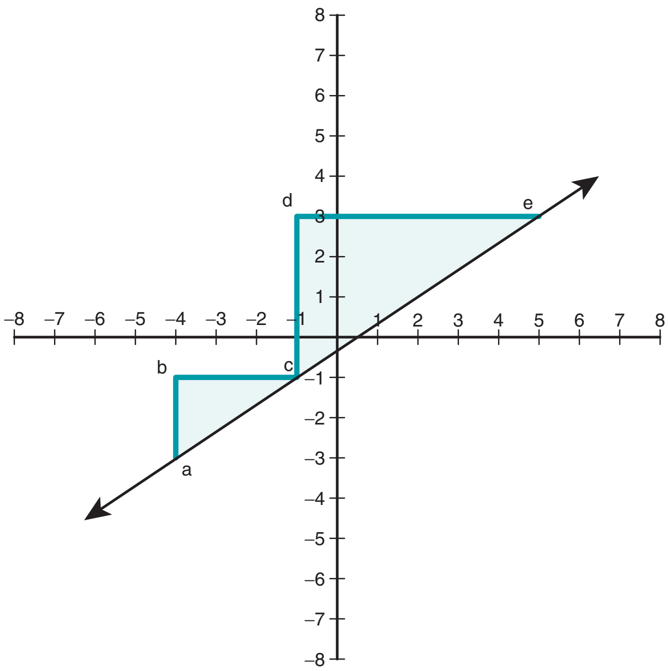
STANDARD 6 (8.EE.B.6)

Use similar triangles to explain why the slope m *is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation* y = mx *for a line through the origin and the equation* y = mx + b *for a line intercepting the vertical axis at* b.

Students gain additional knowledge about slope in this standard as they use similar triangles to explain how the slope *m* of a line is the same between any two points on a given non-vertical line. Students understand positive/negative slopes, 0 slope, and undefined slopes. Through the use of similar triangles, teachers lead students to derive the general equation ($y = mx + b$) of a line and discover that *m* is the slope and *b* is the *y*-intercept.

What the TEACHER does:

- Facilitate a class discussion about two similar triangles as in the example below:



- Explain slope as rise over run (rise is the vertical distance and run is the horizontal distance) by having students see that $\overline{ab}$ (rise) is 2 units and $\overline{bc}$ (run) is 3 units for a ratio of 2 to 3 or written as $\frac{2}{3}$. On similar triangle CED, $\overline{cd}$ is 4 units and $\overline{de}$ is 6 units for a ratio of 4 to 6, which is the same as 2 to 3. Have students create other pairs of similar triangles to convince themselves that the slope of a line is the same between any two points on a non-vertical line.
- Challenge students to find the slope of a horizontal line (0) and a vertical line (undefined) and explain their reasoning.
- Demonstrate how to find a slope using the formula $\frac{y_1 - y_2}{x_1 - x_2}$.
- Lead students to discover the equation $y = mx$ for a line that goes through the origin and $y = mx + b$ for a line that goes through point *b*. Note that when $b = 0$, $y = mx$. The *y*-intercept is *b*.

What the STUDENTS do:

- Explain, orally and/or in writing, using similar triangles, why the slope of a line is the same between any two points on a non-vertical line. Use clear and precise language.
- Discover that *b* is the *y*-intercept and *m* is the slope in the general equation for a line, $y = mx + b$.
- Determine the slope of a line from a graph, table, or linear equation.
- Explain orally and/or in writing how proportional relationships, lines, and linear equations are related.

Addressing Student Misconceptions and Common Errors

A common error students make is to misuse the formula for finding the slope of a line given two points. They use *x* – *y* or use the difference of the *x* coordinates divided by the difference in the *y* coordinates. Look for these common errors. Focus students' attention on the errors by using error analysis tasks. For example, Jed used the following equation to find the slope of a line: $\frac{x_1}{y_1} - \frac{x_2}{y_2}$. Find Jed's mistake and correct it.

Notes

GRADE 8

Expressions and Equations
8.EE.C*

Cluster C

Analyze and solve linear equations and pairs of simultaneous linear equations.

STANDARD 7 **8.EE.C.7:** Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

STANDARD 8 **8.EE.C.8:** Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be* 5 *and* 6.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

*Major cluster

Expressions and Equations 8.EE.C

Cluster C: Analyze and solve linear equations and pairs of simultaneous linear equations.
Grade 8 Overview

Students analyze and solve one variable linear equations for one, zero, or infinitely many solutions, simplifying the equations until they reach $x = a$, $a = a$, or $a = b$ (where a and b are different numbers). Students then apply that knowledge to analyzing and solving pairs of simultaneous linear equations also known as systems of linear equations in two variables.

Standards for Mathematical Practice
SFMP 1. Make sense of problems and persevere in solving them.

Students solve problems with systems of linear equations.

SFMP 2. Reason abstractly and quantitatively.

Students analyze linear equations and systems of linear equations.

SFMP 4. Model with mathematics.

Students model real-world problems with systems of equations.

SFMP 6. Attend to precision.

Students give explanations that are precise and use appropriate vocabulary.

SFMP 7. Look for and make use of structure.

Students use the structure of an equation to know which steps to perform to solve it.

Related Content Standards

6.EE.B.7 7.EE.B.4.a

Notes

STANDARD 7 (8.EE.C.7)

Solve linear equations in one variable.

a. *Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form* x = a, a = a, *or* a = b *results (where* a *and* b *are different numbers).*

b. *Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.*

This standard has students solving linear equations. It is explained by 8.EE.C.7.a and b. It is best to teach a and b together so that they are not considered isolated skills.

These standards provide the foundation for all future work with linear equations. Students solve equations that have one, zero, or infinitely many solutions and relate those solutions to the context. If the solution is in the form $x = a$, there is only one solution. If $a = a$, there are infinitely many solutions. If $a = b$ results (where a and b are different numbers), there are no solutions.

Linear equations can have fractions and decimals as coefficients and can be solved by expanding expressions with the distributive property and/or collecting like terms.

What the TEACHER does:

- Provide pairs of students with three one-variable linear equations: One equation has one solution, one has no solutions, and one has an infinite number of solutions. After students have a chance to solve, facilitate a discussion on the results students found. Demonstrate how when the result is $x = a$, there is only one solution that will make the equation true. Use substitution when student results are $a = b$ (a); ask the students to make sense of the results. Ask questions such as, "*Can* a = b?" Analyze the equation to see why there are no possible solutions. For results $a = a$, use substitution to demonstrate how there are an infinite number of solutions.
- Present students many opportunities to solve linear equations, including those with fraction, decimal, and positive/negative coefficients. Some equations should provide the opportunities to use the distributive property to expand terms and to combine like terms. Use Hands-On Equations™, Algeblocks™, Algebra Tiles™ or a similar set of manipulatives to demonstrate combining like terms and using the distributive property.

What the STUDENTS do:

- Solve and analyze one variable linear equations and explain whether the solution has one, zero, or infinitely many solutions.
- Solve linear equations with rational coefficients. Use the distributive property when appropriate and combine like terms when the equation calls for it.

Addressing Student Misconceptions and Common Errors

A common error students make involves applying the distributive property when negative integers are involved, such as $-2(-x - 4)$. The error occurs when they try to multiply the -2 and the -4. Students need repeated exposure to equations of this type. Prompting students to consider "minus 4" as "plus negative 4" helps correct the misconception. Providing and discussing tasks that involve students analyzing errors helps students self-correct many misconceptions.

Notes

STANDARD 8 (8.EE.C.8)

Analyze and solve pairs of simultaneous linear equations.

a. *Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.*

b. *Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.* For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

c. *Solve real-world and mathematical problems leading to two linear equations in two variables.* For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

This standard has students solving simultaneous linear equations. It is explained by 8.EE.C.8.a–c. It is best to consider a, b, and c together as they are not isolated skills.

Students will understand that points of intersection are the solutions to pairs of simultaneous linear equations (also known as systems of linear equations). Students will solve systems graphically, algebraically, and by inspection. Examples in this standard are in real-world contexts and mathematical problems.

GRADE 8

What the TEACHER does:

- Have students graph two linear equations that share a solution on the same coordinate plane. The lines should intersect. Facilitate a class discussion about what the point of intersection means. Ask students, *"What do you notice about the lines? What are the coordinates of the intersection? Which equation does the point of intersection satisfy?"* When students realize that the point of intersection satisfies both equations, introduce the terms system of equations/simultaneous equations and graphing as a method to estimate solutions to systems of equations.
- Facilitate a discussion about what the graph would look like if there were no solutions to the system of equations and if there are an infinite number of solutions.
- Provide students with simple cases of simultaneous equations that have no solution and ask them to analyze the equations for a solution. A simple example is, "$3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6."*
- Introduce solving a system of linear equations algebraically and checking the results.
- Present opportunities to solve real-world and mathematical problems that are solved by systems of equations. Encourage students to use the most efficient method (graphing, inspection, or algebraic manipulation) to find the solution.

What the STUDENTS do:

- Reason that the intersection of two lines on a graph represents the solution to the system of linear equations. Explain, using clear and precise mathematical language, why this is true. Explain how to recognize if the solution set has one, zero, or an infinite number of points.
- Solve systems of equations graphically, algebraically, and by inspection depending on the problem presented.
- Solve real-world and mathematical problems that lead to pairs of simultaneous linear equations.

Addressing Student Misconceptions and Common Errors

Common errors for systems of equations include students who have trouble accurately graphing and, therefore, cannot correctly estimate the solution. Technology can be helpful as can graph paper with larger than 1-cm squares.

Sample PLANNING PAGE

Standard: 8.EE.A.2. *Use square root and cube root symbols to represent solutions to equations of the form* $x^2 = p$ *and* $x^3 = p$, *where* p *is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that* $\sqrt{2}$ *is irrational.*

Mathematical Practice or Process Standards:

SFMP 7. Look for and make use of structure.

Students use square tiles to see the relationship to square numbers and cubes to cube numbers.

Goal:

Students work with manipulatives and discover that a square root is the side of a square and a cube root is the edge of a cube.

Planning:

Materials: square tiles, cubes (wooden cubes, Unifix cubes™, linking cubes), pencil and paper, 1 sheet poster paper per group

Sample Activity:

- Provide small groups with at least 49 tiles per group.
- Assign each group a square number: 49, 36, 25, 16, or 9.
- Each group makes a square with the number of tiles assigned and answers the following questions on their poster paper along with a drawing of their square: What is the area of your square? How did you calculate it? What is the formula for finding the area of a square? What do you notice about your number? What is the length of one side of your square? How does the length of one side of your square compare to the area of your square? Using your square, describe a square root.
- Repeat the procedure with cubes and adjust the questions to fit cube roots.
- Groups present their finding to the class. After presentations class discussion comes to a conclusion about square roots and cube roots.

Questions/Prompts:

- Are students able to see that they have made a square versus a rectangle? Ask, *"What do we know about a square that makes it a special rectangle?"*
- Are students recognizing that their numbers are perfect squares (or cubes)? Ask, *"What kinds of numbers have we learned about? List all the types and see which labels fit your number."*
- Have students noticed that the square root of the area of their square is the length of a side? Ask, *"How have you used the side of your square? Did you use it for any calculations in this activity? Which one? You may want to look back at the answers you wrote to your previous questions."*

Differentiating Instruction:

Struggling Students: Use heterogeneous groupings. Allow these students to manipulate the tiles and cubes if they seem to be confused. Some students may require their own set of manipulatives.

Extension: Assign students to read a children's book about square numbers such as *Sea Squares* by Joy Hulme or *My Full Moon Is Square* by Elinor Pinczes. After reading, have students outline their own children's book that teaches about cube numbers.

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

PLANNING PAGE

Standard:

Mathematical Practice or Process Standards:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Reflection Questions: Expressions and Equations

1. Discuss how you respond to parents who feel that Grade 6 is too early to study algebraic expressions and equations.

2. Discuss the role manipulatives can play and the types of models that can be used in helping students understand expressions and equations in Grades 6–8.

3. Discuss how the Mathematical Practices "Reason abstractly" and "Critique the reasoning of others" are married to the mathematical standards in this domain.