

# Preface

## A Research Compendium for Building Knowledge and Capacity

The National Council of Teachers of Mathematics (NCTM) will mark its 100th anniversary in 2020. At the same time, the *Journal for Research in Mathematics Education* (JRME) will celebrate its 50th year of fulfilling its charge to publish high-quality research dealing with significant problems in mathematics education. The publication of the *Compendium for Research in Mathematics Education* is an important milestone leading to these significant anniversaries. Over the years, NCTM has published many books and journals, primarily for practitioners, including research-based resources. In addition, NCTM has been, and continues to be, a strong advocate for original research in all aspects of mathematics education, placing great emphasis on building knowledge and capacity in the field. Indeed, alongside JRME, NCTM publishes the JRME monograph series and now this *Compendium for Research in Mathematics Education*.

According to the dictionary, the word “compendium” refers to “a collection of concise but detailed information about a particular subject” that has been “systematically gathered” (Hobson, 2004, p. 84). Three key terms in this definition deserve special attention: systematically gathered, concise, and detailed. This research compendium is precisely a systematically gathered, concise, and detailed volume about research in mathematics education. The nearly 100 authors who have contributed their expertise to its writing have striven to distill the knowledge of the field into a resource that provides the best, most critical evidence about what is known about fostering students’ learning of mathematics. This research compendium represents another investment of NCTM in its effort to achieve excellence for all.

Please note that I deliberately chose to use “for” rather than “of” in the title, because the compendium is not simply a static collection but rather a resource to be used “for” advancing research in mathematics education. Although it is a compendium “of” research in mathematics education, it is to be used “for” research in mathematics

education. This, too, reflects NCTM’s charge to support cutting-edge research in mathematics education that can advance students’ learning of mathematics.

## Structure

The compendium consists of five major sections. The first section, Foundations, comprises six chapters in which the authors examine a variety of fundamental issues in mathematics education research, such as the nature of research, linking research and practice, and the role of funding and policy. Moreover, there are chapters focused on learning (e.g., trajectories), teaching, and pushing the frontiers of research by adopting different theoretical lenses.

The second section, Methods, contains three chapters focusing on qualitative and quantitative research methods, as well as design research. Because research questions circumscribe choices of research methods, the chapters in this section are designed to help readers understand what kinds of research questions each set of methods is able to appropriately address.

Section 3 is called Mathematical Processes and Content. This section covers research on mathematical processes, such as proof and mathematical modeling, as well as students’ sense making. Also, this section examines the state-of-the-art in mathematics education research for mathematical content areas ranging from early number through postcalculus mathematics. Algebra is discussed in two chapters, which together span algebraic concepts and thinking in elementary school through high school. In addition, this section contains chapters that focus on teaching and learning measurement, geometry, probability and statistics, and calculus.

The title of Section 4 is Students, Teachers, and Learning Environments. The first six chapters of this section involve research related to language diversity, race, identity, gender, mathematical engagement, and embodiment. The next three chapters deal with research on teaching and teacher learning with a focus on classroom discourse, core practices in mathematics teaching,

and teacher professional learning. The section ends with two chapters about research in areas that have great potential to support students' learning and teachers' teaching: curriculum and technology.

The final section of the compendium is Futuristic Issues. Rather than a systematic review of literature in an area, each chapter in this section is a short essay discussing challenges and providing forward-looking views about the research in an area. Some chapters discuss burgeoning areas of research where mathematics education intersects with other fields, such as educational neuroscience and gifted and special education. Because of the importance of building capacity for the future of the field, we also include a chapter on doctoral studies in mathematics education. Other chapters in this section consider areas that are increasingly prominent in the research community public eye, including assessment, mathematics education in community college settings, and mathematics education in informal settings.

### Features

This compendium provides the most comprehensive and up-to-date survey of the best research, new developments, and critical analyses of issues in mathematics education. In addition to these common features, this compendium has three unique features that cut across its chapters.

The first is the synthesis of research with an eye to the *historical* development of a research topic, in particular covering historical milestones of the research topic. In most chapters, the research in a given area is traced to examine what is now known in light of what has been done in the past, as well as to look into the future (i.e., what should we do next?) given what is now known.

The second unique feature in this compendium is that, in addition to incorporating a historical perspective, we have placed more emphasis on the *methodological* perspective. Although there are three chapters in the compendium specifically devoted to discussing key research methods in mathematics education, almost every chapter in the compendium addresses methodological issues and implications in a given area of research, including the methods typically used, the contribution of methodology to the generation of knowledge, and how methodology has advanced. In a number of chapters, the authors have also included detailed descriptions of the methods they used to select research articles to be included in their review of research and how they synthesized the research findings.

The third unique feature for this publication of NCTM is a concerted effort to provide an *international* perspec-

tive on research in mathematics education. Although NCTM is situated in the United States, I have striven to include international perspectives whenever possible. First, I have tried to accomplish this by inviting international authors to coauthor a number of the chapters. Second, in each chapter, even those primarily focusing on the United States, the authors have worked very hard to situate findings, issues, perspectives, and future directions in an international context.

### Reading a Compendium Chapter

In each chapter of this compendium, readers will find the most recent, major findings that the authors have highlighted from their area. In particular, the authors have provided insights on where the field is and where it is going in each area of study in mathematics education.

However, compendium chapters can be challenging to read. They tackle long-standing, difficult issues for the field; synthesize large, complex bodies of research; and provide deep insights into an area of research. It can be helpful to approach them with some orienting questions. When reviewing and editing each chapter in the compendium, I kept the following five sets of questions in mind:

How is the chapter structured, why is it structured that way, and how does the chosen structure help the reader grasp the authors' points and the content of the chapter?

What is the conceptual framework, how was it chosen for the chapter, and how does the conceptual framework allow for new insights into the topic or area of study?

How did the author or authors decide what needed to be included (synthesized) and what could be set to the margins (not mentioned explicitly in the chapter)?

How have the research methods evolved over the years for the specific research area reported in the chapter?

What are possible future directions for research in the specific research area in the chapter, and how do the authors elaborate on these future directions?

I found this set of questions extremely helpful, not just as the editor of this compendium, but also as a reader of each chapter. Therefore, I offer them as tools to help the reader dig deeply into what the authors are offering.

### Looking to the Future

And now, finally, I am so pleased to present this compendium to the readers, to the educational research community in general, and to the mathematics education community in particular. I was honored to be invited to edit such a hefty volume. I have had a great sense of

satisfaction editing such a high-quality research compendium and now have a big sense of relief having completed it. That said, this is, I hope, only the first in the research compendium series to be published by NCTM. Possibly every 10 to 15 years we will have a successor—a second, third, and fourth research compendium. Those volumes will be shepherded and authored by future mathematics education scholars. Looking toward this future, I thought it might be helpful to conclude this preface by providing some food for thought about the process of writing a compendium chapter.

Over the past 5 years, I have coauthored three research compendium or handbook chapters, including one in this compendium (Lloyd, Cai, & Tarr, 2017, this volume), one in the *Third International Handbook of Mathematics Education* (Cai & Howson, 2013) and one in the *Second Handbook of Research on the Psychology of Mathematics Education* (Santos & Cai, 2016). As both the editor of this compendium and a chapter coauthor, I have been reflecting on how one goes about writing a compendium or handbook chapter. From my experiences editing this compendium, I have observed that there are different ways of writing a compendium chapter. One way takes a bottom-up approach, which is to systematically search journals or books and then analyze related works using a conceptual framework. The compendium chapter on discourse (Herbel-Eisenmann, Meaney, Bishop, & Heyd-Metzuyanim, 2017, this volume) represents this type of effort. Chapters developed this way are more data-driven. Indeed, the discourse chapter includes a specific section on the methods the authors used to select and code the relevant articles. At the other end of the spectrum is the top-down approach, which involves first building a conceptual framework, followed by synthesizing the research findings based on the organization presented by the conceptual framework. The compendium chapter on curriculum studies (Lloyd et al., 2017, this volume) represents such an effort. There are also many approaches that fall somewhere between these two.

What is the best way to write a compendium chapter? Although I would not claim that there is a single best way to write such a chapter, it is clear that, no matter which method an author chooses, writing a research compendium chapter extends far beyond reviewing the literature to gather and summarize what has been done. Instead, authors must strive to find a novel way to structure the literature (and the chapter) to help readers understand

and navigate the contours of what the field does and does not know for the specific area of research. That novel structure provides insights about the nature of research in a given area and can also serve to show methodological advances and future directions for research in that area. However, the novel structure must be based on a sound conceptual framework.

Assembling a research compendium like this is an important and critical step so the field of mathematics education can take stock of progress and the knowledge that has been accumulated thus far. This volume encodes not only the field's best understandings of the critical issues in mathematics education research today, but also its insights about how researchers can most effectively move forward as a field. This process, in which researchers both reflect on the state of knowledge and explain how they have come to structure their reflections, serves to build the field's capacity to continue to push the frontiers of its knowledge. It is, therefore, my sincere hope and belief that this first *Compendium for Research in Mathematics Education* will serve everyone involved in mathematics education well.

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