Introduction

The cases in *Making Meaning for Operations* concentrate on such questions as "What kinds of actions and situations are modeled by addition, subtraction, multiplication, and division?" and "How do students, as they work with such situations, come to understand the operations?" One seminar facilitator put it this way: "*Making Meaning for Operations* challenges our definition of operation. It brings operation out of a computation framework into a 'meaning' framework."

The casebook begins with a look at young children's counting strategies as they address problems that they will later solve by adding and subtracting. Chapters 1 and 2 examine the different types of situations that are modeled by whole number addition/subtraction and multiplication/division. But as illustrated by the cases in chapter 3, the operation of division introduces a new kind of number, fractions, when the division doesn't come out evenly. Chapter 4 concentrates on how the numerator and denominator of a fraction together determine the size of the fraction.

The latter part of the casebook, chapters 5 through 7, revisits the operations in the context of fractions. What whole number ideas, issues, and generalizations need to be refined or revised once the work is extended to include fractions? For example, how do you now make sense of multiplying when the numbers being multiplied create a product smaller than the numbers you started with?

The cases were all written by elementary and middle school teachers, describing events from their own classrooms. The teacher-authors were inquiring into how their own students come to make meaning for operations. Writing these cases was part of that process of inquiry, and the teachers came together on a regular basis to read and discuss one another' s work.

The casebook concludes with a chapter called "Highlights of Related Research." This essay summarizes some recent research findings that touch on the issues explored in the cases of *Making Meaning for Operations.*

By focusing on central mathematical concepts across the grades, Developing Mathematical Ideas modules support teachers in understanding how these ideas develop and what they look like as students grapple with aspects of these ideas in a variety of problem contexts. It is not enough for teachers to know only the core work of their own grade levels. First, any classroom will include students who are working in a range of different places in their own understanding. Teachers also need to recognize how students might be building ideas expected

before their grade level as well as toward those that follow. Second, teachers themselves should develop as deep and complete an understanding of these concepts as possible—both for their own learning and as a basis for making instructional judgments.

Developing Mathematical Ideas (DMI) was intentionally developed to support the concrete coherence and focus in the professional development of teachers of the elementary and middle grades to which the *Common Core State Standards for Mathematics* (CCSSM) aspires. Focus is provided by the selection for each DMI module of core mathematical ideas that underlie a key segment of mathematics content, while coherence comes from the careful analysis of how these core ideas connect to each other and are developed and applied by students across the grades. DMI is designed to help teachers understand these core ideas more deeply for themselves and to gain extensive knowledge about how students engage with the progression of these ideas.

The material of this module is the substance of the CCSSM domains Operations and Algebraic Thinking (K–4) and Number and Operations—Fractions (3–5), which develop a progression of understanding the operations with whole numbers from kindergarten to grade 4 and expanding that understanding to include fractions, starting in grade 3.

Although all of the Standards for Mathematical Practice (SMP) are illustrated in this module, two of them are emphasized: practice 2, reason abstractly and quantitatively and practice 3, construct viable arguments and critique the reasoning of others. Practice 2 is a focus of the work throughout the module. Reasoning abstractly and quantitatively has to do with decontextualizing problems expressed as situations and contextualizing problems expressed numerically. That is, a central part of understanding the meaning of the operations is being able to see how that meaning is expressed as relationships or actions on quantities in a context, how the meaning is expressed with numbers and symbols, and how to move fluidly between the two. As students describe the relationship between symbols and context, they are often also engaged in practice 3, constructing and critiquing arguments about the quantities in a problem and the nature of the result. In addition, practice 1, make sense of problems and persevere in solving them, is central to the work of all of the DMI modules. Throughout the *Making Meaning for Operations* seminar, you will have opportunities to discuss and reflect on how the Standards of Mathematical Practice appear in the sessions.

When this DMI seminar was first taught, many seminar participants reported that they had to learn how to read the cases:

> "It's different from reading a story. I feel as if I've had to read through each episode with a fine-tooth comb."
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> "I'm reading cases very slowly, and I'm writing down thoughts about what I'm seeing in the text."

> "I find that when I do the mathematics problems that the students in the case are working on, I can better understand what they are doing."

Other participants offered advice to future seminar attendees:

> "Read all the cases in a chapter once and try to write down the mathematical issues they raise. You might focus attention on two or three children that interest you and really figure out the mathematical issues that these few children are facing. Try to really understand how the children are thinking. Here's another way to do it: After reading all the cases in a chapter, go over them again, looking for common threads. What mathematical issues connect these cases together?"
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> "Remember that these are glimpses of real kids dealing with real situations, struggling to make sense of very difficult concepts. Pay particular attention to the natural ways students often solve problems."
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> "Begin with the chapter introduction, which alerts you to the ideas you should pay attention to."
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> "If possible, discuss the cases informally with other participants before the sessions. If you're unable to do any or some of the above, by all means, still come to the seminar!"

As the seminar proceeds, you might talk to other participants about the ways they read the cases to prepare for seminar discussions