



Introduction

Teachers have always included lessons on measurement in their work. In elementary schools, students may learn how to pace off a length, use a ruler, or find the area of a rectangle. Measurement has enormous value as a practical skill used in everyday life; at the same time, the study of measurement can also be an opportunity for students to expand their mathematical thinking. “I used to think that measurement was just about figuring out how big something is,” one teacher commented on her new insights. “But now I see it as the intersection of spatial sense and number sense.”

The cases in this *Measuring Space in One, Two, and Three Dimensions Casebook* reveal how children are thinking as they work on measurement ideas. An analysis of the children’s work provides a vehicle for examining the conceptual bases for measurement. Since the cases span kindergarten–grade 7, they also illustrate the way children’s ideas expand over time.

One important element of measurement that children must work through is that a single object has more than one aspect that can be measured. For instance, consider a rectangle: we can measure its perimeter, its area, and the length of its sides. In order to answer the question “How big is this rectangle?” we must decide what aspect to measure. With three-dimensional objects, there is also volume to consider.

The relationships among all these measures can be puzzling. When the sides of a rectangle are doubled, why isn’t the area doubled as well? Given two boxes, how can it be that the one with more volume actually has the smaller surface area? Working through questions like these is part of understanding what it means to measure in one, two, and three dimensions.

As children begin to measure, they need to determine what unit of measure is appropriate and also how to use those units efficiently to determine a measurement. An integral part of this work is learning how to visualize the space that is to be measured—how to break it into parts and to see how those parts are related to one another and to the whole. For example, underlying the process of finding the area of a rectangle is the idea that we can divide it into rows and columns; underlying the process of finding the volume of a box is that we can see it as layers, slices, or stacks. Analyzing the children’s thinking provides insights into this process of decomposing and recomposing space. These insights lay the groundwork for examining how the area of a triangle (or a trapezoid or a parallelogram) can be compared with that of a rectangle, a process that leads to the development of formulas for area.

Through the *Measuring Space in One, Two, and Three Dimensions Casebook* and seminar, you will explore issues of measurement and how children in elementary and middle school come to understand them. The cases were written by elementary and middle school teachers recounting episodes from their own classrooms. All had inclusive classrooms; the range represents schools in urban, suburban, and rural communities. The teacher-authors, who were themselves working to understand the “big ideas” of the elementary and middle-grade mathematics curriculum, wrote these cases as part of their own process of inquiry. They came together on a regular basis to read and discuss one another’s developing work.

The cases are grouped to present children in classrooms who are working on similar mathematical issues related to measurement. Through the cases, you will study children’s initial ideas as they talk about how big their foot is or which box is bigger; you will discover the cognitive work involved in structuring space in one, two, and three dimensions; and you will see children applying their understandings at ever greater levels of abstraction.

In the cases in chapter 1, children measure or compare objects and consider different aspects of size: length, width, height, area, and volume. Chapter 2 explores a skill at the heart of measurement: decomposing and recomposing space in one, two, and three dimensions. Chapters 3 to 6 explore the measure of length, area, and volume, drawing on previous chapters to point out connections among ideas. Chapter 7 is devoted to the complicated relationships between area and perimeter and between surface area and volume.

Chapter 8, the last in the casebook, is the essay, “Highlights of Related Research”; it summarizes some recent research findings that touch on the issues explored in the cases in chapters 1 to 7.