MATHEMATICALLY *Thinking*

Chapter 1

Number and Operations

Introduction

Doris Mohr and Peter Kloosterman

Number and operations have traditionally been a significant part of the K–12 curriculum. In the Common Core State Standards (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010), work with number and operations begins in kindergarten, with students composing and decomposing numbers from 11 to 19 into ten ones and additional ones. By fourth grade, students are using all four operations to solve word problems. By eighth grade, they are investigating irrational numbers, square roots, and cube roots.

The Number Properties and Operations strand has been a part of the National Assessment of Educational Progress (NAEP) mathematics assessment since the first assessment in 1973, though the emphasis on it has changed over time. In 1990 and 1992, the assessment framework recommended that this strand make up 45 percent of the fourth-grade assessment and 30 percent of the eighth-grade assessment. From 1996 through 2003, the percentage of the assessment consisting of questions from the Number Properties and Operations strand decreased by 5 percentage points for each grade level to 40 percent and 25 percent, respectively. Beginning with the 2005 administration, the emphasis for eighth graders was further reduced to 20 percent (Braswell et al. 2005; National Assessment Governing Board [NAGB] 2017). The Number Properties and Operations strand is the largest portion of the fourth-grade math assessment; at the eighth-grade level, algebra makes up the largest portion of the assessment at 30 percent. Although the grade 12 Mathematics NAEP is not a focus of this volume, this strand is a part of the framework but plays a lesser role, making up only 10 percent of the recommended proportion of items.

The NAEP mathematics framework (NAGB 2017) summarizes the types of skills and understandings assessed at the elementary and middle school levels. Specifically, the Number Properties and Operations framework includes a focus on number sense, estimation, ratios and proportional reasoning, properties of number and operations, and mathematical reasoning using number. According to the 2017 framework:

> In fourth grade, students are expected to have a solid grasp of whole numbers as represented by the decimal system and to begin understanding fractions. By eighth

> grade, they should be comfortable with rational numbers, represented either as decimal fractions (including percentages) or as common fractions, and should be able to use them to solve problems involving proportionality and rates. At this level, numbers should also begin to coalesce with geometry by extending students' understanding of the number line. This concept should be connected with ideas of approximation and the use of scientific notation. Eighth graders should also have some acquaintance with naturally occurring irrational numbers such as square roots and pi. (p. 8)

As noted in this volume's introduction, NAEP scale scores are reported on a scale from 0 to 500. The 33-point increase in scale score on the Number Properties and Operations strand from 1990 to 2017 at grade 4 was the largest increase of the five content areas that NAEP assesses (NAEP Data Explorer 2018). At grade 8, the largest increase was seen in the Algebra strand across the same time period (29 points), and the least amount of growth was in Number Properties and Operations (13 points) (NAEP Data Explorer 2018). Kloosterman and Walcott (2007) estimated that an 11-point gain for grade 4 is roughly equivalent to one grade level of growth, as is an 8-point gain for grade 8. Using this approximation, performance on the Number Properties and Operations strand increased from 1990 to 2017 by about three grade levels at grade 4. There was also improvement at eighth grade, but that improvement was only about one and a half grade levels—substantially less than the improvement of overall NAEP mathematics scores.

The activities in this chapter are designed to build understanding of number properties and of operations. Activities are aligned to specific Common Core State Standards and include suggestions for questions to ask of students, tasks to complete, when to work in groups, how to tie the activities to outside of class experiences, and so forth. Activities are organized by the grade level of the Common Core State Standards they are aligned to, but they are likely to be appropriate review for students one or two grade levels above the level at which they are listed. Because the activities often approach topics from a somewhat different perspective than standard textbook approaches, they may be valuable for students who have had difficulty with other strategies. While the activities include ample guidance on how they should be used in the classroom, teachers should feel free to adapt the tasks and the presentation of those tasks to meet the unique needs of the students in any one class.

Number and Operations Activities

Activity 1.1. **Hundreds, Tens, and Ones [Gr 2].** Students extend their understanding of place value by decomposing the number 124 into various combinations such as 1 hundred and 24 ones or 8 tens and 44 ones. They justify each combination by showing that the value of 124 does not change.

Activity 1.2. **Sums and Differences [Gr 3].** Students use addition and subtraction of two- and three-digit numbers to answer questions such as the following: "The sum of two numbers is 100. Could the larger number be 43?" As part of their answers, they must provide clear and complete justification of their reasoning.

Activity 1.3. **Fractions and Pizza [Gr 3].** Students work together to solve a NAEP item that involves deciding whether a boy can eat $1/2$ of a pizza if he has given $3/8$ of the pizza to each of two friends. Students must use manipulatives or drawings to provide justifications for their claims.

Activity 1.4. **Comparing Unit Fractions [Gr 4].** Students focus on the importance of having the same-size whole when comparing two fractions and find a common multiple of two denominators by using colored rods as models.

Activity 1.5. **Using Hidden Questions to Solve Multistep Word Problems [Gr 4].** Students use the "hidden question" strategy to solve multistep word problems. The strategy involves identifying a sub-question or set of questions that must be answered as part of the process of getting the final answer for a problem.

Activity 1.6. **Multiplicative Comparison [Gr 4].** Students solve a two-step word problem involving multiplicative comparison and represent the multiplicative relationship using pictures, strip diagrams, a table of values, or an equation.

Activity 1.7. **Understanding Whole Number Division [Gr 4].** Students solve word problems that have two- or three-digit dividends and one-digit divisors using base-ten blocks, area models, the build-up-through-multiplication process, and their own invented strategies.

Activity 1.8. **Dividing by Unit Fractions [Gr 5].** Students use the measurement interpretation of division to solve strings of division problems ($3 \div 1/2$, $3 \div 1/3$, etc.). They write word problems for which given division problems would be the representation and communicate their mathematical understanding through pictures, words, and equations.

Activity 1.9. **Sums of Primes [Gr 6].** Students use the Sieve of Eratosthenes to find prime numbers less than 100, and then use examples to build the conjecture that the sum of two primes is always a composite. The lesson concludes with an informal proof of this conjecture.

Table 1.1 provides a summary of the nine activities in this chapter. More information about most of the items is available in tables in Kloosterman, Mohr, and Walcott (2016). The last column of the table indicates where this information can be found.

Table 1.1. Summary of the Number and Operation Activities

Name of Activity	CCSSM Standard aligned with Activity	Focus of Activity	Estimated length of Activity	Table in Kloosterman, Mohr, and Walcott 2016 that includes the NAEP item
1.1. Hundreds, Tens, and Ones	2.NBT.A.1	Decompose three-digit numbers into hundreds, tens, and ones.	Longer	5.6, 5.7
1.2. Sums and Differences	3.NBT.A.2	Use addition and subtraction to solve problems involving the sum and difference of two numbers.	Shorter	5.13
1.3. Fractions and Pizza	3.NF.A.1 3.NF.A.3	Solve a problem involving halves, fourths, and eighths of a pizza.	Shorter	5.22
1.4. Comparing Unit Fractions	4.NF.A.2	Understand the importance of the size of the whole when comparing two unit fractions.	Shorter	5.20
1.5. Using Hidden Questions	4.OA.3	Solve multistep word problems by looking for and solving "hidden questions." Use estimation to check answers.	Longer	5.19
1.6. Multiplicative Comparison	4.OA.A.1 4.OA.A.2 4.OA.A.3	Solve a multiplicative comparison problem using concrete and abstract strategies.	Shorter	5.18
1.7. Whole Number Division	4.NBT.B.6	Students will solve division problems using a variety of approaches that focus on place value and the meaning of division.	Longer	5.15
1.8. Dividing by Unit Fractions	5.NF.B.7 5.NF.B.7.B 5.NF.B.7.C	Analyze strings of equations ($3 \div \frac{1}{2}$, $3 \div \frac{1}{3}$, etc.) to understand what it means to divide whole numbers by unit fractions.	Longer	5.23
1.9. Sums of Primes	6.EE.A.2.C 6.EE.B.6	Students find all prime numbers less than 100 and use that information to investigate the sum of two prime numbers greater than 10.	Shorter	5.5

Activity 1.1

Hundreds, Tens, and Ones

Michael Roach

Original NAEP Item: 2009-4M5 #8

Write a three-digit number using the digits 2, 4, and 6 so that the digit 4 means four tens and the digit 6 means six hundreds.

Solution:

642

Percent Correct (4th Grade)

Year	1996	2000	2003	2005	2007	2009
Correct	58%	64%	65%	68%	69%	69%

Commentary

This item was given to both grade 4 and grade 8 students (the item number for grade 8 is 2009-8M5 #7). Surprisingly, grade 8 students scored 2 to 4 percentage points lower than grade 4 students between 2000 and 2007 and 5 percentage points below grade 4 students in 2009. This suggests that middle school students should be revisiting the concept of place value more than is currently the case.

Hundreds, Tens, and Ones Class Activity

Learning Goal

Students will understand that numbers can be represented in multiple ways using place value.

Performance Goals

Students will use place value to represent a three-digit number in several ways and justify each representation by writing an equation.

Students will complete organized lists of ways to represent three-digit numbers and explain patterns in those lists.

Alignment with CCSSM

2.NBT.A.1. *Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.*

Students decompose the number 124 into various combinations of tens and ones and justify that each combination is 124. Additional numbers are decomposed in Gearing Up and Gearing Down activities.

SMP3. *Construct viable arguments and critique the reasoning of others.*

Students must justify why different combinations of tens and ones represent 124. They must also identify patterns in the combinations and critique the combinations and patterns given by their peers.

SMP5. *Use appropriate tools strategically.*

Students connect various base-ten block representations of 124 with written descriptions of 124 (e.g., 9 tens + 34 ones is 124).

SMP7. *Look for and make use of structure.*

Students examine the place value structure system and patterns in combinations of hundreds, tens, and ones used to create two- and three-digit numbers.

(National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010)

Problem-Based Lesson Plan

Materials Needed

Small whiteboards for students (if available)

Base-ten blocks

Riddles sheet (for projecting)

Tables 1.1.1 and 1.1.2 (to draw on board or project)

Copies of the Making 124 activity sheet for each student

Copies of the Representing Three-Digit Numbers and Representing Two-Digit Numbers activity sheets (for Gearing Up and Gearing Down only)

Pieces of lined paper (for Gearing Up only)

more4U materials for Activity 1.1

Riddles sheet

Tables 1.1.1 and 1.1.2 (blank versions)

Making 124 activity sheet and answer key

Representing Three-Digit Numbers activity sheet

Representing Two-Digit Numbers activity sheet

Launch

Activate Prior Knowledge

Review place value by giving students two or three digits and clues they can use to find a number that answers a number riddle. Students might record their responses on a small whiteboard. They should be prepared to explain how they know their answer is correct. Project and discuss the riddles one at a time.

I have 2 digits: 3 and 9.
The digit 3 means three ones.
The digit 9 means nine tens.
What number am I?
[*93*]

I have 3 digits: 2, 4, and 6.
The digit 4 means four tens.
The digit 6 means six hundreds.
What number am I?
[*642*]

I have 3 digits: 0, 2, and 8.
The digit 2 means two hundreds.
The digit 8 means eight ones.
What number am I?
[*208*]

I have 3 digits: 0, 3, and 4.
The digit 3 means three tens.
The digit 4 means four hundreds.
What number am I?
[*430*]

I have 3 digits: 1, 3, and 5.
I am the largest number possible that can be written with the digits 1, 3, and 5.
What number am I?
[*531*]

I have 2 digits: 2 and 4.
I am the smallest number that can be written with the digits 2 and 4.
What number am I?
[*24*]

To introduce the task for the day, make available base-ten blocks for each group of students. Begin by asking the following:

Do you remember the last number riddle we did? Can you show me 24 with your base-ten blocks?

Students will probably display the common representation of 2 rods and 4 unit cubes. Ask the following questions:

What does the digit 2 represent in 24?

[*Two tens or 20.*]

What does the digit 4 represent in 24?

[*Four ones.*]

Draw or project table 1.1.1. (a blank version to print out or download is available on this book's More4U page, and a filled-in version is on the facing page) and record the number of tens and ones in the first row. Then, with the students' help, justify that 2 rods and 4 unit cubes is indeed 24. Record *20 + 4 = 24* in the last column of the table. Ask the following question:

Is there another way to show 24 using the blocks?

[*Look for 1 rod and 14 unit cubes or 24 unit cubes.*]

If no one responds, provide this hint:

What if you trade in 1 of your rods for some unit cubes?

[*If we trade 1 rod for unit cubes we'll have 1 rod and 14 unit cubes.*]

Then ask the following question:

Can someone explain why 1 rod and 14 unit cubes still represent 24?

[*One rod is 10 plus the 14 unit cubes, which are each worth one, add up to 24.*]

Record the justification in the last column: *10 + 14 = 24*. Ask the following question:

Is there still another way to show 24 using the blocks?

[*Yes. We can trade the other rod for 10 unit cubes and have 24 unit cubes.*]

Record this last representation of 24 in the table. For the justification, record *24 = 24*.

(*Note:* If students are not used to seeing equations with one number on each side, review the fact that the equal sign means "the same as" or "has the same value as.")

Table 1.1.1. Representing tens and ones

Row Number	Tens (Rods)	Ones (Units)	Justification
1	2	4	20 + 4 = 24
2	1	14	10 + 14 = 24
3	0	24	24 = 24

Ask students the following question:

What patterns do you see in the table?

[*Possible responses include the following:*

The number of tens or rods goes down by one.

The number of ones or units goes up by ten.]

If possible, post this table so you can reference it for the next activity. Introduce the activity as follows:

Now represent 124 using base-ten blocks. Use the smallest number of blocks that you can.

[*1 flat, 2 rods, and 4 units.*]

Draw or project table 1.1.2 (a blank version is available on More4U) and fill in the first row. Ask the following question:

How can we be sure that this particular representation has a value of 124?

[*The flat is 100, the two rods are 20, and there are also 4 units making a total of 124.*]

Complete the last column: *100 + 20 + 4 = 124.*

Discuss the possibility of adding one flat to rows 2 and 3 in the previous table (see table 1.1.1) as additional ways to make 124. Record these decompositions in rows 2 and 3 in table 1.1.2.

Table 1.1.2. Representing hundreds, tens, and ones

Row Number	Hundreds (Flats)	Tens (Rods)	Ones (Units)	Justification
1	1	2	4	100 + 20 + 4 = 124
2	1	1	14	100 + 10 + 14 = 124
3	1	0	24	100 + 24 = 124

Ask the following questions:

Are there other ways to make 124, given the condition that we have to use one flat?

[*No, given the constraint of having to use one flat, all the possibilities are shown.*]

What if we can trade in the flat for rods (tens)? Do you think you can find more ways to make 124?

[*The answer is yes. Tell students that their task for the day is to come up with as many different ways as they can to make 124 using no flats.*]

Make Sure the Problem Is Understood

Distribute the Making 124 activity sheet and read the directions at the top of the page. Make sure students understand that the first three rows are from the work they already did and that any new representations they find will have no flats. Remind them to justify each representation of 124 as they record it to make sure it works. If students ask, tell them that they may or may not use all the rows in the table. They should fill in as many as they can. Tell them that base-ten blocks are available for their use, but they may run out of unit cubes at some point, so they should look for patterns in the numbers as they work so they can complete the table without them.

Questions to judge understanding of the task include the following:

> *When will you justify a representation you record?*
>
> [*Immediately after we find and record one.*]
>
> *What should you be looking for as you complete the table?*
>
> [*We should look for patterns in the numbers.*]
>
> *Will you use any flats in your representation?*
>
> [*No, just rods and unit cubes.*]

Establish Clear Expectations

Have the students work in pairs on the Making 124 activity sheet. One student can act as recorder while the other manipulates the base-ten blocks. Then roles should be switched after an appropriate amount of time.

Explore

Listen and Observe

As students are working, walk around the room taking note of which students are using the base-ten blocks and which students are seeing patterns in the table. Make sure students are justifying their representations in the last column of the table and ask them to explain their thinking. For example, if a student has recorded 8 rods and 44 ones and justified that representation by writing 80 + 44 = 124, ask the following:

> *Where does the 80 come from? I thought you only had 8 rods.*
>
> [*Each rod represents one ten, so 8 rods are worth 80.*]

Provide Appropriate Hints

If students have duplicate entries, ask the following:

> *Where else in the table do you see these numbers?*
>
> [*Students should find a row with the same numbers.*]

If students are struggling, ask the following:

> *Can you make any trades?*
>
> [*Help students make additional trades.*]

If students are using a flat, remind them that we found all the ways to make 124 using one flat and are now working on finding ways using no flats. Tell the class that you will fill in the table as students share their findings. Remind them they must provide a justification for each representation they share.

Summarize

Have Groups Share Results

As you call on pairs of students to share, record the results in a drawn copy of table 1.1.2 or in a projected copy of the Making 124 activity sheet. You may want to record findings in the order shown, even though students may not share in any particular order. This will facilitate the noticing of patterns after all representations have been recorded. For example, if the first response is 7 tens and 54 ones, record that in the table on line 9. Tell students they will understand your method once all the findings have been recorded.

As each pair of students share, ask other students to restate what they heard and/or explain why they agree, or disagree, with the justification, as shown in table 1.1.3.

Table 1.1.3. Ways to represent hundreds, tens, and ones

Row Number	Hundreds (Flats)	Tens (Rods)	Ones (Units)	Justification
1	1	2	4	100 + 20 + 4 = 124
2	1	1	14	100 + 10 + 14 = 124
3	1	0	24	100 + 24 = 124
4	0	12	4	120 + 4 = 124
5	0	11	14	110 + 14 = 124
6	0	10	24	100 + 24 = 124
7	0	9	34	90 + 34 = 124
8	0	8	44	80 + 44 = 124
9	0	7	54	70 + 54 = 124
10	0	6	64	60 + 64 = 124
11	0	5	74	50 + 74 = 124
12	0	4	84	40 + 84 = 124
13	0	3	94	30 + 94 = 124
14	0	2	104	20 + 104 = 124
15	0	1	114	10 + 114 = 124
16	0	0	124	124 = 124

If the class has not found all representations, provide additional clues at this time to complete the table.

Wrap Up the Lesson

As a whole class, discuss the completed table. Possible discussion questions include the following:

What patterns do you notice in the numbers?

[*The first 3 rows have a flat. Then after that, it's only tens and ones.*

Starting with row 4, the number of tens (rods) goes down by one and the number of ones (units) goes up by 10.]

Can anyone explain why the number of ones goes up by 10 when the number of tens goes down by one?

[*Because when you trade in a rod, which is a ten, you change it into ten ones.*]

Why did the value of every row come out to be 124?

[*Because we didn't change the number, we just made trades to make different representations of 124.*]

If an exit ticket is desired, have each student respond to the following prompt:

One way to make 78 is 50 + 28. Write down two other ways.

Gearing Up

For students who finish the task early or need a challenge, give them a new number such as 207 or 324 to decompose using the Representing Three-Digit Numbers activity sheet. You may wish to start with a decomposed number such as 1 hundred, 7 tens, and 2 ones. For large numbers, students may need to use a blank piece of lined paper as there may be too many options to fit on the activity sheet. If desired, each student can be given a different starting number.

Gearing Down

Have students who are struggling complete the Representing Two-Digit Numbers activity sheet with two-digit numbers such as 35 or 47.

Activity 1.2

Sums and Differences

Michael Roach

Original NAEP Item: 2007-8M9 #9

The sum of three numbers is 173. If the smallest number is 23, could the largest number be 62?

○ **Yes** ○ **No**

Explain your answer in the space below.

Solution:

If one number is 23 and the other is 62, the third must be 88, which is larger than 62.

OR

23 + 62 + 88 = 173

Largest number is 88.

Sample Student Response:

○ **Yes** ● **No**

23 and 62 is 85. You would need to add 88 to get to 173, so 62 wouldn't be the largest number.

Percent Correct (8th Grade)

Year	1996	2000	2003	2005	2007
Correct	33%	29%	43%	41%	42%

Commentary

The calculations necessary to complete this item are relatively easy, so the fact that less than half of eighth-grade students were successful is likely due to their inability to understand the problem or to decide what calculations to do. The mathematics computation skills needed to complete this item are included in grade 3 standards, and the activity is designed to help students at this level solve this type of problem.

Sums and Differences Class Activity

Learning Goal

Students will understand how to reason about sums and differences.

Performance Goal

Students will identify numbers based on information about their sums and/or differences and justify their thinking.

Alignment with CCSSM

3.NBT.A.2 *Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.*

Students use various addition and subtraction strategies to answer questions similar to those in the NAEP item.

SMP1. *Make sense of problems and persevere in solving them.*

Students must come up with a strategy for determining whether answers for problems are reasonable. If their first approach does not work, they must change course and try a new way to solve the problem.

SMP3. *Construct viable arguments and critique the reasoning of others.*

Many of the tasks in the activity have multiple correct answers. When students present their work to the class, they must explain their thinking and how their solutions are different from other correct solutions. Students will critique the reasoning of their peers.

(National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010)

Problem-Based Lesson Plan

Materials Needed

Copies of the Sums and Differences activity sheet for each student

Ten Frames sheets

Counters

Base-ten blocks (optional)

Gearing Up Problem activity sheet (for Gearing Up only)

more4U materials for Activity 1.2

Sums and Differences activity sheet and answer key

Gearing Up Problem activity sheet and answer key

Ten-Frame Examples sheet

Ten Frames sheet

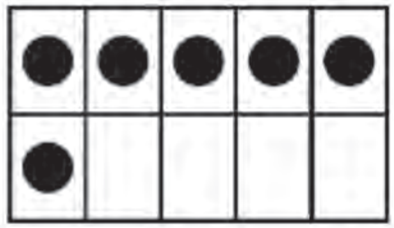
Hundreds Chart sheet

Launch

Activate Prior Knowledge

Display (or have a student display) two ten frames and put or show 6 counters on one of them as shown below. (You will find a downloadable and printable version of the ten frames below as well as blank ten frames on this book's page at NCTM's More4U website.) Ask the following question:

I'm thinking of two numbers whose sum is greater than 10. Could the smaller number be 6?

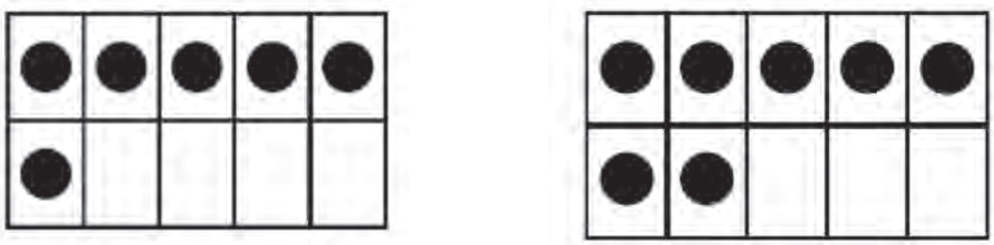


Let students talk in pairs and then, as a whole class, share their thinking. A possible response might be:

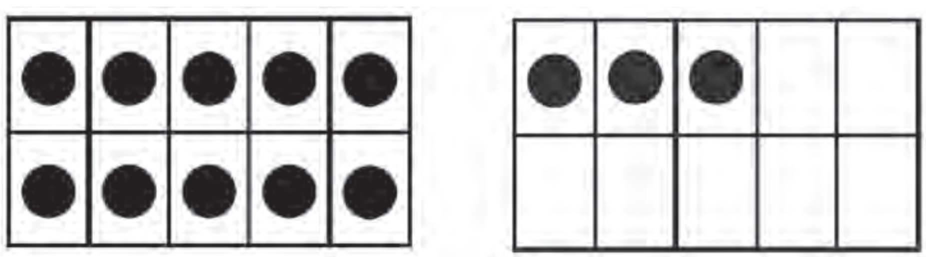
It could be if the other number is 7. Then the sum is 13, which is greater than 10 and 6 is smaller than 7.

Display 7 counters on the second ten frame. Ask the class the following question:

Is 7 more than 6? [*Yes.*]



Move 4 of the counters to the first ten frame to show the sum is 10 + 3 more or 13. After combining the two ten frames, students should see the following:



Make sure everyone understands that the sum, 13, is greater than 10 and 6 is the smaller of the two addends. Then ask the following question:

Does anyone have an example in which 6 is NOT the smaller of the two numbers?

[*Possible response: If you add 5 to 6, then the sum is still greater than 10, but 6 is not the smaller number; five is the smaller number.*]

Have that student come up and show the two numbers on the ten frames and that when you combine them, you get a sum that is more than 10. Next, suggest the following:

So let's go back to the original question: I'm thinking of two numbers whose sum is greater than 10. Could the smaller number be 6?

Have a discussion to make sure students understand that the smaller number could be 6 but it might not be. Because there is at least one instance in which 6 is the smaller number, the answer to the question is yes. Then continue with the following question:

I'm thinking of two numbers whose sum is greater than 10. Could the larger number be 5?

Provide ample time for the students to discuss the question in pairs. Then have students share their thinking with the whole class, using the ten frames to illustrate.

[*Possible response: If there are 5 counters on one of the ten frames, then I have to add more than 5 to get the sum over 10. So 5 will always be the smaller of the two numbers. The answer is no.*]

Explain to students that today's activity will challenge them with questions like these that use larger numbers. If necessary, review addition and subtraction of two- and three-digit numbers. This may include student-invented strategies as well as (or instead of) standard algorithms, depending on the students' current knowledge and skills.

Make Sure the Problem Is Understood

Hand out the Sums and Differences activity sheet, and make sure students understand the directions.

Should you just write YES or NO by each question?

[*No, we have to explain our thinking, too.*]

What does the term "difference" mean?

[*The term "difference" appears in question 2. It is how much bigger one number is than another. To get the difference between two numbers, you can count up from the smaller number to the larger or you can subtract the smaller number from the larger.*]

Be sure that students know that their justifications can include words, drawings, numbers, and symbols, but they should be clear and complete.

Establish Clear Expectations

Ask students to work in pairs and have them cross out, rather than erase, any work that does not lead to a correct solution. Let them know that there are "thinking tools" available for them to use if they wish. Examples of tools that may be useful include hundreds charts, small printed ten frames, and base-ten blocks.

Explore

Listen and Observe

Look for a variety of strategies, including strategies that do not work, to share with the class at the end of the activity. Check to see that students are adding and subtracting correctly, whether they are using invented strategies or standard algorithms. Pay attention to whether students feel they need multiple examples to justify their answers.

[*Multiple examples are not necessary, but they are a good way to check to be sure students have answered the questions correctly.*]

Provide Appropriate Hints

Suggest that students try a few numbers and look for a pattern. For example, possible comments and questions for 1b include the following:

> *Why don't you make a list of numbers you've tried and look for a pattern?*
>
> *What is the largest number you can add to 43 that is still smaller than 43?*
>
> *What number added to 43 gives you 100?*

Summarize

Have Groups Share Results

Bring the class together to share and discuss their answers. Be sure to include students with different strategies for each question. Also, ask the students to share attempts that did not work and explain how they know those examples were not useful.

Wrap Up the Lesson

After all students have shared, bring them together to discuss the big ideas that came out of the lesson. Possible questions include the following:

> *Did the first number you tried always work? Give me an example to support your answer.*
>
> [*Possible response: No, sometimes the first number did not work. For example, in question 1a I added 50 to 43 and got 93. But then I realized that 93 is not greater than 100, so I had to use a larger number.*]
>
> *How many numbers did you have to find that answered the question correctly?*
>
> [*Possible response: Just one number. As long as the problem worked for just one number, then it could be true.*]

Gearing Up

Give students the following additional problem. (This problem is available as a printable sheet on More4U, as is an answer key.)

> *Two numbers are added together and then a third number is subtracted from the sum. The result is 30. Could all three numbers be between 25 and 30? Explain your reasoning.*
>
> [*Yes. The possibilities are 27, 29, 26; 28, 29, 27; 29, 27, 26; and 29, 28, 27.*]

If there is still time available, ask students to find as many possible combinations of three numbers that answer the question as they can.

Gearing Down

Have students who are struggling focus on question 1 and provide manipulatives such as base-ten blocks, ten frames, or hundreds charts.