Standard Algorithm	Use of the Distributive Property	
18	10 + 8	
× 7	× 7	
56	56	(7 × 8)
70	70	(7 × 10)
126	126	

OR 18 • 7 = (10+8)7 = 10□7 + 8□7 = 70 + 56 = 126.

Instruction to Support a Focused Curriculum

Questions to Reflect On

- What does instruction that supports depth of understanding and connections among mathematical ideas "look like"?
- How can questioning be used to support the development of depth of understanding and connections in a focused curriculum?
- What is the role of practice in a focused curriculum?
- What impact does instruction that supports a focused curriculum have on time management?

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

—**The Teaching Principle,** ***Principles and Standards for School Mathematics***

Although NCTM's Curriculum Focal Points can help prioritize and organize mathematics content, teachers and the instruction they provide are crucial to using focal points to improve student learning. Focusing mathematics around a few central ideas at each grade requires skilled teachers who know the content well and can connect mathematical ideas and teach for depth of understanding.

Use of the Process Standards

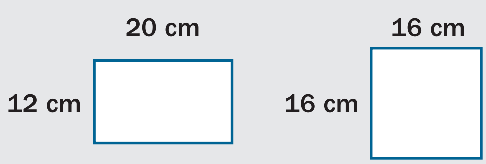
It is essential that teachers incorporate the Process Standards of Problem Solving, Reasoning and Proof, Communication, Connections, and Representation as described in *Principles and Standards for School Mathematics* (NCTM 200) into classroom instruction. Teachers should create a climate that supports mathematical thinking and communication. In this kind of classroom, students are accustomed to reasoning about a mathematical problem and justifying or explaining their results, representing mathematical ideas in multiple ways, and building new knowledge as well as applying knowledge through problem solving. Brief descriptions of the Process Standards can be found in the table below. More detailed descriptions can be found in *Principles and Standards for School Mathematics*.

NCTM Process Standards

Problem Solving. Through problem solving, students can not only apply the knowledge and skills they have acquired but can also learn new mathematical content. Problem solving is not a specific skill to be taught, but should permeate all aspects of learning. Teachers should make an effort to choose "good" problems—ones that invite exploration of an important mathematical concept and allow students the chance to solidify and extend their knowledge. Compare the two versions of a perimeter-and-area task below; whereas Task 1 requires students to do little more than correctly apply formulas, Task 2 engages them intellectually because it challenges them to search for something and is not imediately solvable. The instructional strategies used in the classroom should also promote collaborative problem solving. Students' learning of mathematics is enhanced in a learning environment that is a community of people collaborating to make sense of mathematical ideas (Hiebert et al. 1997).

Task 1

Find the area and perimeter of each rectangle:



Task 2

Suppose you had 64 meters of fence with which you were going to build a pen for your large dog, Bones. What are some different sized and shaped pens you can make if you use all the fencing? What does the pen with the least play space look like? What is the biggest pen you can make—the one that allows Bones the most play space? Which pen size would be best for running?

Source: *Mathematics Teaching Today, Second Edition* (NCTM, 2007a, p. 36). Reprinted with permission.

Reasoning and Proof. For students to learn mathematics with understanding, it must make sense to them. Teachers can help students make sense of the mathematics they are learning by encouraging them to always explain and justify their solutions and strategies as well as evaluate other students' ideas. Questions such as "Why?" and "How do you know?" should be a regular part of classroom discussions. The teacher should respond in ways that focus on thinking and reasoning rather than only on getting the correct answer. Incorrect answers should not simply be judged wrong. Instead, teachers can help students identify the parts of their thinking that may be correct, often leading to new ideas and solutions that are correct.

Communication. Reasoning and Proof goes hand in hand with the process of Communication. Students should have plenty of opportunities and support for speaking, writing, reading, and listening in the mathematics classroom. Communicating one's ideas orally and in writing helps to solidify and refine learning. Listening to others' explanations can also sharpen learning by providing multiple ways to think about a problem. The teacher plays an important role in developing students' communication skills by modeling effective oral and written communication of mathematical ideas as well as giving students regular opportunities to communicate mathematically.

Connections. As students move through the grades, they should be presented with new mathematical content. Students' abilities to understand these new ideas depends greatly on connecting them with previously learned ideas. Mathematics is an integrated field of study and should be presented in this way instead of as a set of disconnected and isolated concepts and skills. Instruction should emphasize the interconnectedness of mathematical ideas and should be presented in a variety of contexts.

Representation. Mathematical ideas can be represented in a variety of ways: pictures, concrete materials, tables, graphs, numerical and alphabetical symbols, spreadsheet displays, and so on. Such representations should be an essential part of learning and doing mathematics and serve as a tool for thinking about and solving problems. Teachers should model representing mathematical ideas in a variety of ways and discuss why some representations are more effective than others in particular situations.

Facilitating Classroom Discourse

The Process Standards, especially the Communication Standard and the Reasoning and Proof Standard, are related to the discourse in the mathematics classroom. "The discourse of a classroom—the way of representing, thinking, talking, agreeing, and disagreeing—is central to what and how students learn mathematics" (NCTM 2007a, p. 46). The teacher plays an important role in initiating and facilitating this discourse and can do so in the following ways:

- posing questions and tasks that elicit, engage, and challenge each student's thinking;
- listening carefully to students' ideas and deciding what to pursue in depth from among the ideas that students generate during a discussion;
- asking students to clarify and justify their ideas orally and in writing and by accepting a variety of presentation modes;

- deciding when and how to attach mathematical notation and language to students' ideas:
- encouraging and accepting the use of multiple representations;
- making available tools for exploration and analysis;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let students wrestle with a difficulty; and
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate. (NCTM, 2007a, p. 45)

The following classroom vignette illustrates a teacher's use of effectively facilitating discourse in the mathematics classroom. In this discussion, a conjecture has been made by some students that the larger a number is, the more factors it has.

Vignette

Drawing on Mathematical Knowledge during Exploration Activities

Although the class period is nearing its end, the teacher invites one group to present to the rest of the class their conjecture that the larger the number, the more factors it has. She suggests that the students record the conjecture in their notebooks and discuss it in class tomorrow. Pausing for a moment before she sends them out to recess, she decides to provoke their thinking a bit more: "That's an interesting conjecture. Let's just think about it for a second. How many factors does, say, 3 have?"

"Two," call out several students.

"What are they?" she probes. "Yes, Deng?"

Deng quickly replies, "1 and 3."

"Let's try another one," continues the teacher. "What about 20?"

After a moment, several hands shoot up. She pauses to allow students to think, and asks, "Natasha?"

"Six: 1 and 20, 2 and 10, 4 and 5," answers Natasha with confidence.

The teacher suggests a couple more numbers, 9 and 15. She is conscious of trying to use only numbers that fit the conjecture. With satisfaction, she notes that most of the students are quickly able to produce all the factors for each of the numbers she gives them. Some used paper and pencil, some used calculators, and some used a combination of both. As she looks up at the clock, one child asks, "But what about 17? It doesn't seem to work."

"That's one of the things that you could examine for tomorrow. I want all of you to see if you can find out whether this conjecture always holds."

"I don't think it'll work for odd numbers," says one child.

"Check into it," smiles the teacher. "We'll discuss it tomorrow."

The teacher deliberately decides to leave the question unanswered. She wants to encourage students to persevere and to not expect her to provide all the answers.

Source: *Mathematics Teaching Today* (NCTM 2007a, p. 23). Reprinted with permission.

Use of Questioning to Focus Learning and Promote Connections

A teacher's use of questioning plays a vital role in focusing learning on foundational mathematical ideas and promoting mathematical connections.

As described in the Introduction and the "Focusing Curriculum" section of this guide, using focal points to organize instruction does not mean teaching less or more content, but instead means directing the majority of your instruction at a smaller number of core areas with the goal of students' gaining a deeper mathematical understanding of those mathematical ideas and the connections among them. To teach for depth of understanding, teachers need to understand what their students are thinking and be able to support and extend that thinking. A teacher's use of questioning plays a vital role in focusing learning on foundational mathematical ideas and promoting mathematical connections. Such reasoning questions as "Why?" and "How do you know that?" posed during a lesson are great starters, but teachers also need to incorporate questioning techniques into their planning by thinking about specific questions to ask related to the particular topic. When planning instruction, teachers must also anticipate the kinds of answers they might get from students in response to questions posed.

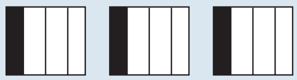
Let us look at the following classroom example related to the grade-3 focal point of "Developing an understanding of fractions and fraction equivalence" to show a teacher's use of questioning to focus learning on essential ideas and promote connections.

Teacher: Suppose there are two groups of students trying to divide brownies equally among themselves. In the first group, there are 3 brownies being shared by 4 students, and in the second group, there are 6 brownies being shared by 8 people. Do the students in each group get the same amount of brownie?

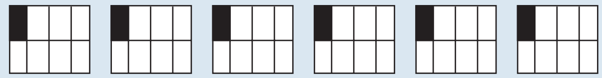
Student: No, the group with 4 people get more because there's only 4 students and 3 brownies, but 8 people only get to share 6 brownies.

Teacher: Can you make a picture or drawing to show me this situation?

Group 1: 3 brownies shared by 4 people



Group 2: 6 brownies shared by 8 people



Teacher: Do the people in Group 1 get the same amount as the people in Group 2?

Student: Yes.

Teacher: But, it looks like to me that the students in Group 1 got 3 pieces of brownie and the students in Group 2 got 6 pieces of brownie each.

Student: But the second group's brownie was divided into more pieces, and 2 of their pieces equal the same amount as one of the other group's pieces.

Teacher: Can you describe the amount of brownie that each person in Group 1 got as a fraction?

Student: 1 out of the four pieces, so 1/4.

Teacher: So each person got 1/4 of each of the three brownies? So how many 1/4's did each person get in the first group altogether?

Student: Each person got 3/4's.

Teacher: What about the second group? What fraction represents the amount of brownie that each person got?

Student: 1/8.

Teacher: OK, so each person got one of the eight pieces, or 1/8, from each brownie. And there's 6 brownies altogether, so each person got how many eighths of a brownie.

Student: 6/8

Teacher: OK, so Group 1 got 3/4 of a brownie each and Group 2 got 6/8 of a brownie each. But, as you said, that's the same amount, since two of Group's 2 pieces can be combined to equal the same amount as Group 1's pieces. So we know now that 3/4 is the same as, or equivalent to, 6/8.

Another student's approach to this same problem might go something like this:

Student 2: The students in Group 2 each get the same amount as in Group 1 because even though there's twice as many people in Group 2, there's also twice as many brownies. So the first four people in the group can share 3 of the brownies and the second four people in the group can share the other 3 brownies just like in Group 1.

Teacher: What an interesting way to look at the problem. You're right. Thinking in that same way, suppose we had 12 people instead of 8 in the second group, how many brownies would they have to share to get the same amount as the students in Group 1?

Student 2: That would be an additional 4 people, so you'd need an additional 3 brownies, or 9 brownies total.

Teacher: Let's put this information in a table to keep track of what we've done and see if we notice any patterns.

Student 2: OK.