CHAPTER

3

Using Curriculum to Develop Mathematical Promise in the Middle Grades

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- Gifted and talented middle-grades mathematics students in the United States have made minimal gains in the last ten years and need a more rigorous, challenging, and articulated curriculum to be able to compete on an international level.
- Gifted and talented students need an in-depth understanding of the middle-grades curriculum as outlined in the *Common Core K–12 Mathematics Standards* and NCTM's *Curriculum Focal Points.* The important mathematics outlined for middle school students in these sources is foundational for an understanding of advanced topics and should not be skipped in an attempt to move more quickly to high school topics.
- Curriculum for the development of mathematical talent in the middle grades needs to consider both students' mathematical abilities and their attitudes towards mathematics.
- Advanced, in-depth curriculum needs four components: creative and complex problem solving, connections within and across mathematical and other contexts, an inquiry-based approach, and appropriate pacing.
- Criteria for assessment of student work need to include an appraisal of the students' mathematical understanding and reasoning, communication, problem solving, and creativity.

In *Principles and Standards for School Mathematics* (NCTM 2000, p. 13), the National Council of Teachers of Mathematics asserts that equity in education includes support for exceptionally talented students:

> All students should have access to an excellent and equitable mathematics program that provides solid support for their learning and is responsive to their prior knowledge, intellectual strengths, and personal interests…. Students with special interests or exceptional talent in mathematics … must be nurtured and supported so that they have the opportunity and guidance to excel.

Exceptionally talented students' needs for support are perhaps especially great in the middle grades, both because of the inherent challenges of adolescence and because of the long history of curricular uncertainty in these grades.

Ever since the beginning of the middle school movement in the early 1980s, what constitutes an appropriate mathematics curriculum for the middle grades has been the subject of debate. International and national test scores have brought into the limelight our problems in developing competence, let alone excellence, in mathematics for our middle school students. This problem is exacerbated for students who are gifted in mathematics.

At the international level, the latest Trends in International Mathematics and Science Study (TIMSS 2008) indicates that although more than 40 percent of eighth graders in Singapore and other Asian countries scored at the most advanced level, only 6 percent of U.S. eighth graders scored at this level. Results from the National Assessment of Educational Progress (NAEP 2008) indicate that even though scores continue to increase, only 7 percent of eighth graders perform at the advanced level. It is at eighth grade that students are expected to use abstract thinking, a cornerstone of high-level mathematics. Moreover, in the period 2000–2007, while the bottom 10 percent of eighth graders showed solid and continued progress (+13 points) on the NAEP, the top 10 percent made minimal gains (only +5 points) (Loveless 2008). Thus, whether we look at international or national measures, our present system of mathematics education, although improving, is not serving the needs of our most capable students.

Why is it that our mathematically promising students are in reality the children who are repeatedly left behind in international and national comparisons? Cossey (1999) suggested that the results of countries other than the United States may reflect the fact that these nations have a more focused mathematics and science curriculum, with concepts covered in greater depth and with more opportunities to apply them. In contrast, the U.S. approach is evident in mathematics texts that cover a wide variety of topics much more superficially, under the assumption that they will be revisited again in future years.

This approach has resulted in a mathematics curriculum that has been labeled "a mile wide and an inch deep" (Schmidt, McKnight, and Raizen 1996). The middle school mathematics curriculum, in particular, has long been criticized for this approach. In fact, Assouline and Lupkowski-Shoplik (2005) share findings from the TIMSS 1998 study that show that the middle school math curriculum (grades 5–8) seems to be a weak link in the U.S. educational system. This curriculum was found to be far less challenging than curricula in other countries where algebra and geometry are included for all students rather than just honors students.

In 2009, the Common Core State Standards Initiative sought to increase the rigor of mathematics standards across the United States by developing a common core of state standards. Also in 2009, NCTM released "Guiding Principles for Mathematics Curriculum and Assessment," noting, "A curriculum is more than a collection of activities: It must be coherent, focused on important mathematics, and well articulated across the grades…. Any national mathematics curriculum must emphasize depth over breadth and must focus on the essential ideas and processes of mathematics" (NCTM 2009, p. 1). In addition to underscoring the necessity of depth in the mathematics curriculum, the draft College- and Career-Readiness Standards from the Common Core State Standards Initiative emphasize the need to develop proficient mathematics students who are experimenters and inventors, who think strategically, and who pursue mathematics beyond the classroom walls. As the draft states, "Encouraging these practices should be as much a goal of the mathematics curriculum as is teaching specific content topics and procedures" (Common Core State Standards Initiative 2009, p. 5).

Overview

In this chapter, we give an overview of the current state of the middle school mathematics curriculum as well as research-based information on how middle school students learn mathematics. This overview serves as a backdrop for issues surrounding developing mathematical talent in middle school students. To develop mathematical talent, we must first recognize the potential in students, and this potential is manifest in different degrees at different levels. In this chapter, we present a model of talent development that is two dimensional, focusing on abilities in and attitudes toward mathematics. Next, we discuss a model for advanced, in-depth curriculum for mathematically talented students. This model has four components: creative and complex problem solving, connections within and across mathematical and other content areas and contexts, an inquiry-based approach, and appropriate pacing. Consideration of all four components is necessary to provide the kind of rigorous curriculum that challenges students to think and act like practicing mathematicians. We then outline each of these components in more detail to give the reader an understanding of how it fits into creating an appropriate curriculum. Examples of sample problems, activities, and classroom vignettes are provided to show the curriculum components in operation. Finally, we discuss how to assess student work and thinking that includes high-level problem solving and creativity.

Development of Mathematical Talent

In determining appropriate curriculum for students, consideration of the level of mathematical talent that they display is important. As identified in the "Report of the NCTM Task Force on the Mathematically Promising" (Sheffield et al. 1995), mathematical promise is a function of four variables: ability, motivation, belief, and experience or opportunity. The report states, "These variables are not fixed and need to be developed so that success for these promising students can be maximized" (p. 310). The reality is that mathematically promising students exhibit a continuum of both attitudes and abilities that should be cultivated and extended (see the model in fig. 3.1). Thus, it is impossible to say that there is a specific mathematics curriculum that meets the needs of all talented students. However, regardless of where students with talent lie on this continuum, we want to provide curriculum and experiences that help them make continuous progress in developing their talent.

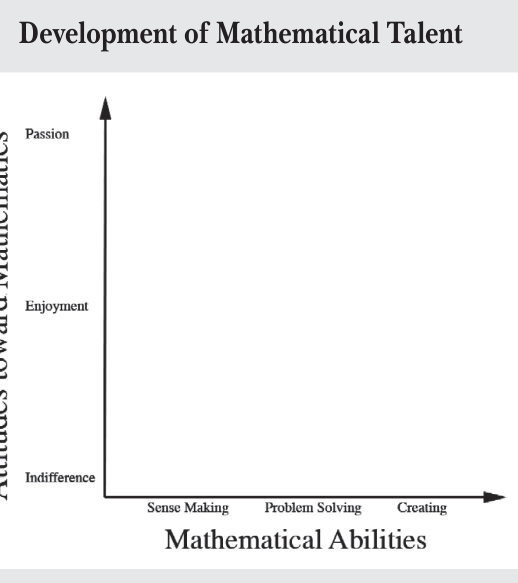

Fig. 3.1. A model for the development of mathematical talent

Mathematical ability in promising and talented students may begin with having a strong number sense, sometimes coupled with strong visual capabilities. Note that this trait, or combination of traits, is different from being fast or accurate at computation but can give students a strong basis for developing computational fluency. Using this number sense, possibly combined with spatial sense, mathematically talented and promising students make sense of the mathematics at hand, sometimes in an intuitive way, and at other times by integrating the new knowledge into their larger understanding of mathematics. To develop their talent, it is important that students build on their previous understanding, making sense of mathematics to help them solve both routine and non-routine problems. The process of problem solving itself has a hierarchy akin to Bloom's taxonomy. Students with talent should be given increasing levels of challenge, with problems calling for application, analysis or synthesis, and evaluation. Problems that require them to use mathematical understanding in real-life situations, compare and contrast mathematical concepts and ideas, and justify their thinking are examples of these hierarchical levels of Bloom's taxonomy. Students move back and forth between sense making and problem solving as they move up in this hierarchy. Moving beyond problem solving, talented students should be encouraged to pose new tasks, questions, and problems, and create new, unique solutions. In fact, these are the activities in which practicing mathematicians engage. Creativity provides both challenge and enjoyment for students with mathematical talent, and they should have opportunities to *play* with mathematics by creating new problems and unique, interesting solutions. This, in effect, is the way in which new mathematics is created, and providing these opportunities can offer an apprenticeship to a budding mathematician.

Some students are quite good at mathematics but apathetic about actually doing math for the sheer joy of learning. Regrettably, for many talented students the goal is simply getting through the required coursework. The phenomenon of a student who is highly successful at algebra or geometry in middle school but does not see this success as something to build on is not uncommon. Rather, such students often see their skill as a means to complete high school graduation requirements in mathematics as quickly as possible, so that they may stop taking mathematics. Some will even get university credit through Advanced Placement Calculus or an International Baccalaureate mathematics course. But then they are happy not to have to take a mathematics course at the university, because they have all the mathematics credits that they need to get a bachelor's degree. We cannot afford to let our best mathematics students think of mathematics as something to get out of the way as quickly as possible. Middle school is a critical time for students to develop an enjoyment in tackling interesting mathematical puzzles and problems as part of a comprehensive, well-articulated curriculum. The ultimate goal is to help as many students as possible become passionate about delving deeply into mathematical concepts and relationships.

Beliefs about Cognitive and Social Development of the Preadolescent

The middle school movement was in part generated by the emergence of the belief that learning plateaus in the early adolescent years and that middle schools need to pay more attention to the affective development of students while limiting their exposure to new content knowledge. The impact of earlier theories of brain functioning and Piagetian studies led some middle school educators to conclude that middle school students are still at the concrete level of thinking and are not capable of higher, more abstract levels of thinking. Thus, they believed that students at this age should practice existing skills and procedures rather than learn new ones (Alexander and George 1981; Toepfer 1990, 1992). This was, and to some extent still is, evident in the middle school mathematics curriculum. *Principles and Standards* (NCTM 2000) notes that some middle school curricula have a "preoccupation with number" (p. 211), even though number concepts and skills are studied in depth in elementary school. In contrast, the development of algebraic and geometric concepts, so important for the study of future mathematics, is limited and often not emphasized in some middle school programs. This certainly does not benefit most middle school students and is definitely harmful to mathematically promising students, who are eager and ready to delve into new and challenging mathematics.

Compounding a repetitive curriculum is the current national emphasis on standardized test scores and the accompanying instructional focus on students who are not meeting grade-level goals. Perhaps even more detrimental is a goal of a single level of proficiency for all students as mandated by the No Child Left Behind Act of 2001. This, in fact, totally ignores the continuous progress of students who have already achieved proficiency.

Today, brain plasticity, the capacity of the brain to change with any type of learning, is well documented. "We now know that the human brain actually maintains an amazing plasticity throughout life. We can literally grow new neural connections with stimulation, even as we age. This fact means nearly any learner can increase their intelligence, without limits, using proper enrichment" (Jensen 2000, p. 149). We know that the brain never stops changing and adjusting—developing and pruning connections, organizing and reorganizing in response to experiences—even growing new neurons.

According to Carol Dweck (2006), a social psychologist and professor at Stanford University, the difference between achieving and not achieving depends on whether a person believes that talent is something inherent that needs to be demonstrated or something that needs to be developed. Seventh-grade students who were struggling with mathematics and were taught a "growth mindset" about intelligence—the belief that intelligence is a potential that can be developed—significantly improved their mathematical performance after they learned about the plasticity of the brain. It is critical that all students realize that they can become far more adept at mathematical reasoning and that the commonly offered excuse, "I don't have a mathematical brain," is not valid.

In *Principles and Standards for School Mathematics* (2000), and more recently, in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006), NCTM outlines a curriculum that moves beyond a repetition of skills and procedures and introduces important new concepts in number, algebra, geometry, measurement, and data analysis and probability for all students. *Principles and Standards* emphasizes the importance of learning significant new mathematics in the middle grades: "Students are expected to learn serious, substantive mathematics in classrooms in which the emphasis is on thoughtful engagement and meaningful learning" (NCTM 2000, p. 213). The new reform curricula have used NCTM's Standards as guidelines in developing material to be studied by all students. Unfortunately, "all students" does not always include gifted and talented middle school students. Differentiation for our gifted students in terms of increased rigor, depth, and complexity is often given short shrift. "Enrichment," as outlined in many textbooks, consists of worksheets that students are expected to do on their own. But are students who can complete these assignments entirely on their own being challenged appropriately? Do these worksheets give students the type of experiences that develop future mathematicians?

In fact, mathematically gifted students are at special risk when a challenging and rigorous curriculum is not offered to them. In his studies of talent development, Bloom (1985) found that highly productive and talented adults in a variety of fields actually began to seek out challenge in content areas and to develop their own high expectations and even intense passion and focus for their work during the middle school years. If mathematically talented students are presented with a curriculum in which they already understand the concepts, do not need to practice the skills, and are bored by the pace, how can we expect them to develop a love of mathematics, let alone learn new and exciting mathematics? It is imperative that we raise the bar and offer these students an intellectually rigorous and stimulating curriculum that will excite them and encourage them to continue their studies in mathematics and eventually enter careers in the field or in another field that is closely related.

Criteria for a Curriculum for Mathematically Talented Students

To create an advanced, in-depth curriculum appropriate for talented students, we propose a model with four components:

1. Creative and complex problem solving
2. Connections within and across mathematical and other content areas and across a wide range of contexts
3. An inquiry-based approach that focuses on processes used by mathematicians
4. Appropriate pacing

The model appears in figure 3.2.

All four components of the model are equally important, serving as the foundational "legs" that support a curriculum that is advanced and in depth. This type of curriculum is necessary to provide appropriate challenge, rigor, and enjoyment for mathematically talented students. The curriculum should be integrated and coherent, encompassing all the content strands identified in *Principles and Standards* and focusing on all the areas of emphasis targeted in *Curriculum Focal Points.* With a focused, coherent curriculum, students have "opportunities to explore topics in depth, in the context of related content and connected applications, thus developing more robust mathematical understandings" (NCTM 2006, p. 4).

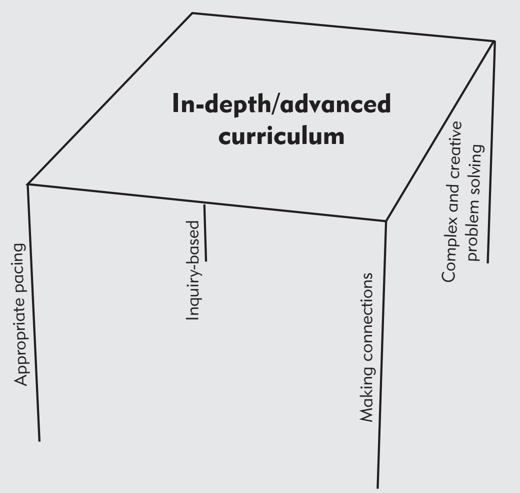

Fig. 3.2. A model for the development of an advanced, in-depth curriculum

In addition to aiding in developing a rigorous curriculum, this model can help in ensuring that students are afforded opportunities to use thought processes akin to those of practicing mathematicians—an instructional strategy recommended by leading experts in gifted education (Renzulli et al. 2000; Tomlinson et al. 2002). Each of these components can be outlined in more detail to provide an understanding of the importance of its contribution to a quality curriculum for talented students.

Creative and complex problem solving

We often teach students that there are four neat steps of problem solving:

1. Understand the problem
2. Determine a strategy to solve the problem
3. Solve the problem by carrying out the strategy
4. Check

However, mathematicians will tell you that when they are struggling with deeper mathematical problems, they do not generally find the solutions by using those four steps. Because a mathematical problem is often defined as something for which we do not have an immediate method of solution, telling students that they should determine and carry out a single solution strategy is a method that might work on exercises or "word problems" but may not be helpful for deeper problems, where creative or complex solutions are required. For these mathematical problems, a much richer heuristic that emphasizes the integration of processes is required. Consider the model in figure 3.3 (Sheffield 2003), for example.

Students who use this model in solving and posing problems may start at any point on the diagram and proceed in any order that makes sense to them. They might do the following:

- Relate the problem to other problems that they have solved, making connections to prior mathematical concepts and perhaps to applications in different contexts.

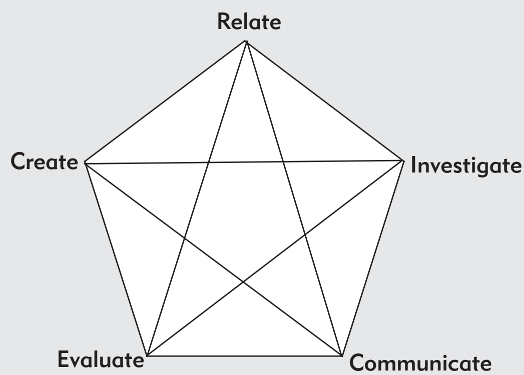

Fig. 3.3. A heuristic for complex and creative problem solving (Sheffield 2003)

- Investigate the problem, perhaps using a variety of strategies and representations.
- Evaluate their findings. What worked, and what did not? Refine and build on aspects that did not seem to work well. Extend and dig more deeply into successful solutions.
- Communicate with peers and others. Discuss strategies as well as results. Can unsuccessful strategies be cultivated or tweaked? Can successful strategies be improved? Is there a more efficient or elegant way to solve the problem? Are there other ways to think about and develop the problem?
- Create new questions and pose new problems to explore as well as create unique and original solutions.

As they begin to think like mathematicians, students might change the order of the steps and the questions that they ask in starting to explore a problem. Throughout the problem-solving process, students should be evaluating their work, making connections, asking questions, communicating results, and creating new problems to investigate.

Connections within and across mathematical and other content areas and contexts

When considering possibilities for students who are mathematically promising, we have customarily asked, "Should we enrich their learning with greater breadth, adding new topics for them to explore beyond the regular curriculum, or should we accelerate them to the next grade level or math course?" This question leads back to the model for the development of mathematical talent (fig. 3.1). Students who are above average in making sense of mathematics and are good problem solvers need to move on to posing new problems and creating mathematics that is unfamiliar to them. This does not mean that we should enrich their mathematics by adding new, disconnected topics or accelerate their mathematics by moving them quickly through a shallow curriculum that emphasizes memorization and basic skills. Genuine enrichment and acceleration do something different. As Schiever and Maker (2003) point out, "without both acceleration and enrichment, more is simply more" (p. 168). They state that the curriculum offered to talented students needs both acceleration and enrichment to such an extent that "more" becomes "different" and the curriculum becomes "qualitatively differentiated" (p. 167). Our model indicates that students who progress along the continuum of talent should have this qualitatively differentiated curriculum. Such a curriculum will be characterized by an increase in the degree of rigorous content and the opportunity to create new questions, new strategies, and new mathematics, along with appropriate pacing commensurate with the students' level of giftedness. And herein lies a key for truly developing mathematical talent and promise. The Common Core State Mathematics Standards (2009) include exemplary performance tasks that are grouped into four categories: exercises, structured tasks, substantial tasks, and target tasks. Target tasks include exemplary problems for challenging mathematically promising students. These tasks "require students to integrate strategic, tactical, and technical skills through connections within mathematics and to the problem context. Some target tasks allow good responses in only 10–20 minutes, though many can stimulate hours of valuable investigation" (p. 2).

An accelerated program that moves middle school students into the next grade-level textbook or high school course designed for all math students is not appropriate for helping students develop their mathematical talents to full potential. In addition, moving rapidly through these courses so that students can take calculus in high school denies students the opportunity to delve deeply into mathematical concepts and does our gifted students a disservice. In fact, it is apparent that this approach has begun to backfire. Rather than instill a love for mathematics and a desire to continue studying mathematics in college, it seems to turn students off from mathematics. We have seen the percentage of students who are enrolled in mathematics classes at four-year institutions steadily decrease relative to the total number of students enrolled in these institutions. In 2005, only a meager 1.02 percent of U.S. college students were enrolled in advanced-level mathematics courses (Bressoud 2009). Clearly, the current way in which we serve our gifted students is not leading them to pursue mathematics as a field of interest or inquiry.

Instead of merely moving rapidly through a shallow curriculum or adding unrelated topics to a curriculum that is "a mile wide and an inch deep," students who are given the opportunity for creative and complex problem solving according to the heuristic discussed earlier (see fig. 3.3) will explore a variety of topics that are often considered "enrichment" or "advanced." Yet, they will also have a solid foundation for connecting this mathematical understanding seamlessly and at a deeper level as they develop into mature mathematicians. They will find that new mathematical concepts are tied to existing mathematical understanding, sometimes with real-world applications or connections to other subject areas. Often, they will discover links among concepts by using a variety of models and representations from a range of mathematical strands—from number theory and computation to algebra, geometry, measurement, and data analysis and probability. This development of mathematical understanding aligns with the recommendations that NCTM has consistently made over time. In *Principles and Standards,* NCTM asserts, “Students’ understanding of foundational algebraic and geometric ideas should be developed through extended experience over all three years in the middle grades and across a broad range of mathematics content, including statistics, number, and measurement” (NCTM 2000, p. 213). In *Curriculum Focal Points,* NCTM continues to emphasize the importance of a coherent, fully articulated curriculum for advanced middle school students:

> Those whose programs offer an algebra course in grade 8 (or earlier) should consider including the curriculum focal points that this framework calls for in grade 8 in grade 6 or grade 7. Alternatively, these topics could be incorporated into the high school program. Either way, curricula would not omit the important content that the grade 7 and grade 8 focal points offer students in preparation for algebra and for their long-term mathematical knowledge. (NCTM 2006, p. 10)

An example from algebra

In a typical algebra 1 program, *slope* is introduced with a definition and the formula, and students are asked to practice finding the slope of a line by working with several computational examples that give two points. The idea of a slope is typically presented in one or two lessons. In contrast, Project M^3: Mentoring Mathematical Minds (Gavin et al. 2008), a new research-based curriculum for talent development, guides mathematically promising students in grades 5 and 6 in exploring algebra concepts related to analyzing change over a six-week period. Set in the motivational context of exploring seemingly wacky world records from the Guinness Book of World Records, the curriculum encourages students to focus on analyzing change in situations from a qualitative perspective. This enables them to look more globally at what is happening throughout the situation. They analyze change, including the rate of change, in various situations and learn how to represent it graphically and describe it as the slope of a line. They study slope, *y*-intercept, and points of intersection in the context of change. Rather than learning the meaning of slope as the formula

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1},$$

students gain an intuitive and conceptual understanding of slope by comparing graphs, tables, and situations, making predictions and then testing those predictions, and creating new situations. After exploring, they are asked to reflect, thinking deeply about the mathematics that they have used or discovered.

These experiences enable the students to make connections to the bigger idea of change in the study of continuous mathematics and to develop mathematical understanding by using processes akin to those of practicing mathematicians. In a study measuring student achievement with the Project M^3 curriculum, Gavin and colleagues (2009) found that mathematically promising students studying this curriculum made highly significant gains in achievement as compared to a group of students of like ability in the same schools.

An inquiry-based approach

The National Council of Teachers of Mathematics (1989, 2000, 2006) promotes inquiry-based curriculum for all students. Indeed, this is the foundation on which the NCTM Content and Process Standards rest and the Curriculum Focal Points were developed. The thought processes that are especially nurtured by experts in the field of gifted and talented education are consistent with those that NCTM promotes in the Process Standards and include critical thinking, creative thinking, and problem solving. However, Tomlinson (1994) cautions that although these processes are laudable goals to develop in all students, it is the *level* at which and *degree* to which students use them that are critical to developing talent. Gifted students need to be encouraged to engage in these processes more frequently and at much higher levels than other students. Challenging and provocative questions and mathematical investigations can provide the encouragement that students need. VanTassel-Baska and Brown (2007) emphasize that inquiry-based strategies that allow students to engage in making choices and that involve complex, creative problem solving and decision making were central to each of nine research-based gifted education curriculum models that they analyzed. They state that emphasis on motivation and student engagement provides an important connection between teacher and learner, and they suggest that this connection may account for greater gains as motivation on the part of both the teacher and the student grow.

In addition, both Usiskin (1987) and Sheffield (1999) state that inquiry-based learning in mathematics, using problem-based strategies rather than simply automatic recall focused on drill and practice, leads to a much deeper understanding among gifted mathematicians. Indeed, research suggests that any review or practice of skills and procedures be spaced rather than concentrated for greater retention of understanding (Dempster 1988; Lupkowski-Shoplik and Assouline 1994). The fact is that talented students need much less