THE CAKE CONTEST

Students used a pinch of this (a 3D printer and geometry software) and a cup of that (various volume formulas) to complete a tiered task.

Colleen Haberern

"Wait, that's what our cake looks like?" seventh-grader Alyssa exclaimed. Her group had designed a three-tier cake that met all the requirements for the Cake Contest (see **fig. 1**). But when they saw the virtual model, it looked nothing like they had imagined. The cake was 36 inches tall but only 8 inches in diameter at its base. It looked like it was going to topple over! When Alyssa's group members asked if they could revise their calculations, I agreed in a heartbeat. My students were asking if they could do more mathematics! It was a math teacher's dream come true.

PRESENTING PROBLEM-BASED TASKS

"Students learn best when they are presented with academically challenging work that focuses on sense making and problem solving as well as skill building" (NRC 2001, p. 335). With the adoption of the

Common Core State Standards for Mathematics (CCSSM), many teachers are changing their classroom structure from teacher-directed to student-centered. When I began designing and using problem-based tasks, like the Cake Contest, I saw a drastic improvement in student engagement and problem-solving skills.

A *problem-based task*, which is also called a *complex task*, is a three-phase lesson (Van de Walle et al. 2013). First, the teacher presents a problem to the whole class in the launch, which is designed to engage students and provide a context for learning. Second, the students work, usually in small groups, to solve the task. Third, the teacher leads a whole-class discussion to help students make connections between their solutions and strengthen their understanding of the mathematical concepts.

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LAUNCHING THE TASK

To engage my students, I presented a YouTube video showing cake designs from TLC's "The Cake Boss," with musical accompaniment from the song "Sugar, Sugar" by The Archies. As students chatted about the creative decorations, I asked them to focus on the structure of the cakes. After I had presented the task, which was to design a cake to fit a set of conditions (see **fig. 1**), the students realized that only the structure would matter in the Cake Contest. Since π would need to be part of their calculations, students were allowed to round to the nearest hundredth because this approximation would have a negligible difference on the size and shape of their cakes. Technology would also play an important role because each group would create a virtual model using a 3D printer and software called Tinkercad®. Of course, every group wanted to create the "best design" because the winning group in each class would get to decorate a full-size model of their cake.

LEARNING THE POWER OF PROBLEM SOLVING

Each group began discussing whether to use rectangular prisms or cylinders or a combination of both. Students were familiar with these shapes, having derived and used volume formulas for both prisms and cylinders in a previous geometry unit. They sketched designs showing the number of tiers and the overall shape of the cakes. Did they know that the design they created would determine the difficulty of the mathematics to follow? If so, it did not prevent groups from being creative. They wanted to win!

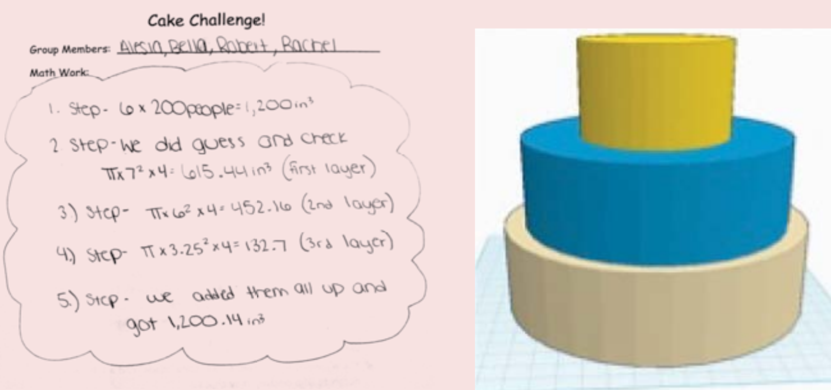
In essence, I did not give the students a problem to solve; each group created its own. Different strategies were needed to determine the exact dimensions of the cakes required to

Fig. 1 Students were given these criteria when designing their cakes.

Design a cake that meets the following conditions:

- A serving size is 6 cubic inches.
- The whole cake must serve between 180 and 200 people.
- The cake must have at least two tiers.
- The tiers must be rectangular or cylindrical.
- Each tier must be the same height.
- The cake must be visually appealing!

Fig. 2 A guess-and-check strategy was used by this group to meet the requirements of the Cake Contest.



Cake Challenge!

Group Members: Alesia, Bella, Robert, Rachel

Math Work:

1. Step- 6 x 200people = 1,200 in³

2 Step- we did guess and check
π x 7² x 4 = 615.44 in³ (first layer)

3) Step- π x 6² x 4 = 452.16 (2nd layer)

4) Step- π x 3.25² x 4 = 132.7 (3rd layer)

5) Step- we added them all up and got 1,200.14 in³

Every group wanted to create the "best design" because the winning group would get to decorate a full-size model of their cake.

create the virtual model. Groups used guess and check, worked backward, or made estimations, or they used a combination of strategies. The only stipulation was that they had forty minutes to figure out the dimensions.

CCSSM emphasizes real-world problem solving. In grades 5 and 6, students find the volume of cubes and other rectangular prisms, then they progress in grade 8 to additional solids, such as cylinders. Various geometry standards are incorporated, depending on each group's design. The Cake Contest also incorporates the Standards for Mathematical Practice. Students are encouraged to "make sense of problems and persevere in solving them"; in the second part of the lesson, they will use computer software to "model with mathematics" (CCSSI 2010, pp. 6–7).

Fig. 3 When the students realized that their cake would be too small, they added a new bottom tier instead of adding a different top tier.

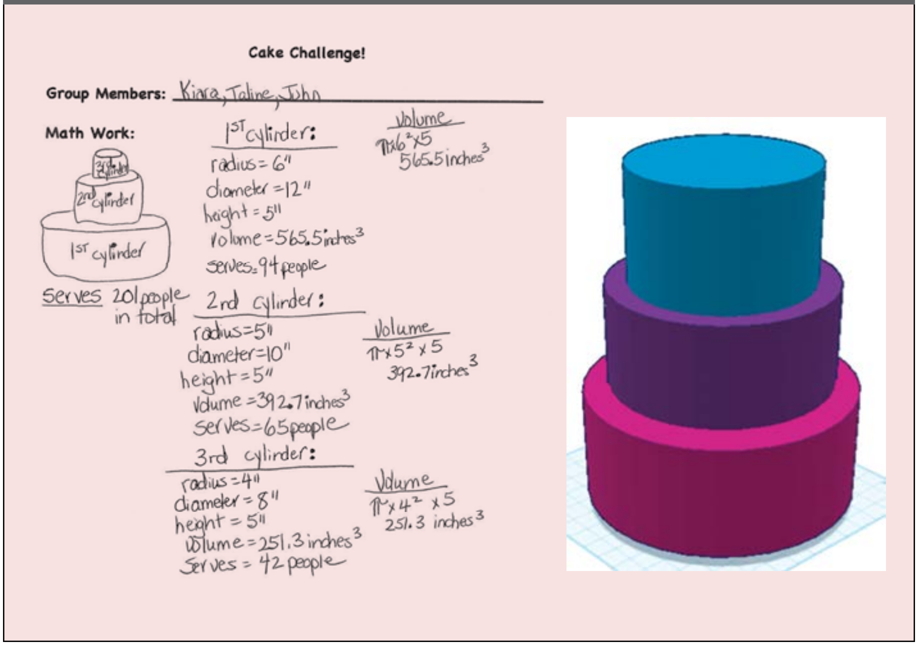
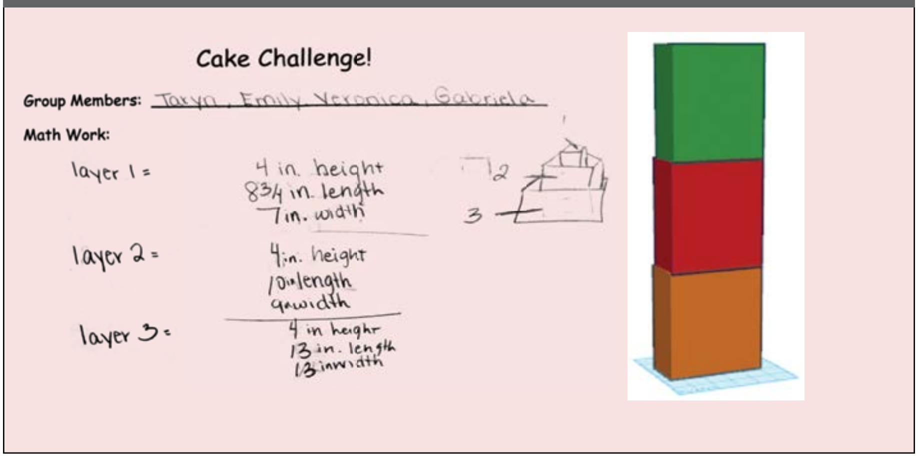


Fig. 4 It was surmised that using rectangular prisms would make the work of cake design less challenging. This treatment led to some in-depth conversations about mathematics.



Using Guess and Check

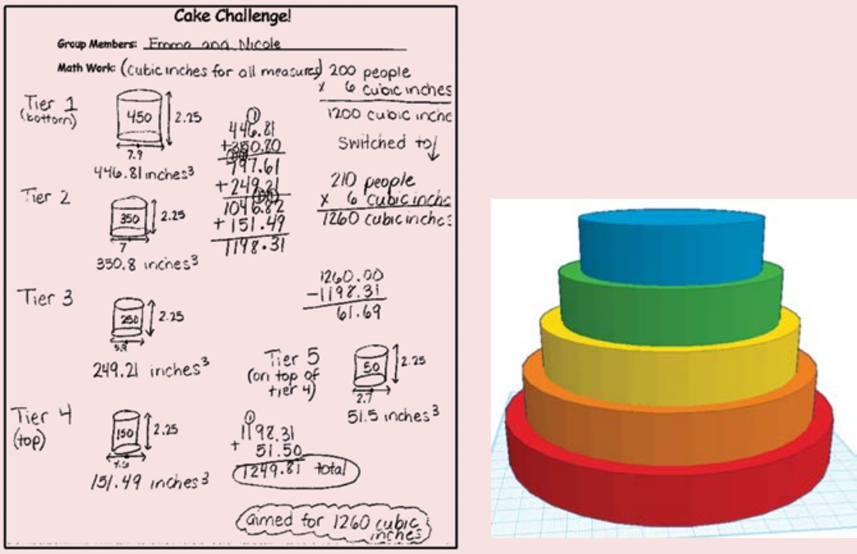
One group began the task by calculating the total volume of cake needed for 200 people by multiplying by 6. Then they sketched a cake with 3 cylindrical tiers, each with a height of 4 inches. The group used calculators and substituted values for the radius of each tier until the sum was approximately 1200 in.^{3}.

In **figure 2**, the radius of the top tier is much smaller than those of the other 2 tiers. Perhaps students were too focused on limiting the volume to 1200 in.^{3} and did not realize that they could add another 120 in.^{3} of cake by increasing the number of people to 220.

Making Modifications

Another group's guess-and-check strategy looked much different, although they also created a cake with 3 cylindrical tiers. Instead of focusing on the volume of the entire cake, the students divided the volume of each tier by 6 to determine the number of people that each tier would serve. After creating the bottom and middle tiers, they realized that the cake would be too small. One student

Fig. 5 A working-backward strategy was used to create a cake with 5 cylindrical tiers.



suggested renaming those tiers as the middle and top, then creating a larger tier for the bottom. Since the radius of each tier was 1 inch less than the tier beneath it, the cake had the balanced look apparent in **figure 3**.

Working Backward

A third group used a working-backward strategy to create a cake made from 3 rectangular prisms. They began by multiplying 180 people by 6 cubic inches per person to get a total volume of 1080 cubic inches. Then they divided by 3 to find the average volume of each tier. The discussion below explores their thought processes:

Taryn: So if we make one 350, one 360, and one 370, that adds to 1080. Now we just have to figure out our dimensions.

Emily: Let's find the dimensions for the 360 tier first because that seems like the easiest to do. . . . If we divided 360 by 3, we get 120, but there has got to be an easier way. Maybe something by 10 . . . 9 × 4 × 10 equals 360. Right?

Gabriella: Yeah.

Emily: I say we should make 10 the height because it is a number they all have in common.

Taryn: How?

Emily: They all end in 0, so they are all multiples of 10.

The group questioned whether or not a 30 inch tall cake was acceptable. They concluded that it was reasonable because the cakes on the "Cake Boss" videos are quite large. Then they went back to work calculating the dimensions of the other tiers. Their work appears in **figure 4**.

Taryn: The top one is 35, so we could do 7 and 5.

Emily: Hey, that would be hanging off the cake! Is there anything else that we could multiply by? Or we can have a fraction of an inch on the side.

Emily realized that they had to consider how the tiers would fit

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I thought it was impressive for seventh graders to devise this calculation, especially considering that formal algebra had not been covered at that point in the school year.

Fig. 6 Group members estimated the dimensions of a real cake, aided by a pastry chef's daughter.

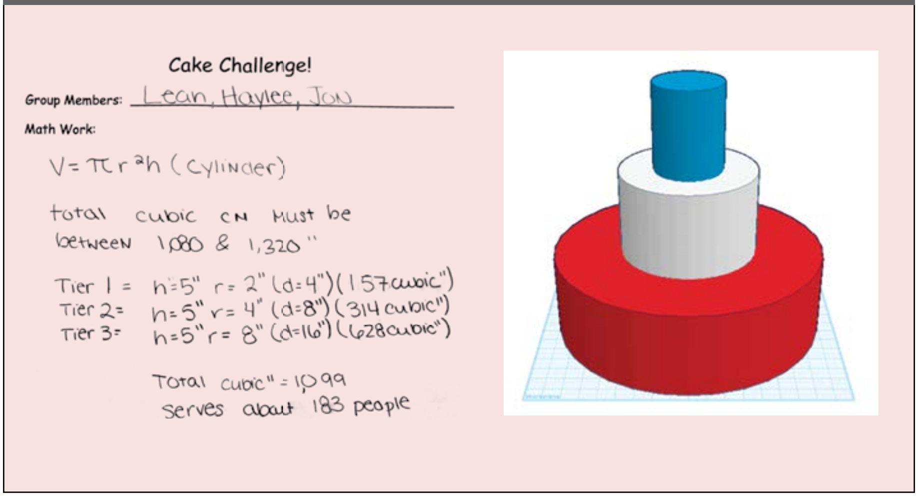
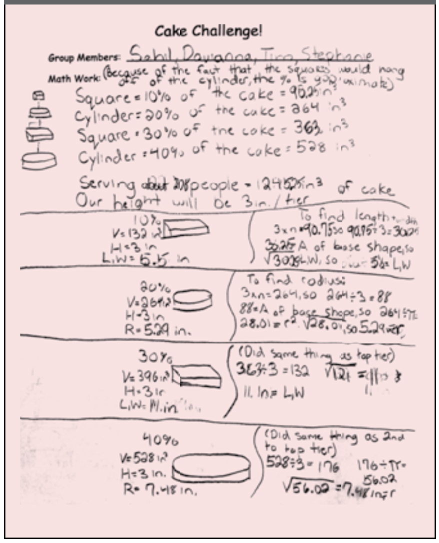


Fig. 7 This group used both cylinders and "square" rectangular prisms in their design. (See their virtual image in **fig. 10.**)



together to create the cake. She knew that the length and width of each tier must be less than or equal to the corresponding dimensions beneath it. Since there are no other integral factors of 35 between 4 and 9, the group decided to make the width of each tier constant, then use division to calculate the missing lengths.

Creating a Need for Algebra

Working backward looked different for the group that created a cake with 5 cylindrical tiers (see **fig. 5**). They originally wanted 4 tiers and began by calculating the total volume of cake required for 200 people then divided by 4. They started with 450 cubic inches on the bottom tier and decreased the volume of each tier by 100 in.^{3}. This constant change in size created a roughly constant slope along the profile of the cake.

Once students determined the volume of each tier, the group decided that the height of each tier would be 2.25 inches. At this point, the group worked backward to calculate the radius of each tier. The students knew that $\pi r^2 \times 2.25 = 450$, so they used calculators to divide 450 by 2.25π, then took the square root to find the radius of the base. Although we had previously covered square roots and cube roots, I thought it was impressive for seventh graders to devise this calculation, especially considering that formal algebra had not been covered at that point in the school year. They used the same process to find the radius of the other 3 tiers.

The students checked their work by substituting the radius and height into the formula for the volume of a cylinder. They added to find the total volume of the cake and realized that they could add another tier. That turned out to have been a smart choice because this group was voted "best design" in their class.

Using Estimation

As I checked in on another group, I could see that they used "ballpark estimation" to find the minimum and maximum volume of cake. Because all their dimensions were whole numbers, I assumed that they had used guess and check. I asked the group to tell me about the process.

"I just thought about how big a real cake would be," explained Leah, holding out her hands to approximate the size. The other group members showed me how they used rulers to measure the height of the tier that Leah was representing with her hands. They had measured all three tiers, and the total volume fell within their range on the first try. Apparently, Leah's mother was a chef and often made wedding cakes at home. (See **fig. 6**.)

Employing Multiple Math Concepts

The other winning group used both rectangular prisms and cylinders in their design. They chose to use the maximum number of people, so that they would have the greatest volume of cake, which was 1320 in.^{3}. A student suggested that the 4 tiers, shown in **figure 7**, should contain 10 percent, 20 percent, 30 percent, and 40 percent, respectively, of the total volume of cake to create a smooth look. They multiplied the decimal form of each percentage by 1320 in.^{3} to calculate the volume of each tier.

The students thought it would be easier to find the dimensions of the rectangular prisms versus the cylinders, but they were unsure how to proceed. The group wanted the top and bottom faces of the prisms to be

Fig. 8 Alyssa's group made last-minute revisions to their cake.

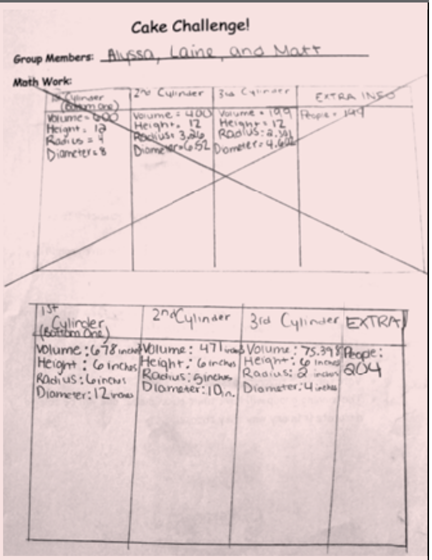


Fig. 9 Reducing the height of each tier made the original cake (a) more realistic (b).

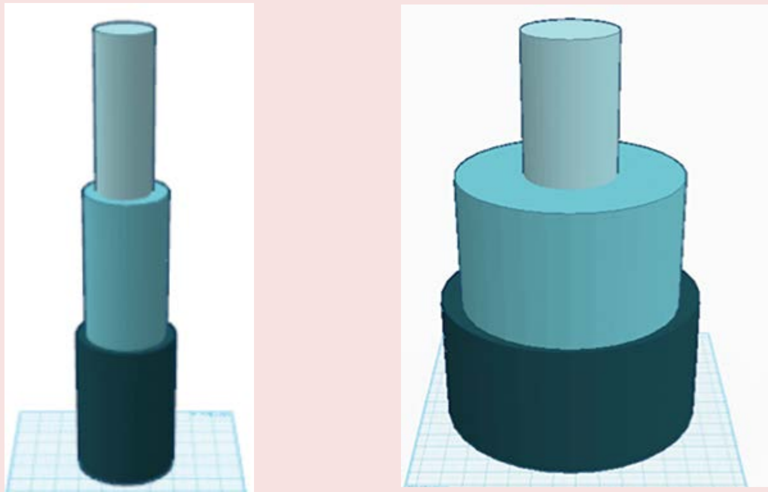
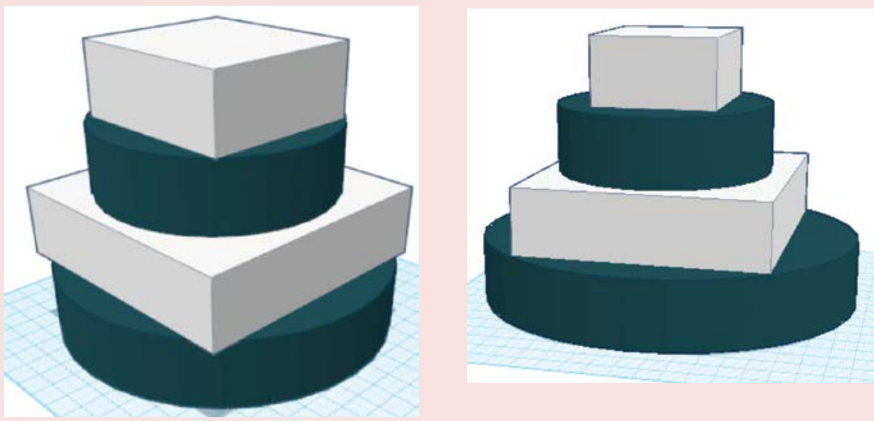


Fig. 10 In this design, the corners of the "square" tiers were hanging over the edges of the cylinders. This group corrected their error, and their cake was voted "best design."



square, which added to the difficulty level. After a short discussion, one student suggested taking the cube root of the volume of both rectangular prisms, giving side lengths of approximately 5.09 inches for the top tier and 7.34 inches for the third tier.

At this point, the group asked me for help with the cylinders. I reminded them that the Cake Contest rules stated that all tiers had to be the same height. Not only did my comment help the students realize that the cubes they created were incorrect, it also gave them a starting point for the cylinders.

The students chose a height of 3 inches for each tier. They divided the volume of the large square tier by 3 to get 44, which was the area of the base of the rectangular prism. Then they took the square root of 44 to find the length and width. They repeated the process for the top tier. To find the dimensions of the cylinders, they divided the volume by the height to get the area of the circular faces. Next, they divided by π to find the radius squared. By taking the square root of that number, they were able to determine the length of the radius.

After working through obstacles to find the dimensions, the group felt a sense of accomplishment. However, they would face yet another challenge when they created a virtual model of their cake.

VIRTUAL MODELS

On the second day of the Cake Contest, students used computer-aided design (CAD) software, called Tinkercad, to create virtual models. The students had no prior experience using this software but quickly learned after viewing Tinkercad's video tutorial. When "modeling with mathematics," students often draw diagrams as they imagine them to look. Using the Tinkercad software, students were able to see what their cakes would actually look like. It became a powerful learning tool.

After working through obstacles to find the dimensions, the group felt a sense of accomplishment.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LEARNING FROM MISTAKES

Two groups did not like what the Tinkercad software generated, so they asked me if they could make changes. Alyssa's group originally designed tiers with a height of 12 inches. They gradually decreased the volume of each tier from 600 to 400 to 199 in^3. They divided by the height, divided by π, and took the square root to calculate the radius of each tier. The virtual image was much taller and narrower than they expected, so they reduced the height of each tier from 12 inches to 6 inches. In turn, they increased the radius of each tier to keep the volume within the given range, as seen in **figure 8**.

Perhaps because they were running short on time in the contest, the group did not continue to use the same strategy. Instead they selected a radius for each tier and calculated its volume. So as to keep the total number of people served close to 200, they made the top tier much smaller. This last-minute correction made the cake more stable but changed the appearance to a less even look, as shown in **figure 9**.

Overhanging Tiers

When the group that alternated cylindrical and "square" tiers created their virtual cake image on Tinkercad, they noticed that the corners of the "square" tiers were hanging over the edges of the cylinders (see **fig. 10**). They had planned carefully to ensure that the length of each square was less than the diameter of the cylinder beneath it. They did not understand the problem, so they called me over for help.

I told students that they had not taken into account the fact that the diameter is the widest part of a circle but that the corners of the "square" tiers were overlapping a narrower part of the circle. One student asked indignantly, "How were we supposed to know that?" To answer, I explained that virtual models serve a purpose. If there is an error in the design, it could be corrected. He remembered that they had made a cake with the greatest volume, so they reduced the length and width of the rectangular prisms to make them fit on the cylinders.

Differentiating Instruction

This problem-based task was completed by seventh-grade students working in groups of three or four. The student work shown was selected from two advanced classes with approximately 20 students in each class. However, this task can be differentiated and used with other grade levels or abilities; see the **sidebar** on the next page.

When presented with an open-ended problem like the Cake Contest, students are able to choose an appropriate difficulty level. Even though I required a minimum of two tiers, all groups made a cake with three or more tiers. I heard one student suggest using two tiers but her group convinced her that adding an extra one could help them win. In addition, eight out of eleven groups included cylinders, even though the formula is more difficult than the formula for rectangular prisms. I structured the task so that the groups could not produce a cake design that they did not have the ability to determine dimensions for.

FINDING CONNECTIONS

In the past, I taught mathematical concepts in isolation by focusing on one objective at a time. Having

Suggestions for Differentiation

These suggestions for making the task more or less challenging for various ability levels will help this lesson become accessible for all students.

1. Vary the number of people.
2. Allow a wider (or narrower) range.
3. Assign a height for each tier.
4. Change the number of tiers required.
5. Assign the types of solids to be used.

Let's Chat about the Cake Contest

On Wednesday, January 18, 2017, at 9:00 p.m. EST,

we will expand on "The Cake Contest" (pp. 274–82), by Colleen Haberern. Join us at #MTMSchat.

We will also Storify the conversation for those who cannot join us live. Our monthly chats will always fall on the third Wednesday of the month.

been taught this way as a student, I only recently recognized how many connections there are among mathematical concepts. Now one of my goals is to help my students make those connections by combining various areas of mathematics in one problem-based task. "Problem-based learning (PBL) works well with all students, making its strategies ideal for heterogeneous classrooms where students with mixed abilities can pool their talents collaboratively to invent a solution. . . . By allowing children to direct their own activities and by giving them greater responsibilities, teachers show them how to challenge themselves and learn on their own" (Delisle 1997, p. 7).

The Common Core State Standards Writing Team stated, "Problems involving areas and volumes extend previous work and provide a context for developing and using equations" (2011, p. 18). Although this was a geometry task, my students used measurement, estimation, percentages, square roots, and algebra. They are beginning to understand the need for various types of mathematics. Tasks like the Cake Contest help reinforce concepts previously learned and provide a real purpose for using them.

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