
I noticed...
I disagree...
I think it could be...

PROMOTING MATHEMATICAL ARGUMENTATION

These evidence-based instructional strategies can lead to deeper mathematical conversations in upper elementary school classrooms.

Chepina Rumsey and Cynthia W. Langrall

The Standards for Mathematical Practice (SMP) in the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) highlight the mathematical habits that educators should be fostering in mathematics classrooms throughout K–grade 12 education. That argumentation and discourse are important components of developing mathematically proficient students has been well established, and this fits well with SMP 3, which states that students will *construct viable arguments and critique the reasoning of others* (CCSSI 2010, p. 6). Given that this practice is essential, how do teachers effectively incorporate mathematical argumentation into their upper elementary-level lessons? What does this practice look like, and what can teachers expect from students who have had minimal experience with this form of instruction? How do teachers strategically embed argumentation into the appropriate mathematical content? We address these questions as we present evidence-based instructional strategies for promoting argumentation.

FROM LEFT TO RIGHT: RON CHAPPLE STOCK/THINKSTOCK, RIDOFRANZ/THINKSTOCK, 3BUGSMOM/THINKSTOCK, AMOS MORGAN/THINKSTOCK

Mathematical argumentation

Many opportunities for discussion and communication take place in most classrooms: Students may share a computational answer, disagree with an answer, list the steps in a procedure, explain a solution strategy, compare two strategies, or notice a pattern. Argumentation—in mathematics and other subject areas—goes beyond these types of communication. We view *mathematical argumentation* as a process of dynamic social discourse for discovering new mathematical ideas and convincing others that a claim is true. Within an instructional setting, justifications are part of mathematical arguments because students provide evidence and reasoning to convince others that their claim is valid. Sometimes students base their claims on generalizations or patterns that they notice. With this process, mathematical authority and ownership shift from the textbook or teacher to the community of learners. We have found that students in the elementary grades are able to look at patterns and conjecture about mathematical ideas; develop mathematical claims; justify, critique, and challenge claims; and modify their claims on the basis of feedback from others (Rumsey 2013). Moreover, the inclusion of instruction promoting mathematical argumentation can positively affect the development of students' mathematical understanding (Rumsey 2012).

TABLE 1

The authors identified these general instructional strategies to effectively promote students' use of argumentation within the context of exploring the arithmetic properties.

Instructional strategies and sample tasks	
Instructional strategy	**Sample task or question**
Provide language supports	Make a claim and ask students to complete one of three language frame statements.
Discuss rich, familiar content	"What do you know about even and odd numbers?"
Specify conditions	$A \times \_\_\_ = B \times \_\_\_$
Introduce false claims	"Every time you multiply two numbers, you are always going to get an even number as the product."
Manipulate familiar content to be unfamiliar	$(a + b) + c = a + (b + c)$ $(a - b) + c = a - (b + c)$

WE AIMED TO STRENGTHEN STUDENTS' SKILLS OF ARGUMENTATION AS A MATHEMATICAL PRACTICE, WHILE DEEPENING STUDENTS' UNDERSTANDING OF MATHEMATICAL CONTENT RELATED TO THE ARITHMETIC PROPERTIES.

Background and context

We have drawn the ideas in this article from a research project in which the first author taught an eight-lesson unit of instruction to a class of fourth-grade students, emphasizing the development of students' mathematical argumentation skills. As with all mathematical practices, mathematical argumentation exists in the service of investigating mathematics content. In our study, the mathematics content under investigation pertained to the arithmetic properties. We integrated content and practice standards within our lessons to highlight both in a meaningful way, thus engaging students in the type of instruction that Russell described as a "*constellation* of Content Standards and Practice Standards" (2012, p. 52). In doing so, we aimed to strengthen students' skills of argumentation as a mathematical practice, while deepening students' understanding of mathematical content related to the arithmetic properties. The students in this project had not previously experienced a classroom culture in which conjecturing, justifying, and exploring one another's ideas were the norm. However, the mode of instruction for our lessons emphasized those practices, and we found that students were accepting of the format of instruction and were excited to learn mathematics in this environment.

Key instructional strategies

Through a detailed analysis of the instructional unit (Rumsey 2012), we identified strategies that effectively promoted students' use of argumentation. We present these general instructional strategies (see **table 1**) to help others integrate mathematical argumentation into their instruction. Although these general instructional strategies would apply to mathematical argumentation within various mathematics topics, the specific examples we use to illustrate the strategies are all within the context of exploring the arithmetic properties.

Provide language supports

We found that students needed support in developing the discourse of mathematical argumentation. We supplied this communication support through the introduction of language frames (Ross, Fisher, and Frey 2009) and demonstrated their use during whole-group discussions (see **fig. 1**).

When examining a false claim, we modeled how students' ideas could be placed into the language frames that we had displayed in the classroom. For example, the teacher said, "So, some of you are saying 'I disagree, because . . .' and then you're giving me some examples."

To put the language frames into practice, in the next lesson, the students were given a claim and asked to write a response on a recording sheet where three choices of language frames were prepared (see **fig. 2**). Using the language frames as a tool, students made some specific verbal statements during subsequent lessons:

- "I noticed another pattern: Anytime, when you multiply by two, you're doubling."
- "I disagree because you'd have one leftover."
- "I agree, because you're just adding the other two numbers first, then you're just adding the two different numbers."

FIGURE 1

Some of these sample language frames were most useful in helping students develop the discourse of mathematical argumentation.

I agree with ________ because ________.

I noticed ________ when ________.

I wonder why ________.

I have a question about ________.

I disagree because ________.

Based on ________, I think ________.

FIGURE 2

This is a sample written language frame activity.

Read the following claim. Decide if you agree, disagree, or are unsure; then complete one of the sentences below.

If you take any number and multiply it by zero, you will always get a number larger than 100.

If you **agree**, complete this sentence:

I agree because ____

If you **disagree**, complete this sentence:

I disagree because $9 \times 0 = 0$ any number multiplied by zero is zero.

If you **are unsure**, complete this sentence:

I have a question about ____

Discuss rich, familiar content

Early in the instructional unit, we asked students to state what they knew about even and odd numbers. Although the concepts of even and odd might appear unrelated to the arithmetic properties, this classification is an important component of our number system, with a complexity that reaches into college-level number theory. We knew this was a familiar topic to the students and, as we had hoped, they offered a variety of statements that we used to introduce a key component of mathematical argumentation—the claim. We asked students to convince the class that their statements were true and also asked their classmates to comment on the statements. Students' claims and the ensuing discussions were a meaningful entry to mathematical argumentation and also served as a bridge to some of the arithmetic properties (see **table 2**).

Specify conditions

When mathematicians write proofs, they state a claim and then precisely outline conditions under which the claim is true. We found it beneficial to provide students with opportunities to modify claims and explain the conditions being put on the values of numbers they were using. For example, we asked students to conjecture whether the sum of three numbers would be even or odd. This task prompted students to recognize that more information was needed to make a claim about the sum and that the answer depended on the conditions of the addends. For example, one student asked, "Wait, are there two evens or two odds?"—a question that launched a discussion of the various possibilities (two evens; two odds; three evens; three odds). As students tested their conjectures and formulated claims, they considered how to efficiently add the three addends. Some students noted that for a number sentence like 17 + 9 + 1, it made sense to add the second two addends first, and that whether they added the first two addends or the second two addends together as a first step, the sum would be the same; that is, $(17 + 9) + 1 = 17 + (9 + 1)$. Thus, this open-ended problem that required students to specify conditions offered not only an opportunity for them to make and justify claims but also a meaningful context for students to explore the associative property.

We presented multiple opportunities for the students to consider precision and to specify the conditions for which claims were valid. For example, the task in **figure 3** required them to negotiate the values of *A* and *B* and to consider whether all blank spaces should represent the same value. Some students wanted *A* and *B* to be the same quantity, which meant that the blank spaces would both be replaced with any number, as long as both blanks were the same. Others suggested that if *A* and *B* were not equal, then the blanks could be replaced with a zero to make the number sentence true. Based on the specific conditions assumed, multiple correct solutions exist for this equation. This task engaged students in justifying their ideas to the class and convincing others that their claims about the statement, given their assumptions, were true. It also helped them to value precision in mathematical argumentation.

FIGURE 3

An open-ended question promotes the need to specify conditions.

$$A \times \_\_\_\_ = B \times \_\_\_\_$$

FIGURE 4

This open-ended question highlights the identity properties.

$$100 \ \_\_\_\_ \ \_\_\_\_\_\_\_\_ = 100$$

How can we fill in the blanks with an operation (on the smaller line) and a number (on the larger line) so that the number sentence is true? Is there another way to complete this number sentence?

TABLE 2

Throughout the instructional unit, the authors revisited the context of even and odd numbers when discussing the arithmetic properties.

Sample student claims and verbal justifications

Students' claims	Students' verbal justifications
An even number plus an odd number gives an odd sum.	Eight plus one equals nine; that's odd. Well, I didn't know all the odds, so I put zero plus one, and then I added this, one—which is odd. Then I did two plus three, which equals five, and still odd. If I kept going, it would all equal odds.
If you add two even numbers, you'll get an even number for the sum.	Because if you add two plus two, that equals four; and four plus four equals eight. And eight plus eight equals sixteen and so on. So, it keeps [equaling] even because if you divide four into halves . . . you'd each get two and two.

WE NEEDED TO PROVIDE OPPORTUNITIES FOR STUDENTS TO DEVELOP THEIR OWN IDEAS AND TO HAVE THE CONFIDENCE TO VALIDATE OR CHALLENGE THE CLAIMS OF OTHERS.

Another open-ended task (see **fig. 4**) engaged the students in meaningful discourse that led to the exploration of the identity properties. Initially, students argued that zero was the only value that would make the number sentence (100 __ ____ = 100) true, essentially ignoring the conditions of different operations. With some prompting from the teacher, they recognized other ways to complete the equation.

After students had presented equations for each of the four operations, the teacher directed them to look for patterns in the equations listed on the board. As a class, they concluded that they could perform different actions on a quantity yet still have the value remain unchanged, and the teacher introduced the concept of identity. Later, when the class was asked to state and justify a claim, several students choose to focus on the identity property of multiplication (see **fig. 5**).

Introduce false claims

One of our goals in promoting mathematical argumentation was to shift mathematical authority from solely the textbooks or the teacher and to encourage students to become producers of mathematical understanding and knowledge (Bay-Williams et al. 2013). Thus, we needed to provide opportunities for students to develop their own ideas and to have the confidence to validate or challenge the claims of others. By presenting false claims, we were able to break down the barriers of ownership in the classroom and enable students to recognize that invalid claims could be modified and improved.

In one of the lessons, the students were presented with an overgeneralized false claim based on a subset of numbers. We presented a variety of number sentences, such as 2 × 5 = 10, where the 5 and the 10 could be replaced with positive whole numbers. For this series of number sentences, we presented the following overgeneralized claim to the students:

- "Every time you multiply two numbers, you are always going to get an even number as the product."

Some students assumed that the claim was correct because the teacher had introduced it, whereas a few challenged it with counterexamples (e.g., 1 × 3 = 3, 7 × 7 = 49; "Two times seven and a half—that would be fifteen").

One student defended the false claim because he had assumed the teacher meant that every time you multiply two numbers, *and one of them is two*, you are always going to get an even number as the product. He stated, "But, she's telling you *times* two [*pointing to the example on the board*]; you times it by two."

After more discussion of students' counterexamples, the consensus of the class was that the claim was not true. The teacher encouraged students to modify the conditions of the claim, which resulted in the following two claims:

- "Any number multiplied by two gives an even product."
- "Any number multiplied by an even number gives an even product."

As evidence for the new claims, students gave specific examples and exhibited such reasoning as "When you multiply by two, you're doubling. Everyone has a partner." No one challenged the first claim, even though one of the counterexamples that had been shared previously—that the product of two times

FIGURE 5

A student states and justifies a claim.

Claim:

any number x 1 = that number.

Why it's true:

you will have 1 group of that number.

As students became more adept at using mathematical argumentation, they were better able to support their claims with examples and to challenge others' claims with counterexamples.

SHIRONOSOV/THINKSTOCK

seven and a half is not an even number—would have served as a counterexample to this claim as well. To be accurate, the claim should have been further modified to state, "Any *whole* number multiplied by two gives an even product." This was an issue the teacher decided to explore in a subsequent lesson. Nevertheless, during this brief discussion, students revisited the idea that the specific conditions of a claim must be presented with precise language; they considered ways of determining the validity of a claim, explored the use of counterexamples, and began to go beyond specific examples in their justifications.

FIGURE 6

A question explored the associative property of addition.

If you had three numbers a, b, and c, would this statement be true? Why?

$$(a + b) + c = a + (b + c)$$

Is this statement true or false:

$$(a - b) + c = a - (b + c)$$

What is it about addition that makes it work when it doesn't work for all operations or combinations of operations?

Manipulate familiar content to be unfamiliar

Another strategy we employed was to examine a familiar property in an unfamiliar way. This required students to unpack the statement, explore the concept, and decide whether the statement was true. For example, when discussing the associative property of addition, we investigated the task in **figure 6**. The students, who were in the seventh lesson, had many skeptical responses:

- "It depends on the numbers. . . . If they're all the same, it *could* be true."
- "I think it could be true sometimes."
- "You're using the same numbers, but they're not in the same order; so I don't think it'd be the same."

After seeing counterexamples that prompted a consensus that the second statement is not always true, students made claims about conditions for which the statement *would* be true. Manipulating familiar content in this way created a rich context for argumentation; it also showed students that mathematics involves a playful curiosity. Encouraging students to ask "What if—?" enabled them to take ownership of the questions under investigation and to lead the discussions that ensued. Often it deepened the level of the mathematics being explored. For example, one student explored the commutative property with the operation of subtraction ($A - B = B - A$) to find that it *is* true if A and B are the same quantities. This raised the question of whether the use of different variables (here A and B) meant that the numbers they represented must be different. By manipulating familiar content, the student playfully explored a new idea and shared his claim and justification with the class. This type of activity can help prepare students to confidently encounter and explore unfamiliar mathematical situations in the future.

Discussion

Through instruction promoting mathematical argumentation, the students in our study had the opportunity to take ownership of the mathematics that they were learning and to playfully engage in mathematics, exploring, conjecturing, and justifying their ideas. Moreover, by having opportunities to confront such issues as being specific about the conditions of the numbers, critiquing the claims of others, and considering unfamiliar claims confidently, the students gained a conceptual understanding of the arithmetic properties, rather than only a procedural understanding (Rumsey 2012). That is, the students went beyond learning what the properties *say*, to understanding what they *mean* and *why they are true*.

Although before this study, students had received little instruction focused on mathematical argumentation or discourse, they quickly adapted to our instructional approach and were eager to share their ideas. Some students were willing to share right away in whole-class discussions, whereas others shared more readily in small-group settings. The small-group time allowed more students the opportunity to share out loud than was possible during whole-class discussions, and we believe this was as valuable as speaking during the class discussions.

Teaching with an emphasis on mathematical argumentation is a powerful tool that can be embedded into many mathematical content areas as well as other subject areas. We hope that the instructional strategies presented in this article will help teachers incorporate this important mathematical practice in their classrooms.

STUDENTS WENT BEYOND LEARNING WHAT THE PROPERTIES SAY, TO UNDERSTANDING WHAT THEY MEAN AND WHY THEY ARE TRUE.

Common Core Connection

SMP 3

REFERENCES

Bay-Williams, Jennifer M., and Maggie McGatha with Beth M. McCord Kobett, and Jonathan A. Wray. 2013. *Mathematics Coaching: Resources and Tools for Coaches and Leaders, K–12*. Boston: Pearson.

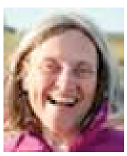
Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics (CCSSM). Washington, DC: National Governors Association Center for Best Practices and Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Ross, Donna, Douglas Fisher, and Nancy Frey. 2009. "The Art of Argumentation." *Science and Children* 47 (November): 28–31.

Rumsey, Chepina. 2012. "Advancing Fourth-Grade Students' Understanding of Arithmetic Properties with Instruction That Promotes Mathematical Argumentation." PhD diss., Illinois State University.

———. 2013. "A Model to Interpret Elementary-School Students' Mathematical Arguments." In *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, edited by A. M. Lindmeier and A. Heinze. Kiel, Germany.

Russell, Susan Jo. 2012. "Keeping the Teaching and Learning Strong." *Teaching Children Mathematics* 19 (August): 50–56.



Chepina Rumsey, chepina@ksu.edu, is an assistant professor at Kansas State University, where she teaches graduate and undergraduate mathematics education courses. She is interested in studying arithmetic properties and number sense, mathematical argumentation, and sociomathematical norms that promote discourse, specifically at the elementary school level. **Cynthia W. Langrall, langrall@ilstu.edu**, a professor at Illinois State University in Normal, teaches graduate research and theory courses and elementary and middle school preservice teacher methods courses. Her research interests include the development of elementary and middle school students' probabilistic and statistical reasoning and teacher professional development.