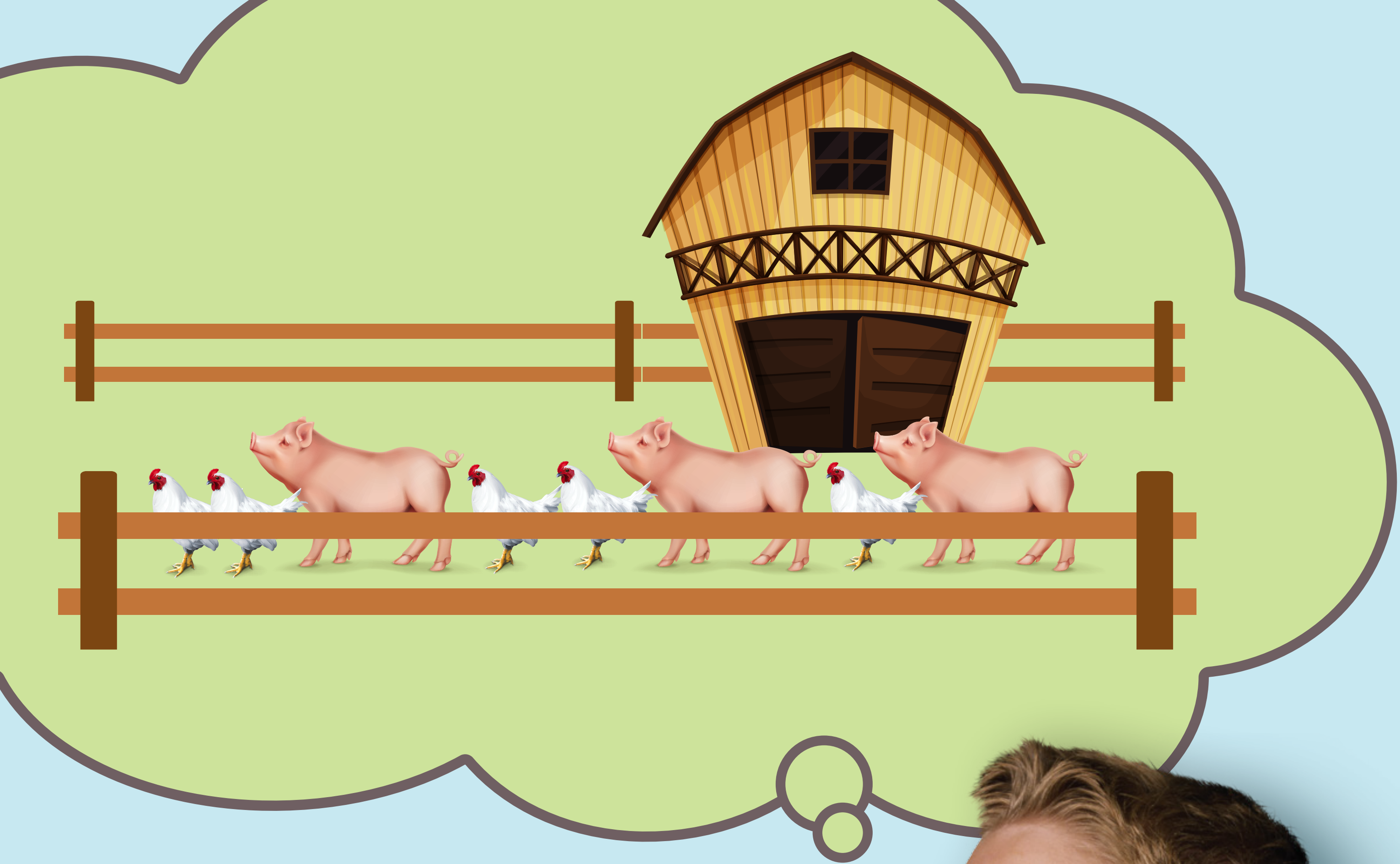
Chickens have 2 legs. Pigs have 1 head and 4 legs. How many chickens and pigs altogether will total 22 legs?

Are Your Students Problem Performers or Problem Solvers?

Examine these three strategies, which offer scaffolds for enhancing understanding and lead toward investigations that are more meaningful.



Angela T. Barlow, Matthew Duncan,
Alyson E. Lischka, Kristin S. Hartland,
and J. Christopher Willingham

Ms. Henry wanted to use a problem-solving context for introducing division with fractions to her students. She selected the Peach Tarts problem (Chapin, Connor, and Anderson 2003, p. 31):

> Ms. Stangle wants to make peach tarts for her friends. She needs two-thirds of a peach for each tart, and she has ten peaches. What is the greatest number of tarts that she can make with ten peaches?

After reading the problem, Henry engaged her students in a discussion of the context, the idea of two-thirds of a peach, and the purpose of the problem. Next, she asked students to think of strategies they could use to represent and explore the problem. After a moment of discussion with partners, students offered the following strategies:

- "We could take the numbers 10 and 2/3 and add them."
- "Ten minus two-thirds, because it is like she is taking away."
- "You could change the fraction to a decimal."
- "The problem says *each*, so you could multiply."

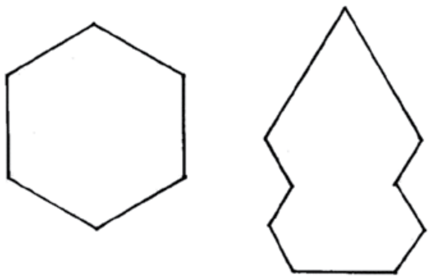
JBRYSON/THINKSTOCK (CHILD, LEFT); COLEMATT/THINKSTOCK (BARN); ALLEVINATIS/THINKSTOCK (ANIMALS); PHOTOPLOTNIKOV/THINKSTOCK (FENCE); AMOS MORGAN/THINKSTOCK (CHILD, RIGHT)

FIGURE 1

Authors Barlow, Gerstenschlager, and Harmon (2016) based their Quilt problem on one presented by Hartweg (2003).

Which quilt covers the greatest area?

Quilt A Quilt B



Recognizing that her students had not identified problem-solving strategies, Henry knew that they were not ready to explore the problem. She was unsure, though, of how to support them without robbing them of this problem-solving opportunity.

Henry's lesson appeared to have the necessary components. She engaged students in a discussion of the problem context and the mathematics within it, fundamental actions for teaching through problem solving (Fi and Degner 2012; Jackson et al. 2012; NCTM 2014). Further, she followed this discussion with sharing potential problem-solving strategies. It seemed, however, that students were functioning as problem performers rather than problem solvers (Rigelman 2007). That is, rather than understanding the problem, students focused on using an operation to complete it. Students' tendencies to act as problem performers seemed to prevent them from suggesting problem-solving strategies (e.g., draw a picture or act it out) that would support exploration of the problem. Therefore, they needed additional scaffolding during this process of setting up the problem.

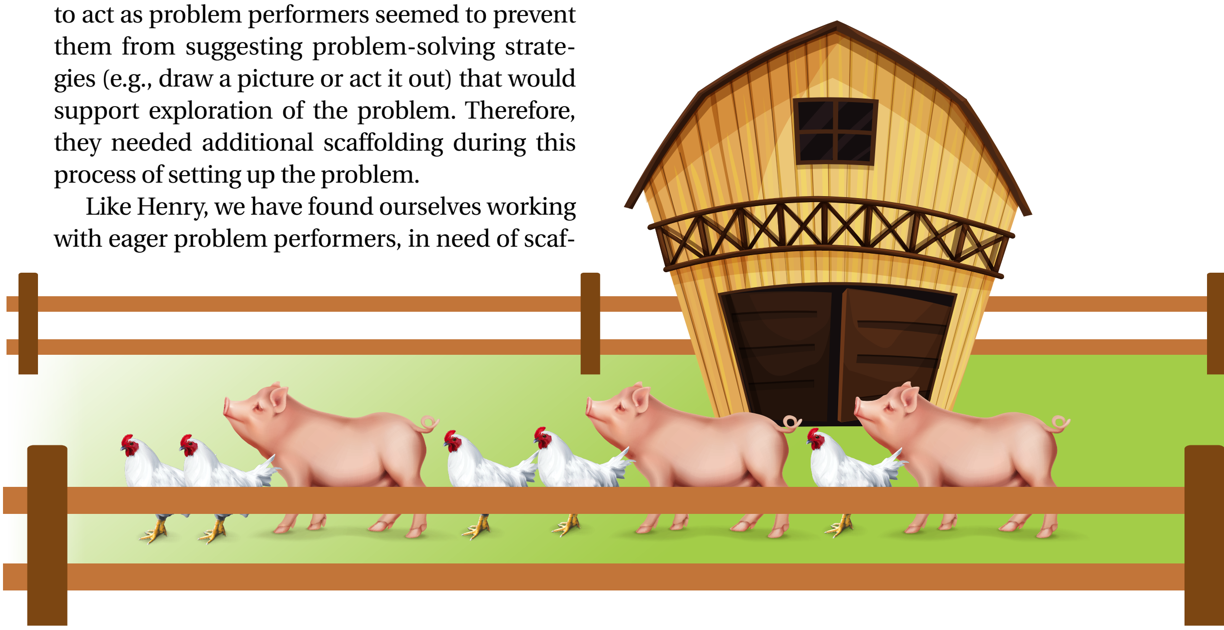
Like Henry, we have found ourselves working with eager problem performers, in need of scaffolds to support their exploration of a problem. Opportunely, we work in a context that allows us to teach a single lesson multiple times, thus providing the chance to explore potential scaffolds. Typically, Barlow, the first author, teaches a lesson while the remaining authors observe. Afterward, the group discusses how to improve the lesson, often with emphasis on scaffolding problem exploration. Therefore, the purpose of this article is to share three strategies that we have found to be useful in scaffolding students toward exploration of a problem. For each strategy, we give a general description along with its application to two sample problems.

Strategy 1: Introduce a simpler problem

The first strategy involves presenting a simpler problem before presenting the primary problem. We refer to this as the *Introduce-a-Simpler-Problem* strategy. The simpler problem is a related problem that is less complex than the primary problem and presents an opportunity for students to develop key understandings and potential strategies that will likely be useful.

Example 1: The Quilt problem

The Quilt problem (Barlow, Gerstenschlager, and Harmon 2016) is designed as a problem-solving context that provides the opportunity for students to manipulate pattern blocks to think about using nonstandard units to compare areas (see **fig. 1**). When we have used



COLEMATT/THINKSTOCK (BARN); ALLEVINATIS/THINKSTOCK (ANIMALS); PHOTOPLOTNIKOV/THINKSTOCK (FENCE) this problem in the past with problem performers, the class discussion typically indicates that students understand the problem context and what it means to cover the greatest area. However, when we have requested strategies for exploring the problem, strategies typically have included comparing the quilts, choosing the taller one, choosing the bigger one, adding up the sides, and seeing which quilt has the most sides. None of these strategies will support students in exploring the problem. Therefore, now when we use this problem, we scaffold students toward exploration of the problem by using the Introduce-a-Simpler-Problem scaffolding strategy.

Before presenting the Quilt problem, we actually use two simpler problems (see **fig. 2**). When asked which figure in the first problem covers the greater area (see **fig. 2a**), students readily identify the hexagon, stating that it is bigger than the triangle, confirming that they understand the meaning of the phrase *covers the greatest area*.

The second simpler problem (see **fig. 2b**) generates more discussion because it supports students in brainstorming ways to compare area. For these figures, we ask, “How might we determine which figure covers the greater area?” Recently, we implemented this problem with a group of third graders. By eliciting a list of strategies (see **fig. 3**) for the second simpler problem (see **fig. 2b**), we facilitated students’ development of possible plans for solving the Quilt problem, which we would present next.

Example 2: The Pigs and Chickens problem

The Pigs and Chickens problem is another task for which we have used the Introduce-a-Simpler-Problem strategy.

> Ray helps raise chickens and cows on his family’s farm. When he looks over the fence, he counts eight heads; when he looks under the fence, he counts twenty-two legs. Do you think that he has more chickens or cows? How many of each animal does he have? (NCTM, n.d.)

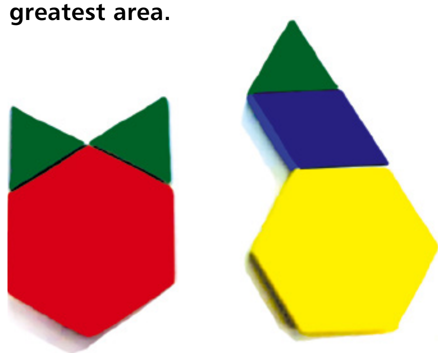
Often, problem performers tend to focus on one condition (e.g., the number of heads) without considering the second condition (e.g., the number of legs). To help students understand

FIGURE 2

The authors typically present two simpler problems before introducing the Quilt problem.

(a) The ability to readily identify the hexagon below as bigger than the triangle verifies that students understand the meaning of the phrase *covers the greatest area.*

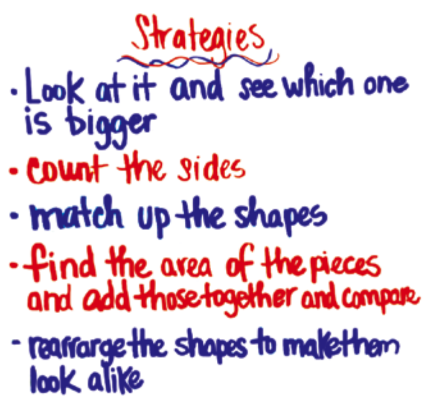
(b) The second problem prompts class discussion, brainstorming, and strategies for how to determine the greatest area.



ANGELA BARLOW

FIGURE 3

By prompting a list of strategies for the second simpler problem (see **fig. 2b**), the authors can facilitate students’ development of possible plans for solving the next problem, the Quilt problem.



the need to focus on both conditions, we introduce the following simpler problem.

> Suppose that you see a chicken and two pigs on a farm. How many legs will you see? How many heads will you see?

With this simpler problem, students have an opportunity to recognize that two conditions are involved. It also provides a platform for discussing ways of representing the problem situation. In turn, this supports students as they begin to explore the primary Pigs and Chickens problem.

Strategy thoughts

Looking back at the list of strategies in **figure 3**, you will notice that not all of them are appropriate for comparing the two figures. When we implement the Introduce-a-Simpler-Problem strategy, we have found that before moving to the primary problem, it is best to support students in reflecting on which strategies were helpful in solving the simpler problem and in eliminating the strategies that were not useful. Then the Introduce-a-Simpler-Problem strategy ignites students' thinking and allows students to discuss and explore their strategies, thus bridging them toward success with the primary problem.

Strategy 2: Elicit prior knowledge

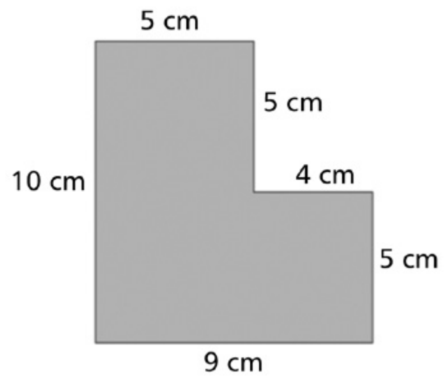
The second strategy is to elicit prior knowledge. Often, particularly at the beginning of a new unit, students need an opportunity to situate a problem in their prior experiences. By asking such a question as "What do you know about measurement?" we set the stage for a variety of connections to be made. Connections to prior mathematics, between student responses and representations, and across key mathematical ideas can all be framed by introducing a problem in this fashion.

Example 1: The L problem

The L problem (Watanabe 2008) presents students with a problem-solving opportunity to apply their knowledge of area to a nontypical shape (see **fig. 4**). When identifying strategies for this problem, problem performers tend to suggest multiplying all the numbers. However, when we have used the Elicit-Prior-Knowledge strategy before presenting this problem, problem performers have suggested appropriate problem-solving strategies.

FIGURE 4

The L problem (Watanabe 2008) gives students a chance to apply knowledge of area to a nontypical shape. When problem performers find strategies for this problem, they often include multiplying all the numbers. ***Note:*** Figure should be drawn to scale.



In our initial attempts to elicit prior knowledge for this problem, we asked, "What do you know about area?" In response, students focused on the area formula for a rectangle ($A = l \times w$), which was not helpful for exploring the problem. We then changed the prompt to this: "Using the grid paper, draw a rectangle that has an area of twelve square units." Unlike our first question, this prompt enabled students to think about the *concept* of area and supported them in identifying useful problem-solving strategies. Once the L problem was introduced, strategies appeared to build on the process of drawing the rectangle and included drawing a grid, laying the figure on top of a grid, and drawing lines to make rectangles.

Example 2: The Stolen Blocks problem

The Stolen Blocks problem (adapted from Barlow 2010) offers students a chance to think about the decomposition of numbers.

> On Thursday, Tara was at home representing numbers with base-ten blocks. The value of her blocks was 304. When she wasn't looking, her little brother grabbed two longs and a flat. What is the value of Tara's remaining blocks?

When presented with this problem, problem performers respond with such ideas as, “She had extra blocks that she didn’t know about,” or “That’s not possible, because she doesn’t have any longs.” As a result, scaffolding is needed to support their engagement with the problem.

To solve the problem, students must recall that multiple ways exist to represent 304 with base-ten blocks (e.g., 30 longs and 4 units). Recognizing this key mathematical idea, we ask, “What are three ways that you can represent 127 with base-ten blocks?” Discussing students’ responses to this question serves to elicit the prior knowledge needed to begin exploring the problem at hand. With this information identified, students are ready to tackle the Stolen Blocks problem.

In working with problem performers, we have noticed their tendency to select a problem-solving strategy, or often an operation, on the basis of the question—without first attempting to understand the problem.

Strategy thoughts

This fundamental exercise of eliciting prior knowledge may appear trivial, but it serves two essential roles. First, it activates students’ prior knowledge of key content for the lesson and primes the type of thinking they will use when the problem is introduced. Second, it filters the initial classroom discussion through a pertinent mathematical lens. The questions should specifically target the mathematical knowledge needed for the problem. For example, for the Stolen Block problem, a general question, such as, “What are three things you know about base-ten blocks?” may activate prior knowledge without provoking the idea of decomposition, which is necessary for the problem. With the necessary mathematical knowledge identified, such resources as *Good Questions for Math Teaching* (Sullivan and Lilburn 2002) can be helpful for developing questions.

Strategy 3: Delay the question

In working with problem performers, we have noticed their tendency to select a problem-solving strategy, or often an operation, on the basis of the question—without first attempting to understand the problem. As a result, our final strategy involves sharing the problem without revealing the question. We refer to this as the *Delay-the-Question* strategy. Delaying the question gives students the opportunity to focus on understanding the relationships in the problem without the distraction of attempting to answer a specific question.

Example 1: The Fruit Scales problem

The Fruit Scales problem (Lappan and Smith 2012) consists of three pictures of fruit scales and asks how many bananas will balance one apple (see **fig. 5**). When working with problem performers, we present only the first two pictures of the problem and ask, “What are some relationships that you see represented with these scales?” In response, students will note the obvious relationships: Two pineapples balance ten bananas, and one pineapple balances two bananas and an apple. However, when pressed for additional ideas, they share less obvious relationships, such as one pineapple balances five bananas, and an apple is heavier

FIGURE 5

When working on the Fruit Scales problem (Lappan and Smith 2012) with problem performers, the authors present only the first two of the three pictures.

than a banana. Once these relationships have been recorded, students are prepared to continue exploring the problem individually or in pairs as the question (the third picture) is revealed.

Example 2: The Peach Tarts problem

For the Peach Tarts problem that we presented at the beginning of the article, without additional scaffolding, Henry's students focused on applying an operation to the problem (see **fig. 6**). In subsequent lessons involving the same problem, we have applied the Delay-the-Question strategy by posing the following problem:

> Ms. Stangle wants to make peach tarts for her friends. She knows two-thirds of a peach will make one tart.

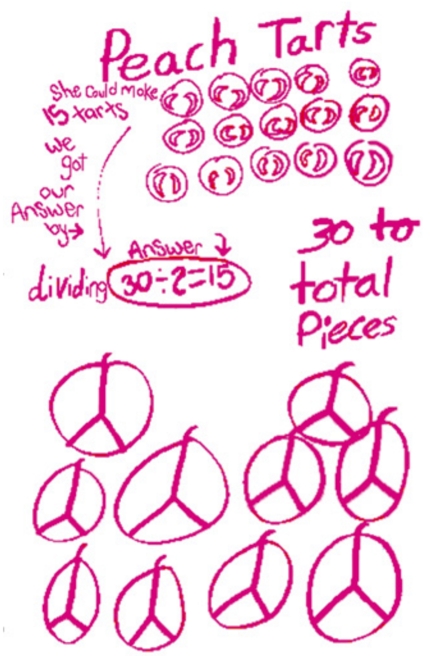
In this instance, the strategy selection was based on the idea that students are able to represent fractions, and if they take the time to represent the problem without rushing to perform a computation, they will be able to successfully analyze their representation as a means for solving the problem. In delaying the question, we have been able to elicit from students the various relationships represented in the problem. These include that each peach is broken into three pieces, that it takes two pieces to make a tart, and that each peach will have an extra piece. Having taken the time to think about this information, students have been better prepared to represent and explore the problem when they are then given the question (see **fig. 7**).

FIGURE 6

Without additional scaffolding, this student incorrectly focused on applying an operation to the Peach Tarts problem.

FIGURE 7

This child's solution process built on previously identified relationships: By drawing, she found that ten peaches provide thirty total pieces to use in making tarts.



More strategy thoughts

The Delay-the-Question strategy gives the teacher a chance to use questions as a means for focusing students' attention on the relationships between the relevant quantities in the problem. To help students see these relationships, consider the following moves.

- Ask, "What are some things that we know about this problem?" After students have given the obvious information, follow up by asking questions that illuminate the types of relationships that will be helpful. For example, for the Fruit Scales problem, you might say, "On the basis of the first picture, I am wondering if we can figure out how many bananas will balance one pineapple." Then ask, "Are other relationships represented in these pictures?"
- Let "If you don't know what to do, draw a picture of what you know" (Buschman 2003, p. 30) be your class motto. Representing

the problem in a different way will help students organize their thoughts and identify relationships.

Conclusion

In *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014), two of the Mathematics Teaching Practices presented are (1) to implement problems that promote reasoning and problem solving, and (2) to support productive struggle in learning mathematics. Engaging students in problems that encourage reasoning and sense making promotes deep understanding of mathematical concepts. In working with problem performers, however, teachers must decide how to engage students in reasoning about the problem without reducing the productive struggle, recognizing that students are inclined to bypass the reasoning process to avoid the struggle.

The featured strategies support student exploration of a problem without depriving them of engaging in the problem-solving process. The strategies enable and support students to not only engage in reasoning about the problem but, over time, also develop as problem solvers. The strategies support students in attaining the first Standard of Mathematical Practice: "Make sense of problems and persevere in solving them" (CCSSI 2010, p. 6). Designing classroom instruction that successfully engages all students, including problem performers, in problem solving requires scaffolding so that students can engage in mathematical reasoning. The scaffolding strategies shared here will aid teachers in this process.

Common Core Connections

SMP1 **2.NBT.7**
3.MD.5 **5.NF.7**
6.G.1

BIBLIOGRAPHY

Barlow, Angela. T. 2010. "Building Word Problems: What Does It Take?" *Teaching Children Mathematics* 17, no. 3 (October): 140–48.

Barlow, Angela T., Natasha E. Gerstenschlager, and Shannon E. Harmon. 2016. "Learning from the Unknown Student." *Teaching Children Mathematics* 23, no. 2 (September): 74–81.

Buschman, Larry. 2003. *Share and Compare: A Teacher's Story about Helping Children become Problem Solvers in Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Chapin, Suzanne H., Catherine O'Connor, and Nancy Canavan Anderson. 2003. *Classroom Discussions: Using Math Talk to Help Students Learn* (*Grades 1–6*)*.* Sausalito, CA: Math Solutions Publications.

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: The National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Fi, Cos D., and Katherine M. Degner. 2012. "Teaching through Problem Solving." *Mathematics Teacher* 105, no. 6 (February): 455–59.

Hartweg, Kim. 2003. "Problem Solvers: Pattern Block Quilts." *Teaching Children Mathematics* 10, no. 2 (September): 52–53.

Jackson, Kara J., Emily C. Shahan, Lynsey K. Gibbons, and Paul A. Cobb. 2012. "Launching Complex Tasks." *Mathematics Teaching in the Middle School* 18, no. 1 (August): 24–29.

Lappan, Glenda, and Margaret S. Smith. 2012. *Rich and Engaging Mathematical Tasks.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

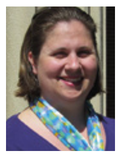
———. "Problems from *Teaching Children Mathematics.*" Accessed January 10, 2014.

Rigelman, Nicole. "Fostering Mathematical Thinking and Problem Solving: The Teacher's Role." 2007. *Teaching Children Mathematics* 13, no. 6 (February): 308–14.

Sullivan, Peter, and Pat Lilburn. 2002. *Good Questions for Math Teaching: Why Ask Them and What to Ask* (*K–6*)*.* Sausalito, CA: Math Solutions Publications.

Watanabe, Tad. "Can Fifth Graders Derive Area Formulae? Implications from Japanese Curriculum Materials." Presentation at the Georgia Mathematics Conference, Rock Eagle, GA, October 15–17, 2008.

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On **Wednesday, May 10, 2017**, at **9:00 p.m. EDT**, we will discuss "Are Your Students Problem Performers or Problem Solvers?" by Angela T. Barlow, Matthew Duncan, Alyson E. Lischka, Kristin S. Hartland, and J. Christopher Willingham.

Follow along using #TCMchat. Unable to participate in the live chat? Follow us on Twitter@TCM_at_NCTM and watch for a link to the recap.