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Teacher Characteristics Associated With Mathematics Teachers' Beliefs and Awareness of Their Students' Mathematical Dispositions

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This study investigates relationships between teacher characteristics and teachers' beliefs about mathematics teaching and learning and the extent to which teachers claim awareness of their students' mathematical dispositions. A professional background survey, a beliefs and awareness survey, and a teacher mathematical knowledge assessment were administered to 259 novice upper-elementary and 184 novice middle-grades teachers. Regression analyses revealed statistically significant relationships between teachers' beliefs and awareness and teachers' mathematical knowledge, special education certification, race, gender, and the percentage of their students with free and reduced meal status. This report offers interpretations of findings and implications for mathematics teacher education.

Key words: Teacher beliefs; Teacher characteristics; Teacher knowledge

Over the past several decades, educational researchers have engaged in considerable conceptual and empirical work focused on exploring the relationships between teachers' professional experiences, teachers' characteristics, teachers' instructional practices, and their students' achievement (Brophy, 1986; Darling-Hammond, 2000; Wayne & Youngs, 2003). Quantitative studies have sought to model relationships between students' performance on standardized achievement tests and the characteristics and professional qualifications of the teachers of those students (Rockoff, Jacob, Kane, & Staiger, 2011). The measures of teacher characteristics and qualifications typically listed in those models include certification status, amount of completed coursework, attainment of an advanced degree, and number of years of teaching experience. However, those studies have been unable to establish a strong relationship between proxy indicators of teachers' qualifications and student achievement, and those studies have been inconsistent when addressing middle-grades mathematics (Lubienski, Lubienski, & Crane, 2008; Rivkin, Hanushek, & Kain, 2005; Wayne & Youngs, 2003).

Recent research, however, has documented a positive relationship not only between direct measures of teachers' knowledge of mathematics content and pedagogy and student achievement (Baumert et al., 2010; Hill, Rowan, & Ball, 2005) but also between teachers' beliefs about mathematics teaching and learning and student achievement (Love & Kruger, 2005). In this study we shift the focus from investigating the influence of teacher beliefs on teaching and learning to examining potential influences on teacher beliefs. We contend that teacher characteristics, teacher qualifications, and teaching contexts may differ in the degree to which teachers possess varying levels of teacher knowledge and hold specific beliefs about mathematics teaching and learning. Findings from such explorations may provide empirical evidence supporting or refuting assumptions that currently define teacher preparation programs, accreditation standards for those programs, and teacher professional development efforts. This is particularly relevant as proposed standards for accreditation of teacher preparation programs in the United States are calling for evidence of the effectiveness of graduates of teacher education programs as judged by these graduates' impact on their students' learning (Council for the Accreditation of Educator Preparation, 2013).

Recent studies situated in mathematics teaching and learning contexts reveal that mathematics teachers draw on a range of cognitive and affective resources when teaching, including their knowledge (Ball, Thames, & Phelps, 2008), beliefs about mathematics teaching and learning (Beswick, 2007), and awareness of mathematics classroom conditions and interactions (Sherin, Jacobs, & Philipp, 2011). Over the

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past decade, there has been considerable concentration on the study of mathematics teacher knowledge and potential influences on teacher knowledge growth and development (Baumert et al., 2010; Hill et al., 2005; Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007). In addition, there have been calls to acknowledge and better understand the role of other resources mathematics teachers draw on to teach, such as teachers' beliefs related to mathematics teaching and learning (Hill, 2007) and influences on the development of those beliefs (Voss, Kleickmann, Kunter, & Hachfeld, 2013). Furthermore, with emerging conceptualizations of mathematical proficiency explicitly identifying students' mathematical disposition as critical to students' performance and success in mathematical contexts (Kilpatrick, Swafford, & Findell, 2001), there is growing interest in the role mathematics teachers play and the resources they draw on in the development of students' mathematical dispositions (Clark, Badertscher, & Napp, 2013; Gresalfi & Cobb, 2001). As an initial step in framing aspects of this role and potentially related teacher characteristics, this quantitative study is an analysis of survey data measuring teachers' claimed awareness of students' mathematical dispositions as well as teachers' claims of the steps that they take to ascertain those dispositions.

This study explored the following research question: *How do teacher characteristics, professional qualifications, and teaching contexts relate to teachers' beliefs about mathematics teaching and learning and teachers' awareness of their students' mathematical dispositions?* The analysis reported herein focuses on a specified set of mathematics teacher beliefs about mathematics teaching and learning and a construct characterizing the extent to which teachers claim they are aware of their students' mathematical dispositions.

Theoretical Perspectives

In our efforts to frame this study, we drew on theoretical perspectives related to teachers' beliefs formation, conceptualizations of teachers' beliefs about mathematics teaching and learning, and conceptualizations of students' mathematical dispositions.

Teacher Beliefs Formation

This investigation of relationships between teacher characteristics, teaching conditions, and novice mathematics teachers' beliefs and levels of awareness is grounded in the perspective that teachers' beliefs about mathematics teaching and learning are shaped by teachers' personal and professional experiences. The theoretical consensus across the literature on beliefs formation suggests that "beliefs, attitudes and values are the consequence of an evolutionary process that involves all of an individual's experiences with mathematics throughout their entire life" (Maasz & Schlöglmann, 2009, p. vii). Beliefs formation, therefore, is guided by a confluence of cognitive, cultural, and social factors and experiences (McGarty, Yzerbyt, & Spears, 2002), and teachers' beliefs about teaching and learning are strongly influenced by both socialization in the profession and experiences as students (Richardson, 1996). Analyses that explore relationships between who teachers are and what they believe are therefore needed and important as researchers build claims connecting teacher identity to teacher practice (Day, Sammons, Stobart, Kington, & Gu, 2007).

We recognize that when researchers use quantitative analyses in which positioning human characteristics, particularly race and gender, are positioned as potential influences on teachers' beliefs, behavior, and performance, those researchers run the risk of reducing very complex experiences (such as living as a female and/or minority in the United States) to simple categories that need no theorizing or explanation, resulting in significant discursive consequences (Parks & Schmeichel, 2012). Too often, discussions of research findings that suggest race and gender effects do not make attempts at interpretation or explanation of why such effects might emerge. However, as exemplified by researchers exploring mathematics teacher identity (Martin, 2007; Van Zoest & Bohi, 2005), discussions of relationships between teacher characteristics and teacher beliefs and practices can move beyond presenting teacher characteristics as fixed, static constructs. With this perspective, studies that consider teachers' beliefs and awareness and identify influences on them, including the potential influence of teacher knowledge, may prove useful as researchers attempt to make sense of teachers' instructional decisions and may inform teacher educators' efforts to prepare and produce highly effective mathematics teachers.

Conceptualizing Teachers' Beliefs

The research literature characterizes beliefs as "psychologically held understandings, premises, or propositions about the world" (Philipp, 2007, p. 259), potentially serving as mediators between teachers' knowledge and teacher practice (Murphy, Delli, & Edwards, 2004; Pajares, 1992). Although Leder and Forgasz (2002) contend that attitudes and beliefs of students and teachers must be explored in order to enhance mathematics teaching and learning, others suggest that teachers' beliefs may significantly influence students' opportunities to learn rigorous, engaging mathematics (Gellert, 2000). Moreover, there is growing evidence that teachers' beliefs about mathematics teaching and learning are consistent with teachers' pedagogical practices (Cross, 2009).

A review of frameworks in the literature on mathematics teacher beliefs reflects tendencies toward organizing mathematics teachers' beliefs about teaching and learning into roughly two categories. In one category are beliefs that reflect behaviorist transmission theories of learning and the teaching practices that support such theories, while the other category portrays beliefs that reflect conceptualizations of mathematical learning and knowing emphasizing conceptual understanding, problem solving, reasoning, and sense-making and reflect ambitious teaching practices that support such learning (Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010; Ross, McDougall, Hogaboam-Gray, & LeSage, 2003; Voss et al., 2013).

Behaviorist transmission theories of learning, and the teaching practices that support such theories, suggest that learning is a process of information transmission with students operating as more or less passive recipients (Voss et al., 2013). Teachers who strongly hold beliefs aligned with behaviorist transmission theories of learning, therefore, may focus on mathematical facts and procedures during instruction and dedicate less time to developing students' conceptual understanding. Teaching practices associated with behaviorist transmission theories of learning emphasize repetition, automatization, and skill mastery.

In contrast, beliefs about mathematics teaching and learning that support ambitious teaching practices (Lampert et al., 2010) acknowledge broader and more complex conceptualizations of mathematical knowledge and proficiency, including conceptual understanding, strategic competence, reasoning, and sense-making (Kilpatrick et al., 2001). Furthermore, such beliefs reflect learning theories that emphasize knowledge construction as an active process of synthesizing preconceptions and prior knowledge with the learning content at hand. Teachers who hold these beliefs, therefore, may engage in teaching practices that promote students' active engagement with challenging mathematical problems and tasks that lead to deepening students' conceptual understanding.

Conceptualizing Teachers' Awareness of Students' Mathematical Dispositions

Although educational psychologists have considered the definition and role of disposition for some time, only recently have mathematics education researchers begun addressing the critical role that a student's mathematical disposition may play in his or her capacity to engage in and learn from mathematical tasks (Kilpatrick et al., 2001). A *productive mathematical disposition* refers to

> the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. If students are to develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning abilities, they must believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and that they are capable of figuring it out. (Kilpatrick et al., 2001, p. 131)

Researchers have also begun conceptualizing students as having a *mathematics identity* (Anderson, 2007; Martin, 2000) in which self-perceptions and dispositions play major roles. From a pedagogical perspective, therefore, an important aspect of mathematics instruction includes influencing students' mathematics dispositions and helping students develop perceptions of themselves as members of a community of mathematics learners (Boaler, 1999). Researchers suggest that teachers' awareness and understanding of students' prior mathematical experiences, dispositions, and self-perceptions may allow mathematics teachers to better meet the needs of students as learners (Anderson, 2007; Boaler & Greeno, 2000; Martin, 2000). Evidence of teachers' awareness of their students' mathematical dispositions is also emerging as an important data point on which to assess mathematics teacher effectiveness and quality (Stanford Center for Assessment, Learning, and Equity, 2012).

From an identity development perspective, mathematics instruction consists of both socializing students into the norms and discourse practices of the mathematics classroom (Cobb & Yackel, 1996) and influencing students' perceptions of themselves as members of a community of mathematics learners (Boaler, 1999; Boaler & Greeno, 2000). This perspective suggests that the resources teachers draw on to teach mathematics may include an awareness and understanding of students' mathematics dispositions and identity formation and development. Frameworks through which teachers might identify and consider a student's mathematics disposition are scarce in the mathematics education research literature; however, three frameworks have been developed that organize mathematics identity into a cohesive set of dimensions or features. The three frameworks are Martin's (2000) four dimensions of mathematics identity, Anderson's (2007) four faces of mathematics identity, and Cobb, Gresalfi, and Hodge's (2009) interpretive scheme. A synthesis of these three frameworks suggests that teachers may gain an awareness of their students' mathematics disposition through gathering information and paying attention to students' (a) perceptions of their mathematics ability and the ways these perceptions influence their mathematics performance, (b) perceptions of the importance of mathematics inside and beyond their current experiences in the mathematics classroom, (c) perceptions of the engagement in and exposure to particular forms of mathematical activity and the ways these engagements influence students seeing themselves as mathematics learners, and (d) motivations to perform at a high level and attributions to their success or failure in mathematical contexts. As students' mathematics disposition and identity constructs are emerging as important contributors to students' engagement and performance in mathematical contexts, it may prove useful to gauge teachers' awareness of students' mathematical dispositions and attempt to get a sense of relevant influences on such awareness.

Potential Influences on Teachers' Beliefs and Awareness

In keeping with the aims of this study, it is important to consider potential influences of teachers' beliefs and awareness identified in the research literature. Four interrelated categories emerged in our review of the literature as potential influences on teachers' beliefs and awareness of classroom conditions and interactions: (a) teachers' professional background and experiences, (b) teacher knowledge, (c) teaching contexts, and (d) students' experiences.

Teachers' Professional Background and Experiences

Coursework. Mathematics content courses and mathematics methods courses may have differing influences on mathematics teachers' beliefs about mathematics teaching and learning. In their study of 276 prospective elementary teachers, responding to a mandate by a state board of education, Smith, Swars, Smith, Hart, and Haardörfer (2012) measured the effects of enrolling in four mathematics content courses and one methods course, as opposed to enrolling in three mathematics and two methods courses. Although these researchers found no statistically significant difference in prospective teachers' mathematical knowledge for teaching resulting from an increased number of content courses, they did find that those with higher knowledge scores showed an increase on the teacher efficacy belief scales and also showed growth in the belief that students can construct their own mathematical knowledge. At the same time, this study found that those prospective teachers who enrolled in two methods courses, as opposed to those who only enrolled in one, showed more positive change on a subscale that measured the belief that they could teach mathematics effectively and improve student learning.

Preservice elementary teachers have also been found to change their beliefs toward a more constructivist orientation about the learning of mathematics during their mathematics methods course (Smith et al., 2012; Vacc & Bright, 1999) or toward beliefs consistent with reform efforts in mathematics education at the conclusion of an integrated mathematics content and instructional methods course (Hart, 2002). Quinn (1997) noted similar changes in preservice elementary teachers' attitudes and beliefs, but he found no change in beliefs of preservice secondary mathematics teachers as a result of engaging in the mathematics methods course. In contrast to Quinn's findings, Conner, Edenfield, Gleason, and Ersoz (2011) found that secondary preservice teachers experienced a shift in their beliefs about pedagogical approaches and the role of mathematics teachers at the end of their methods courses. Most indicated that they intended to engage in more student-centered practices when they entered the classroom, and many of them redefined the role of a mathematics teacher as "facilitators of learning rather than transmitters of knowledge" (Conner et al., 2011, p. 495). Similarly, focused reflection on teaching has also been shown to improve preservice teachers' awareness of what is going on in the classroom, or "withitness" (Snoeyink, 2010).

In contrast to these reports of shifts in beliefs about mathematics teaching and learning and awareness of classroom conditions and interactions, there is evidence that many preservice teachers do not exhibit such change in beliefs regarding students and the nature and structure of mathematics (Conner et al., 2011). Mathematics methods courses may, therefore, have limited influence on preservice teachers' persistent view of mathematics as either a set of fixed, universal truths or a body of rules and procedures.

Certification and advanced degree status. Due to the widely disparate nature of state licensing standards, the assumption that teacher certification implies teacher quality has been questioned (Darling-Hammond, 2000, 2009). There is evidence that certified teachers of secondary mathematics have a statistically significant positive influence on their students' mathematics achievement scores compared with the achievement of students whose teachers either hold private school certification or are not certified in mathematics (Goldhaber & Brewer, 2000). However, it is unclear how certified and uncertified teachers might differ in terms of their beliefs about mathematics teaching and learning and their awareness of classroom conditions. Furthermore, the teacher salary schedules of many school districts encourage teachers to pursue advanced degrees, reflecting the assumption that teachers holding advanced degrees possess desirable attributes. Yet, it is unclear what those attributes might be. It may prove useful, therefore, to consider whether practicing teachers who differ in terms of their certification status and level of degree attainment, reflecting differing levels of professional background, also differ in terms of their beliefs about teaching and learning mathematics and their awareness of students' mathematical dispositions.

Special education certification. Within teacher preparation institutions, mathematics education and special education programs are frequently perceived as having contrasting pedagogies. Many contemporary mathematics education programs emphasize teaching strategies that are intended to foster students' mathematical reasoning and understanding; however, special education programs are more likely to promote instructional routines emphasizing procedural skill and attainment of specific measurable objectives (Boyd & Bargerhuff, 2009). Although all special education certified teachers may not reject consideration of what may be termed mathematical processes and practices, the distinct manner in which the fields of special education and mathematics education conceive of student learning and effective instruction (Kroesbergen & Van Luit, 2003) may be associated with teachers' beliefs.

Teacher race and gender. Elements of racialized and gendered experiences pervade our perspectives and associated behaviors (Rothenberg, 2007) in ways that make race and gender relevant teacher characteristics to consider when exploring teachers' beliefs about mathematics teaching and learning. Beliefs, including beliefs about the role and function of teaching and learning, are rarely neutral. Despite the acknowledgement that there is considerable heterogeneity in relation to life experiences, values, and practices within racial and gender groups, researchers have suggested that teacher race and teacher gender influence teachers' perspectives, including teachers' goal orientation (Rubie-Davies, Flint, & McDonald, 2011) and teacher expectations for and perceptions of capacities of underrepresented minorities (Beady & Hansell, 1981; Pigott & Cowen, 2000).

Of particular relevance to this line of work is the conceptualization of some African American teachers as "warm demanders" (Ware, 2006), characterized by a stance that simultaneously communicates warmth and a "tough-minded, no-nonsense, structured, and disciplined classroom environment" (Irvine & Fraser, 1998, p. 56). A recent study (Den Brok, Levy, Rodriguez, & Wubbels, 2002) of the perceptions of Asian American teachers and Hispanic American teachers indicated that teacher ethnicity significantly influenced teachers' perceptions and beliefs related to their role and classroom environments. While not focusing solely on mathematics teaching or mathematics classrooms, these studies suggest that, although not deterministic, teacher ethnicity and culture are factors that potentially shape teachers' perceptions and beliefs about teaching and learning. Teacher race and ethnicity have been shown to influence teachers' beliefs related to collective efficacy (Goddard & Skrla, 2006) and perceptions of their relationships with their students (Saft & Pianta, 2001), yet little is known about the role of teacher race and teachers' instructional belief systems, including their belief systems about mathematics teaching and learning in particular (Clark, Johnson, & Chazan, 2009).

Teacher Knowledge

Thompson (1992) cautioned, "To look at research on mathematics teachers' beliefs and conceptions in isolation from research on mathematics teachers' knowledge will necessarily result in an incomplete picture" (p. 131). Scholars have distinguished between knowledge of mathematics content and knowledge of how to teach school mathematics, while noting the importance of both forms of knowledge (e.g., Shulman, 1986). Shulman's initial definition of teacher knowledge has since been critiqued and refined (Grossman, 1990; Leinhardt & Smith, 1985; Marks, 1990), with recent characterizations proposing categories of mathematical knowledge for teaching spanning pedagogical content knowledge (PCK) and subject matter knowledge (Ball et al., 2008). Researchers have argued the importance of both PCK and mathematics content knowledge in the design and implementation of mathematics instruction (e.g., Ball, Lubienski, & Mewborn, 2001; Eisenhart et al., 1993; Hill et al., 2008; Ma, 1999; Thompson & Thompson, 1996) and have empirically linked components of teacher knowledge to student achievement in mathematics (Baumert et al., 2010; Hill et al., 2005).

Ball (1990) posited that teachers with very similar mathematical knowledge may teach very differently depending on their views and beliefs related to the teaching and learning of mathematics. Current thought regarding the relationship between teacher knowledge and beliefs suggests that knowledge and beliefs do not operate independently or in isolation; rather, teacher beliefs can act as a mediator between teacher knowledge and teacher practice (Pajares, 1992; Wilkins, 2008). Further, instructional practice is influenced by a teacher's unique interaction of teacher knowledge and beliefs (Cooney & Wilson, 1993; Philipp, 2007; Wilkins, 2008).

Teaching Contexts

Teaching contexts may influence teachers' beliefs about mathematics teaching and learning (Brown & McNamara, 2005), as the institutional environments of schools may shape teachers' ways of teaching (Peressini, Borko, Romagnano, Knuth, & Willis, 2004; Valli, Croninger, Chambliss, Graeber, & Buese, 2008) in ways that differ from the practices promoted within university teacher education programs (Putnam & Borko, 2000). Case study research suggests that teachers react to these pressures differently at different points in their teaching careers (Peressini et al., 2004). Furthermore, teachers' instructional assignments may influence their beliefs about mathematics teaching and learning. For example, research studies have documented the relationship between teachers' instructional placements and their beliefs regarding use of cognitively demanding tasks (Lipman, 2003) and their beliefs about providing structured, low-level instruction as a means of classroom management (Zohar, Degani, & Vaaknin, 2001).

Opportunities to engage in professional development can be perceived as a contextual factor or resource and may influence teachers' beliefs and perceptions (Marzano, 2003). A frequently cited study of primary teachers noted that professional development focused on students' thinking in mathematics influenced teachers' beliefs about mathematics teaching and student learning over a 3-year period (Fennema et al., 1996). Similarly, when reflecting on the implications of a 3-year case study of a beginning teacher, Potari and Georgiadou-Kabouridis (2009) concluded that professional development opportunities, wherein practicing teachers would have the time and support to study and reflect on their students' mathematical thinking, could influence teachers' instructional beliefs and practice.

Students' Experiences

Because there is evidence to suggest that teachers' instructional practice may be associated with student-level experiences such as their students' socioeconomic status, their students' perceived ability level, and their students' prior achievement (Knoblauch & Hoy, 2008; Lipman, 2003; Solomon, Battistich, & Hom, 1996), it is plausible that teachers' beliefs about instruction are similarly associated. In their study of teachers' beliefs and perceptions related to teaching students of ethnic and cultural backgrounds dissimilar from their own, Taylor and Sobel (2001) concluded that much work must be done to help teachers "see their students' backgrounds and abilities as resources not problems" (p. 499). The experiences students bring to the classroom, including experiences associated with being members of racial, ethnic, and socioeconomic communities, potentially influence teachers' beliefs and perceptions of what is instructionally possible with or necessary for particular students.

Context of Study

This study reports a secondary analysis of the relevant data collected during an investigation of the potential relationships between student achievement, teachers' mathematical content and pedagogical knowledge, and teachers' beliefs and awareness, while accounting for teacher qualifications and specific contextual variables (Campbell et al., in press). In the next sections, we describe the instruments developed for the primary analysis and illustrate how the nature of the data and results from the primary analysis make this data set well suited for an investigation of this study's research questions. Because the survey of teachers' beliefs and awareness provides the measure of the dependent variables in the secondary analysis defining this study, we describe that survey in more detail, referencing the factor analysis underlying its development.

Prior Instrument Development and Data Collection

Several instruments were developed to collect data for the primary analysis, including two teacher knowledge instruments, a professional background and teaching experience survey, and a beliefs and awareness survey.

The teacher knowledge instruments. The teacher knowledge instruments were designed to measure both the mathematical content knowledge and the PCK of teachers that might most directly influence students' mathematics achievement, as assessed on high-stakes standardized state tests in Delaware, Maryland, and Pennsylvania. Separate instruments were developed to assess upper-elementary and middle-grades teachers of mathematics.

The mathematical content items in each of these instruments were developed through a two-step process. First, teacher content knowledge for upper-elementary and for middle-grades teachers were separately specified by identifying aspects of teachers' mathematical knowledge noted in either professional licensure frameworks (e.g., Educational Testing Service, 2008; National Council for Accreditation of Teacher Education/National Council of Teachers of Mathematics [NCTM], 2003) or in compendia compiled by mathematicians and mathematics educators (e.g., Bush et al., 2005; Conference Board for Mathematical Sciences, 2001; NCTM, 1991). Then, this listing of expected teacher knowledge was intersected with the Delaware, Maryland, and Pennsylvania curriculum standards for student mathematics in either Grades 4–5 or in Grades 6–8 in order to identify overlapping topics. Multiple-choice content items were then identified, developed, piloted, and revised prior to final evaluation using classical educational measurement procedures (reliability, item difficulty, distractor analysis, point-biserial correlation). This process yielded 80 items measuring the mathematical content knowledge of upper-elementary teachers and 80 items measuring the mathematical content knowledge of middle-grades teachers.

Multiple-choice items addressing four differing PCK domains were written across the mathematics content areas upon which students in the three states were assessed. These PCK domains were (a) common student errors and misconceptions; (b) mathematics, models, representations, and contexts; (c) sense of order of mathematical content; and (d) understanding of students' interpretations of mathematical structures. The 40 upper-elementary and 40 middle-grades PCK items were subjected to the same review, modification, and piloting procedures reflecting educational measurement standards described previously for the mathematical content items.

Because these assessments of teacher mathematical content and pedagogical knowledge were designed to address understandings associated with teaching the particular domains of school mathematics upon which students in three states are assessed and the understandings a teacher may draw on to teach that content, there is no claim that these teacher knowledge assessments or the data resulting from administration of these assessments are exhaustive. These teacher knowledge assessments do differ from other empirical measures of teacher knowledge (e.g., Baumert et al., 2010; Hill et al., 2005) in that these measures of teachers' mathematical content and pedagogical knowledge aligned with expectations for student achievement, as expressed in state mathematics curriculum standards and as measured in state assessments.

Professional background and teaching experience survey. This survey asked teachers to provide information on their certification status, type of certification, education level, titles of completed mathematics content and mathematics education courses, and years of teaching experience as well as race and gender. In addition, teachers were asked to provide the number of hours of professional development completed and to indicate their teaching assignment, including whether or not they taught an above-grade-level mathematics curriculum to students. Teachers were asked to report this information for the most recently completed academic year.

Beliefs and awareness survey. Beliefs items originated from a 20-item Likert-format instrument developed by Ross et al. (2003). However, there was concern that in the decade since the development of the Ross et al. instrument, shifting educational assessment policies could lead to decreasing variance in teachers' responses to this survey. For example, one item on the Ross et al. survey stated, "I teach students how to explain their mathematical ideas" (p. 349). Because state assessments in the mid-Atlantic region ask elementary students to compose brief constructed responses, it was expected that most of the teachers being sampled would agree with this statement. In response to these concerns, all but four of the Ross et al. items were rephrased, resulting in the development of many new items to reflect current dynamics in education and reflecting both the behaviorist transmission theories of learning and the perspective of ambitious mathematics teaching (Lampert et al., 2010). Awareness items were developed to assess the extent of teachers' perceived sense of their students' mathematics identity by drawing on the four features emerging from a synthesis of three relevant theoretical frameworks (Anderson, 2007; Cobb et al., 2009; Martin, 2000). The awareness items were framed to identify the degree to which teachers claimed they knew or explicitly gathered information about their students' mathematical dispositions.

The 40 items in the beliefs and awareness instrument reflected a variety of perspectives and crossed multiple theoretical constructs. The intent of the survey was not to determine whether teachers' beliefs were aligned with a specific theory of instructional practice or learning theory. In other words, there were no preconceived factors to be extracted from the survey data. Rather, the question of interest was whether any underlying factor structure might emerge across the variety of perspectives that could be present in teachers' beliefs about the teaching and learning of mathematics and their claimed awareness of students' mathematical dispositions. Consequently, in order to allow for the clustering of items in ways that would reflect more nuanced teachers' beliefs and claimed awareness, an exploratory factor analysis was conducted as a component of the primary analysis.

Factor analysis. Exploratory factor analysis accessed 459 teachers' responses on the beliefs and awareness survey. After examining the scree plot, three factors were extracted. An oblique rotation was conducted, resulting in relatively small correlations. A subsequent varimax rotation yielded loadings for all items. Applying the criteria of the absolute value of an item loading .4 or above on only one factor, the rotated solution yielded three interpretable orthogonal factors, with 21 items in total loading on two beliefs factors (the belief that students should be allowed to struggle [6 items] and the belief that teachers should model for incremental mastery [7 items]) and one awareness factor (awareness of students' mathematical dispositions [8 items]). Each of the 21 items, along with their factor loadings, is listed in Appendix A. A total variance of 35.5% was explained across these three factors.

Belief that students should be allowed to struggle. The first factor, Teacher Allowance for Student Struggle with Problems (TASSP), reflects the belief that mathematics teaching and learning should include periods of time when students struggle, grapple, and solve problems on their own, making sense of mathematics without relying on teacher intervention. This period of time may occur at any point, or at multiple points, during a lesson. These elements of the TASSP factor align with beliefs about mathematics teaching and learning that support ambitious teaching practices (Lampert et al., 2010). Items that loaded on the TASSP factor reflect the belief that teachers should not necessarily answer students' questions immediately but rather let students do the work of figuring out how to solve many mathematics problems without being told what to do. Additionally, one item within the TASSP factor loaded negatively, indicating that teachers who believe in allowing for student struggle also disagreed with the characterization of mathematics teaching as the instructor sequentially modeling the activity, providing guided practice, and clarifying the student assignment. Cronbach's alpha for the TASSP factor was .662.

Belief that teachers should model for incremental mastery. The second factor, Teacher Modeling for Incremental Mastery (TMIM), reflects a belief consistent with elements of Battista's (2001) "universal script" (p. 43) for a traditional classroom, including teacher modeling of activities and approaches followed by student practice. From this perspective, memorization is critical, and instruction should emphasize the incremental mastery of procedural skills prior to solving application problems. Some aspects of the TMIM factor are consistent with behaviorist transmission theories of learning. Items that loaded on the TMIM factor reflect a belief that an integral part of mathematics learning is "mastery of a fixed set of facts and procedures" (Lloyd, 2002, p. 149). It should be noted that agreement with items that loaded on the TMIM factor does not necessarily imply a belief that teaching for meaning or conceptual understanding is unimportant. Cronbach's alpha for the TMIM factor was .653.

Awareness of students' mathematical dispositions. Factor analysis identified a third cluster of items that may be characterized as a factor addressing Teachers' Awareness of their Students' Mathematical Dispositions (TASMD). The TASMD factor reflects the extent to which teachers claim to know about their students' mathematical dispositions, such as their students' motivations for wanting to succeed in mathematics, their perceived challenges to their mathematics performance, and whether they make connections between mathematics in class and their everyday lives. Further, this factor reflects the extent to which teachers claim to take explicit actions to learn about their students' mathematical dispositions, including asking students to write about or discuss their mathematical experiences. Two additional items that loaded on the TASMD factor measured the degree to which teachers claimed to highlight multiple approaches to solving a problem and to include problems that have multiple solutions in their instruction. It may be that these two items loaded on the TASMD factor because teachers perceived these instructional practices as being responsive to their understandings and awareness of their students' mathematical dispositions. Cronbach's alpha for the TASMD factor was .657.

Implications Drawn From the Primary Analysis

The primary analysis applied hierarchal linear modeling techniques that indicated a relationship between some aspects of teachers' beliefs about mathematics teaching and learning, teachers' awareness of their students' mathematical dispositions, and their students' performance on high-stakes achievement measures. At the upper-elementary level (Grades 4 and 5), TASMD was positively and significantly associated with student achievement. Furthermore, the primary analysis revealed a statistically significant interaction effect between elementary teachers' mathematical knowledge and their TASMD scores (Figure 1). In other words, if those elementary teachers who had high knowledge also scored highly on the TASMD scale, then their students were more likely to have standardized

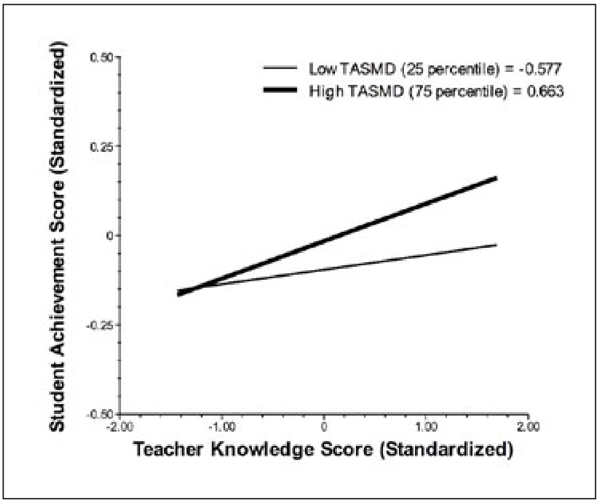

Figure 1. Interaction between upper-elementary teachers' awareness of their students' mathematical dispositions and teachers' mathematical knowledge on student achievement.

achievement scores that were statistically significantly higher than those of the students of teachers possessing comparable mathematical knowledge but scoring lower on TASMD.

At the middle-grades level, the primary analysis identified a statistically significant interaction between teachers' TMIM scores (Figure 2). The analysis suggested that those middle-grades teachers who had high mathematical knowledge scores and also scored highly on TMIM had students who were likely to have statistically significantly higher standardized mathematics achievement scores as compared to the students of teachers with comparable mathematical teacher knowledge scores but scoring lower on TMIM. In contrast, those middle-grades teachers who had low mathematical knowledge and also had high TMIM scores had students who were likely to have statistically significantly lower standardized mathematics achievement scores as compared to those who had teachers with comparable low mathematical knowledge and lower scores on the TMIM scale.

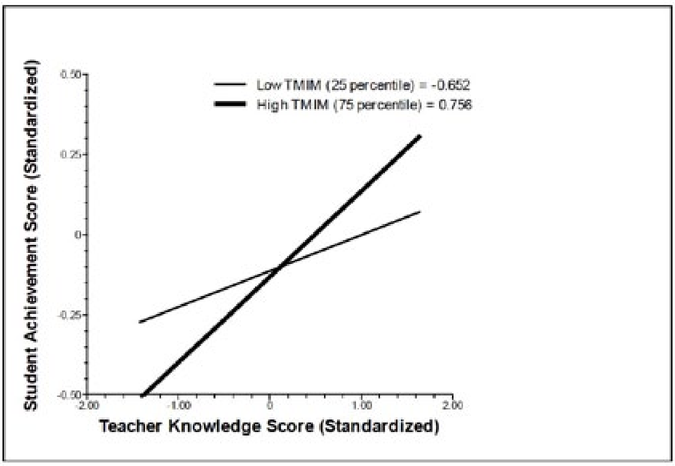

Figure 2. Interaction between middle-grades teachers' beliefs related to incremental mastery and teachers' mathematical knowledge on student achievement.

These results from the primary analysis suggest that teachers' beliefs and awareness may act as a mediator between teacher knowledge and instructional practice, supporting previous claims made by researchers (Murphy et al., 2004; Pajares, 1992). These findings support an examination of the same data to investigate whether relationships exist between teacher characteristics, including knowledge and qualifications as well as teacher beliefs and awareness. The study reported in this article is a secondary analysis of the teacher-level data utilized in the primary analysis, data that are ideally suited for the purpose of this study because teachers' scores on the scales emerging from factor analysis of the beliefs and awareness survey significantly interacted with teacher knowledge to influence student achievement. Although the primary analysis treated the three extracted beliefs and awareness factors as independent variables in models examining student achievement, this study treats those extracted factors as dependent variables in regression models.

Methodology

Subjects

The teachers in this study were 259 upper-elementary (ES) and 184 middle-grades (MS) teachers from 23 districts in Delaware, Maryland, and Pennsylvania. Table 1 presents demographic and professional characteristics of the teachers in

Table 1
Demographics and Professional Background of Participating Teachers

	Grade 4 or 5 *n* = 259	Grade 6, 7, or 8 *n* = 184
Gender (%)		
Female	86.9	78.3
Race-Ethnicity (%)		
White	80.7	75.0
Black/African American	13.9	17.9
Asian American, Pacific Islander or Hispanic	5.3	7.1
Certified (%)	93.8	91.3
Special Education Certification (%)	16.6	20.1
Holding Master's Degree (%)	45.2	44.6
Mean Number of Math Education Courses (SD)	1.0 (0.8)	1.2 (1.2)
Mean Number of Mathematics Courses (SD)	2.4 (1.3)	5.2 (3.9)
Mean Years of Teaching Experience (SD)	3.4 (1.6)	3.7 (1.7)
Taught Students an Above-Grade Curriculum (%)	32.0	65.8
Mean Hrs. of Mathematics Prof. Development (SD)	37.6 (32.2)	61.9 (44.2)
School District Location (%)		
Large City	37.8	35.9
Mid-Sized or Small City	21.1	20.1
Suburb	31.3	30.4
Rural	9.7	13.6
Proportion Special Education Students Taught	14.5	11.6
Proportion ELL Students Taught	5.1	3.4
Proportion FARM Students Taught	60.4	61.6

the sample, including some variables excluded from our final analytical regression models, either because of misalignment with our theoretical framework or for parsimony. Due to its focus, the primary analysis limited the participants sampled to teachers who were in their first 6 years of teaching experience.

The analytic samples include upper-elementary and middle-grades teachers from school districts that were solicited because they were representative of the demographic characteristics of school districts in these states. The districts and individual schools represent a variety of teaching contexts, ranging from large urban locales to distant rural locales according to the classification of school locale codes by the National Center for Education Statistics (Keaton, 2012). Across participating school districts, there were wide ranges in proportions of students attending Title I schools (from 13% to 81%), of students of color (from 14% to 92%), and of students receiving free and reduced meals (FARM) (from 21% to 81%). Although English language learners (ELL) were enrolled in many of these school districts, they were never in the majority (from 2% to 12%).

Teachers came to a local site that was not on school property for one day of testing during non-school hours. During that time they alternated between completing subsets of items from the teacher knowledge assessment, for either ES or MS teachers, and completing surveys of teachers' beliefs and awareness and of professional background and experience. These volunteer teachers were paid $350 for the day of testing. The stipend was set high to attract teachers who did not feel mathematically confident as well as teachers who did.

Independent Variables

Data collected via the aforementioned teacher knowledge instruments and professional background and teaching experience survey represented the independent variables for this study's regression models. In addition, two measures of aggregated classroom data indicating prior student achievement and the proportion of FARM students were included as independent variables in the regression models employed in this study.

Professional background, teaching context, and teacher and student experiences. The professional background and teaching experience survey produced the following data for each teacher: years of teaching experience, number of mathematics courses and mathematics education courses completed, certification status, highest degree earned, race and gender, whether assigned to teach an above-grade-level mathematics curriculum to students, and the hours of mathematics professional development attended. Additional aggregated classroom data identified for each teacher included students' mean prior performance on their state's standardized mathematics achievement test and the mean percentage of students accessing free and reduced meals funding. The aggregate mean drawn from state achievement test data served as an indicator of the prior mathematics achievement of a teacher's students. The free and reduced meal percentage served as a student poverty indicator.

The means and, if applicable, standard deviations for these independent variables are summarized separately in Table 1, noting values for the sampled ES and MS teachers. In addition, the number of mathematics courses completed by sampled ES teachers ranged from zero to eight; the number of mathematics education courses completed by sampled ES teachers ranged from zero to five courses. Some of the sampled MS teachers were certified for middle-grades mathematics instruction but were not secondary certified; thus, they would have fewer mathematics classes on their transcripts than secondary-certified teachers. The number of mathematics courses completed by sampled MS teachers ranged from 0 to 18 courses. The number of mathematics education courses completed by sampled MS teachers ranged from zero to six courses.

Teacher knowledge. Item response theory (IRT) scaled the teachers' responses on the knowledge instruments using a two-parameter model. Both the ES and MS teacher knowledge instruments contained 120 items (80 content and 40 PCK). Empirical reliability values for the measures respectively were .932 (ES) and .941 (MS). Although IRT analyses elicited separate content knowledge (CK) and PCK scores as well as combined teacher knowledge scores, to reduce the number of predictors in the regression models and to avoid issues of multicollinearity, the independent variables measuring teacher knowledge in this study accessed the scaled standardized scores on the combined teacher knowledge measure. These values were produced separately for ES and MS teachers via IRT scaling.

Dependent Variables

The three extracted beliefs and awareness factors (TASSP, TMIM, and TASMD) represent the dependent variables of interest for this study. Because different teacher knowledge instruments were administered to the ES and the MS teachers, separate models predicting the beliefs and awareness factors as a function of teacher characteristics and qualifications, including teacher knowledge, were utilized for the ES and MS teacher samples. This resulted in a total of six regression models (three dependent variables being investigated with each of two grade bands). In each of the models, the dependent variables are standardized variables, as derived by factor analysis procedures.

This study does have limitations associated with its measure of beliefs and awareness factors. First, while the reliabilities of the factors meet the reliability criteria of .650 (DeVellis, 2003), this level is not robust. Second, the sample is restricted to teachers with 6 or fewer years of teaching experience in an effort to better control for the effects of teaching experience on both teacher knowledge and student achievement. We realize, however, that novice teachers may conflate their beliefs about mathematics teaching and learning and their efficacy to engage in a particular practice expressed in the beliefs item. We attempted to reduce this limitation during the administration of the beliefs and awareness measure by verbally reminding participants that the items they would be responding to measured their beliefs and not the extent to which they feel that they could successfully implement the practices or create the learning environments that may be associated with the perspectives expressed in the items.

In addition, although a survey presents the most efficient method for collecting snapshot data measuring the beliefs and awareness of a large number of teachers, the resulting data is limited to those characteristics identified in advance during development of the instrument. Yet, instruments with Likert-type items represent one of the most commonly used formats in contemporary survey design and survey research (Babbie, 2010) and are viewed as an acceptable means for testing quantitative hypotheses, particularly when the survey is limited to a single administration.

Regression Analysis

Regression analyses were conducted to investigate the relationship between teachers' professional and demographic background, teaching context, mathematical knowledge, and their beliefs and awareness. Because the unit of analysis for both the independent and dependent variables in this study was teachers, and because of the lack of a nested structure among teachers and individual students or schools, we employed the method of ordinary least squares regression analysis to investigate the aforementioned relationship. The method of entering all independent variables at once allowed for examination of the degree to which this collection of independent variables explained the variance in teachers' beliefs and awareness. This also allowed for determination of the statistical significance of each variable when controlling for the other variables in the model.

The variables associated with teachers' professional and demographic background, as entered into the regression models, included number of years of experience (EXP); number of mathematics courses completed (MATH); number of mathematics education courses completed (MATHED); an indicator of endorsement as a certified teacher (CERT) or certified special education teacher (SECERT); an indicator of completion of a master's degree (MDEG); an indicator that a teacher was of a minority race (RACE); and an indicator that a teacher was male (GEN). The value of the EXP variable for each teacher was defined as the number of years of experience minus 1, in order to establish teachers with 1 year of experience as the reference group. The values of the MATH and MATHED variables were standardized. Variables CERT, SECERT, MDEG, RACE, and GEN were treated as dichotomous variables, with a 1 indicating that the teacher had the characteristic listed. The teacher knowledge variable (TK) referenced the teacher's standardized IRT-scale score on the teacher mathematical and pedagogical knowledge assessment.

Variables associated with teaching context included an indicator that the teacher taught an above-grade-level mathematics curriculum to students (ABVEGL) and the number of hours of mathematics professional development attended in the past 12 months (HRSPD). The ABVEGL variable was treated as a dichotomous variable, with a 1 indicating that the teacher taught an above-grade-level curriculum. Number of hours of professional development was entered as a scaled score, which was centered on the mean. Variables associated with each teacher's classroom data included the proportion of a teacher's students who received free and reduced meals (FARM) and the mean of a teacher's students' mathematics achievement scores on the 2007–2008 state standardized test (ACH). The ACH variable reflected students' mathematics achievement in the year prior to entering a teacher's classroom.

The general analytical model used was:

$$Y_i = \beta_0 + \beta_1\mathrm{EXP}_i + \beta_2\mathrm{MATH}_i + \beta_3\mathrm{MATHED}_i + \beta_4\mathrm{CERT}_i + \beta_5\,\mathrm{SECERT}_i + \beta_6\mathrm{MDEG}_i + \beta_7\mathrm{RACE}_i + \beta_8\mathrm{GEN}_i + \beta_9\mathrm{TK}_i + \beta_{10}\mathrm{ABVEGL}_i + \beta_{11}\mathrm{HRSPD}_i + \beta_{12}\mathrm{FARM}_i + \beta_{13}\mathrm{ACHi} + \varepsilon_i$$

Y_i represents the beliefs or awareness score of ith teacher, β_0 (intercept) is the mean belief or awareness score of the reference group, and ε_i is an error term associated with ith teacher. β_n is the coefficient for each variable. The referent group specified by this analytical model was teachers who scored 0 for all dichotomous measures, with 1 year of teaching experience and mean scores for the scaled variables. That is, the referent group specified by this analytical model was made up of White, female, noncertified first-year teachers who did not hold master's degrees and who had mean scores on the teacher knowledge assessment and mean-scaled scores for numbers of mathematics and mathematics education courses and for hours of professional development.

Post-Hoc Analysis

Because a review of the results of the regression analyses for ES and MS teachers indicated a possible grade-band distinction, post hoc analyses were conducted to further investigate the potential differences across the two grade bands in relationships of interest.

Results

Because the values of all continuous analytic variables were standardized and all other analytic variables were dichotomous, the regression coefficients for all measures in Tables 2 and 3 are equivalent to standardized coefficients (betas). In other words, for a one-standard-deviation increase for continuous independent variables or for a teacher possessing the respective characteristic for dichotomous variables, the coefficient represents the change in the dependent variable in terms of standard deviations. Thus, the effects on the dependent variables are comparable across all predictors.

Trends Within Upper-Elementary Teachers

Table 2 presents results from the regression analysis examining the relationship between ES teachers' professional and demographic characteristics, teacher knowledge, teaching contexts, students' experiences, and teacher beliefs (TASSP, TMIM) and awareness (TASMD) scores. After entering all independent variables, the variance explained (R-squared) for TASSP, TMIM, and TASMD was 15.8%, 9.0%, and 7.7%, respectively.

Table 2
Models of TASSP, TMIM, and TASMD for Upper-Elementary Grade Teachers

	TASSP		TMIM		TASMD	
Measure	Coefficient	*SE*	Coefficient	*SE*	Coefficient	*SE*
Intercept	-.635	.328	-.408	.341	.352	.344
Teacher Characteristics						
Years of Teaching Experience (EXP)	.099*	.039	.028	.040	-.065	.040
Number of Mathematics Courses Completed (MATH)	.053	.062	-.009	.064	-.035	.064
Number of Mathematics Education Courses Completed (MATHED)	.025	.065	.039	.068	-.100	.069
Certified (CERT)	.355	.283	.438	.294	.055	.296
Certified in Special Education (SECERT)	-.404*	.174	.065	.181	-.179	.182
Master's Degree (MDEG)	.043	.130	-.173	.136	-.071	.136
Male (GEN)	-.116	.192	.451*	.200	-.210	.201
Minority (RACE)	-.308	.169	.399	.176	.077	.177
Teacher Knowledge						
Combined CK PCK score (TK)	.179**	.063	-.199**	.066	-.130	.066
Teaching Context						
Teaching Above-Grade Curriculum (ABVGL)	.137	.141	-.306*	.146	.123	.147
Hours of Professional Development (Standardized) (HRSPD)	.141*	.063	.100	.065	.095	.066
Student Experiences						
Proportion of FARM students (FARM)	.228	.236	-.051	.246	-.347	.247
Mean Score of Prior Student Achievement (ACH)	.070	.073	.063	.076	.001	.077

*$p < .05$; ** $p < .01$

As indicated in Table 2, there was a statistically significant negative relationship between being certified in special education and scores on the TASSP factor (effect size = -.404; p = .021). Across all surveyed upper-elementary teachers, there was a positive relationship between their number of hours of professional development and their TASSP scores (effect size = .141; p = .026). This was also the finding for teaching experience, as for each year of teaching experience ES teachers' TASSP scores increased by approximately 9.9% (p = .011) of a standard deviation. There was a statistically significant positive relationship between ES teachers' mathematical knowledge scores and their TASSP scores (effect size = .179; p = .005). Neither number of mathematics or mathematics education courses completed, certification, attainment of a graduate degree, gender, assignment to teaching an above-grade curriculum, nor proportion of FARM students had a statistically significant effect on the TASSP scores of the ES teachers.

In contrast to the findings for TASSP, both gender and teaching assignment were related to ES teachers' TMIM scores. Male teachers from Grades 4 and 5 were more likely to have high TMIM scores (effect size = .451; p = .025). ES teachers who were teaching an above-grade mathematics curriculum to students had TMIM scores that were statistically significantly lower that the TMIM scores of the ES teachers who did not teach students above-grade-level mathematics content (effect size = -.306; p = .038). There was a statistically significant negative relationship between ES teachers' mathematical knowledge score and their TMIM scores (effect size = -.199; p = .003). No other teacher or teaching context variables in the model had an effect on TMIM scores. There were no statistically significant relationships between teacher or teaching context variables and the TASMD scores for ES teachers.

Trends Within Middle-Grades Teachers

Table 3 presents results from the regression analysis examining the relationship between MS teachers' professional and demographic characteristics, teacher knowledge, teaching contexts, students' experiences, and teacher beliefs (TASSP, TMIM) and awareness (TASMD) scores. After entering all independent variables, the variance explained for TASSP, TMIM, and TASMD was 14.9%, 21.2%, and 16.6%, respectively.

In contrast to the findings of the ES teacher analysis, there were no statistically significant relationships between any of the teacher or teaching context variables and the TASSP scores for MS teachers. Those MS teachers who were male or had minority status were more likely to have higher TMIM scores (Male: effect size = .495; p = .005; Minority: effect size = .446; p = .020). Consistent with the findings in the analysis of upper-elementary data, there was a statistically significant negative relationship between MS teachers' mathematical knowledge score and their TMIM scores (effect size = -.350; $p < .001$). As MS teachers completed more mathematics courses, they were more likely to have higher TMIM scores (effect size = .203, p = .020). Across all analyses, the only classroom variable that had a statistically significant relationship with teachers' beliefs and awareness scores was the proportion of FARM students in middle-grades

Table 3
Models of TASSP, TMIM, and TASMD for Middle-Grades Teachers

Measure	TASSP		TMIM		TASMD	
	Coefficient	*SE*	Coefficient	*SE*	Coefficient	*SE*
Intercept	.103	.359	.401	.346	-.059	.356
Teacher Characteristics						
Years of Teaching Experience (EXP)	-.029	.048	-.019	.046	-.042	.048
Number of Mathematics Courses Completed (MATH)	.140	.090	.203*	.086	.162	.089
Number of Mathematics Education Courses Completed (MATHED)	.125	.084	-.037	.081	-.008	.083
Certified (CERT)	.131	.285	-.264	.274	.157	.282
Certified in Special Education (SECERT)	.029	.200	.054	.193	.325	.198
Master's Degree (MDEG)	.121	.152	-.116	.146	.362*	.151
Male (GEN)	.015	.181	.495**	.174	-.248	.179
Minority (RACE)	-.129	.197	.446*	.189	.407*	.195
Teacher Knowledge						
Combined CK PCK score (TK)	.066	.096	-.350***	.092	-.297**	.095
Teaching Context						
Teaching Above-Grade Curriculum (ABVGL)	-.117	.073	-.033	.292	-.007	.073
Hours of Professional Development (Standardized) (HRSPD)	.144	.176	.188	.170	.059	.174
Student Experiences						
Proportion of FARM students (FARM)	-.161	318	-.665*	.306	-.461	.315
Mean Score of Prior Student Achievement (ACH)	.121	.096	-.047	.093	.083	.095

*$p < .05$; ** $p < .01$; *** $p < .001$

teachers' classrooms. There was a statistically significant negative relationship between MS teachers' TMIM scores and teaching a high percentage of FARM students (effect size = -.665; $p = .031$). No other teacher or teaching-context variables in the model had an effect on MS teachers' TMIM scores.

In contrast to the findings for ES teachers, three teacher variables were related to scores on the TASMD factor. There was a statistically significant negative relationship between MS teachers' mathematical knowledge score and their TASMD scores (effect size = -.297; $p = .002$). Those MS teachers who had minority status or who possessed a master's degree were more likely to have higher TASMD scores (Minority: effect size = .407; $p = .038$; Master's: effect size = .362; $p = .017$). No other teacher or teaching-context variables in the model had an effect on MS teachers' TASMD scores.

Contrasting Upper-Elementary and Middle-Grades Teachers

Post hoc analyses of variance indicated that there was a statistically significant difference between how ES and MS teachers scored on the TASSP factor ($F = 19.096$, $p < .001$) but not on the two other factors of TASMD and TMIM. ES teachers were less likely to hold the belief that students should be provided opportunities to struggle than were MS teachers.

Discussion

This study investigated relationships between teacher characteristics, including teacher knowledge, and teachers' beliefs about mathematics teaching and learning and the extent to which teachers claim awareness of their students' mathematical dispositions. Analytic findings included relationships between teacher knowledge and each of the three beliefs and awareness variables (TASSP, TMIM, and TASMD). Differing measures reflecting teachers' qualifications were significantly related to each of these teacher perception variables. Whereas teachers' race was related to TMIM and TASMD but not TASSP, the socioeconomic compositions of teachers' classrooms were related to TMIM and no other perception variable. Although the statistical analyses in this study identified those relationships, they do not explain why such relationships exist or how these relationships emerge in instructional practice. Our discussion will interpret these statistically significant relationships identified between teacher characteristics and teachers' beliefs and awareness.

Interpretations of Relationships Between Teacher Characteristics and TASSP

The positive relationship between upper-elementary teachers' mathematical knowledge and their TASSP scores suggests that more mathematically knowledgeable upper-elementary teachers believe more strongly that students should struggle with problems prior to teacher intervention as compared to their less mathematically knowledgeable colleagues. More mathematically knowledgeable teachers may feel more confident in facilitating periods of student struggle than do less knowledgeable teachers and, therefore, more strongly hold the belief. There is a growing body of research literature that suggests more knowledgeable mathematics teachers, particularly at the elementary level, have the capacity to teach mathematics in ways that engage students in challenging tasks and can maintain the cognitive demand of mathematical tasks once they have introduced tasks to their students (Charalambous, 2010; Hill et al., 2008).

This relationship between teachers' mathematical knowledge and TASSP, however, did not hold at the middle-grades level. Contextual factors frequently associated with middle-grades teaching, such as fast curriculum pacing, heightened accountability pressures, and prevalent tracking policies, may influence middle-grades teachers' TASSP beliefs, including those of more knowledgeable teachers. In particular, the demands of fast curriculum pacing and heightened accountability pressures produce time constraints that may lead middle-grades teachers to discount the practicality of supporting student struggle. Schoenfeld (2002) suggested that the increase in standardized testing and accountability pressures have lessened the complexity of mathematics instruction, and Oakes (2005) claimed that teachers of lower-tracked students often hold preconceived notions of student ability. The contexts of middle-grades teaching, particularly in relation to testing, content coverage, and tracking policies, may discourage middle-grades teachers from perceiving it to be worthwhile to allow their students to struggle with mathematical tasks for extended periods of time.

Upper-elementary teachers certified in special education, on average, did not score as high on the TASSP scale as did other upper-elementary teachers. This may be due to the program preparation and instructional experiences of special education teachers. Existing literature suggests that teachers with special education licensure hold distinctly different pedagogical stances as compared to other certified teachers (Boyd & Bargerhuff, 2009). For example, current instructional practice in special education applies a behaviorist model emphasizing task analysis, the instructional practice of breaking tasks down into small attainable parts (Boyd & Bargerhuff, 2009). This teacher-directed instructional approach does not incorporate student struggle or align with TASSP. Facilitating instructional environments that integrate students identified as having special needs with students that are not so identified (Zigmond, Kloo, & Volonino, 2009) requires considerable coplanning and coinstruction between special education teachers and teachers who do not hold special education certification. Given that teacher beliefs regarding mathematics teaching and learning influence decisions defining instructional practice (Philipp, 2007), the results of this study provide empirical evidence of the potential dissonance that general upper-elementary teachers and special education teachers may experience when coplanning. This finding highlights the need for preservice teacher education and professional development that emphasizes collaboration between mathematics education and special education. In particular, mathematics teacher educators should work with the special education community to consider the potential of reexamining task design and instructional practice from a cognitive, rather than a behaviorist, frame.

Lastly, a post-hoc analysis revealed an additional statistically significant relationship: Upper-elementary teachers' mean TASSP scores were significantly lower than those of middle-grades teachers. Mathematics teacher educators, professional developers, and mathematics coaches should consider the implications of this finding. When they address strategies for promoting student engagement with mathematics as delineated in the Common Core State Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), especially those related to perseverance, they may find elementary teachers particularly resistant. More than their middle school counterparts, elementary teachers may question the need for allowing students to struggle, even for short periods of time. Further, comparative analysis of the Common Core State Standards for Mathematics (CCSSM) and existing state mathematics standards suggest that for many states, there will be considerable increase in the cognitive demand of tasks suggested by the more rigorous content of the CCSSM, particularly in the areas of demonstrating understanding and solving nonroutine problems (Porter, McMaken, Hwang, & Yang, 2011). High-level cognitive tasks are characterized as not having a predetermined solution path and having the tendency to create frustration for students (Stein, Smith, Henningsen, & Silver, 2000). These increases in cognitive demand will likely require teachers to allow for sustained periods of time wherein students struggle and grapple with rigorous mathematics tasks. If the promise of the CCSSM is to be realized, then teacher beliefs related to the importance and necessity for student struggle will need to be addressed, particularly with elementary teachers. Past curricular and pedagogical reforms have demanded substantial paradigmatic shifts (Swars, Smith, Smith, & Hart, 2009); the CCSSM movement will be no exception.

Interpretations of Relationships Between Teacher Characteristics and TMIM

Over the past several decades there has been considerable debate about the extent to which mathematics learning environments should be characterized by teachers modeling and demonstrating formal symbolic procedures followed by students' repeated practice until mastery (Battista, 2001), as suggested in the TMIM belief. Recent discourse related to this instructional approach suggests that it is shortsighted to characterize highly directed teacher-centered instruction of this form as either good or bad (National Mathematics Advisory Panel, 2008). Various instructional approaches have different purposes and expected outcomes, and each should be considered in relation to the specific mathematics learning goals at hand (Hiebert et al., 1997).

In both the upper-elementary and middle-grades contexts, teachers who had higher scores on the teacher mathematical knowledge assessment had, on average, lower TMIM scores compared to teachers with lower scores on the mathematical knowledge assessment. This negative relationship may be reflective of less mathematically knowledgeable teachers' low efficacy and an expression of their comfort with staying within an instructional pathway that consists of guiding students incrementally through a set of skills, but further research is needed to support this hypothesis.

Despite our limited capacity to interpret this finding, the negative relationship between teachers' mathematical knowledge and TMIM scores has important implications for teacher education. This finding suggests that when introducing either practicing or prospective teachers to instructional strategies contrary to those characterized by TMIM, teacher educators need to be aware of and address teachers' mathematical knowledge. The strength of the negative relationship between TMIM and teacher knowledge revealed in this study suggests that questioning teachers' beliefs regarding TMIM without attending to teachers' knowledge may not be productive. Teacher education efforts focused on influencing prospective and practicing teachers' beliefs that integrate teachers' opportunities to deepen their mathematical knowledge may prove to be more effective than those that do not address content knowledge. This finding also lends credence to the hypothesis that when teachers learn mathematics in professional development that reflects instructional strategies contrary to TMIM (Simon & Schifter, 1993), the professional development may influence both their knowledge and their teaching beliefs.

At the middle-grades level, this analysis revealed two interesting findings associated with teachers' mathematical knowledge: The number of university mathematics courses completed by teachers was positively related to TMIM scores, and, simultaneously, a negative relationship emerged between the teachers' mathematical knowledge assessment scores and their TMIM scores. In other words, a proxy for teachers' mathematical knowledge (number of university mathematics content courses) did not relate to TMIM in the same way that a direct measure of teachers' mathematical knowledge related to TMIM. At first glance, these findings appear contradictory; however, it is important to consider that the number of mathematics courses a teacher has experienced and his or her teacher knowledge assessment score may reflect distinctly different measures of teachers' knowledge.

The mathematical knowledge assessment administered in this study measured two components of teacher knowledge: mathematical content knowledge and pedagogical content knowledge. Further, this teacher knowledge assessment measured only the mathematical knowledge teachers might draw upon to teach students the mathematics assessed on their state achievement tests. Thus, although overlapping, domains of knowledge generated through university mathematics coursework and knowledge assessed on the teacher knowledge assessment may be structured differently, particularly in relation to teachers' beliefs. Contemporary conceptualizations of the mathematical knowledge teachers use in practice support this interpretation. For example, Ball et al. (2008) suggested that the mathematical knowledge base that is directly accessed and used for teaching is not exclusively developed through university mathematics courses.

The analysis also revealed a negative relationship between a middle-grades teacher's TMIM score and his or her percentage of FARM students after controlling for other analytic variables. This finding contradicts literature characterizing teachers who teach mathematics to large numbers of students from high-poverty homes as holding instructional beliefs privileging memorization and modeling for incremental mastery of skills (Diamond & Spillane, 2004; Lipman, 2003). Over the past two decades, mathematics instructional reform initiatives at the state and district levels, frequently accompanied by increased accountability demands, have targeted schools in high-poverty communities. A common goal of these reform initiatives has been to transform mathematics teachers' instructional practices as well as their beliefs about the capabilities of students in high-poverty communities (Borman, 2005; Kitchen, Roy, Lee, & Secada, 2009). It is possible that through either reform efforts or school accountability demands, middle-grades mathematics teachers are learning that students from economic disadvantage need to be engaged in mathematics learning environments that do not focus on instructional practices solely aligned with the TMIM belief. Although this study did not track participants' engagement and involvement in these specific reform efforts and initiatives, it is important to consider how and why middle-grades teachers who taught higher numbers of students qualified for free and reduced meals had lower TMIM scores, on average, than did middle-grades teachers who taught fewer students who qualified for this economic support.

Interpretations of Relationships Between Teacher Characteristics and TASMD

Prior mathematics education research has established that students' mathematical dispositions influence their performance and participation in mathematics classrooms (Boaler, 1999; Pajares & Graham, 1999). Further, as stated, the primary analysis of this data set showed that the interaction between upper-elementary teachers' mathematical knowledge and TASMD had a statistically significant effect on student achievement (Campbell et al., in press). In the current analysis of middle-grades teachers' responses to the TASMD items, teachers' knowledge assessment scores and attainment of a master's degree each emerged as statistically significantly related to teachers' TASMD scores, yet this was not the case in the analysis of the upper-elementary teacher data.

There is little existing research that helps explain why middle-grades teachers with less mathematical knowledge, on average, would score higher on the TASMD scale than middle-grades teachers with higher mathematical knowledge. Future research is needed to explore this connection. The result associated with attainment of a master's degree is equally difficult to interpret in light of the fact that master's degree programs in education vary greatly in terms of coursework and perceived quality (Greenberg, McKee, & Walsh, 2013). Future research characterizing the nature and content of master's degree programs is needed in order to consider the role master's programs play in shaping teachers beliefs and awareness beyond what occurs during a teacher's undergraduate experience.

Teacher Race and Beliefs

A few key findings associated with teacher race, beliefs, and teaching contexts emerged at the middle-grades level:

- As the percentage of students qualifying for free and reduced meals increased, teachers' scores on the TMIM scale decreased (less supportive of TMIM);

- Middle-grades teachers of color were more likely to teach students of color, students qualifying for free and reduced meals, and students with lower standardized mathematics achievement than were White middle-grades teachers; and
- Middle-grades mathematics teachers of color, on average, had higher TMIM and TASMD scores than did White middle-grades teachers.

Taken together, these findings suggest an interesting implication: Middle-grades mathematics teachers of color may structure mathematics learning environments that reflect the TMIM belief and simultaneously work to obtain a deeper awareness of their students' mathematical dispositions more so than White middle-grades mathematics teachers. Furthermore, across the middle grades, teachers placed less emphasis on incremental mastery of basic skills as the percentage of FARM students in their classrooms increased. This trend is not as pronounced with middle-grades teachers of color. Non-White teachers' insistence on classroom order and structure (Irvine & Fraser, 1998; Ware, 2006) may partially explain the relationship between teacher race and teacher TMIM score in this analysis. Yet, the TMIM items were very specific to teachers' beliefs about mathematics teaching and learning and were not designed to measure teachers' perspectives on classroom management or general classroom structures.

A different interpretation of these findings relies on Delpit's (1986) assertion that many teachers of color, particularly African American teachers, are protective of and feel a sense of responsibility to their students of color, and this stance manifests in an insistence that their students master the basic skills of the academic discipline. Critical perspectives on progressive forms of mathematics instruction, including reforms that deemphasize rote memorization and mastery of skills, argue that such reforms could exacerbate achievement gaps and fail to provide underserved populations with a solid mathematics foundation (Apple, 1992; Brantlinger, 2011; Lubienski, 2000). It is possible, therefore, that non-White middle-grades mathematics teachers in our sample viewed the mastery of skills to be critical for the future success of their students, despite working side by side with teachers who may have held the belief less strongly. Delpit (1986) stated, "Black teachers . . . see the teaching of skills to be essential to their students' survival" (p. 383).

In keeping with Delpit's (1986) assertion, non-White middle-grades teachers' higher TASMD scores may be interpreted as responding to their minority non-White students' specific needs, namely protections and supports in racialized environments that place students of color at the bottom of the "racial hierarchy of mathematics ability" (Martin, 2009, p. 315). Middle-grades teachers of color, through their experience of being a minority in the United States, may begin their teaching career with a heightened awareness and sensitivity to their students' mathematical dispositions. Kohli (2009) suggested that, through personal experience, teachers of color are likely aware of the challenges that many students of color experience and that this awareness should be explicitly acknowledged as a resource and asset in schools. Further study is needed to better understand if and why mathematics teachers of color bring different belief and awareness systems to their practice and the ways these systems may emerge from personal experiences, historical narratives, and contextual conditions (Clark, Frank, & Davis, 2013).

Future Directions

Several significant relationships emerged that would require further research to better interpret. An important dynamic to consider as a potential influence across the three beliefs and awareness constructs is what Lortie (1975) described as the *apprenticeship of observation*. The teachers in this study all spent thousands of hours as schoolchildren and college students, and, through observation of and participation in the educational process, formed beliefs about mathematics teaching and learning based on their own experiences. It is possible, therefore, that participant responses on the beliefs and awareness scales may reflect interactions with teachers, those teachers' teaching styles and approaches, and the schooling structures participants have been exposed to throughout their entire lives. Future research can and should develop and incorporate measures of participants' apprenticeship of observation in an effort to better explain potential influences on teachers' beliefs about mathematics teaching and learning and awareness of their students' mathematical dispositions.

A second area of interest for future analysis is the inclusion of interaction terms that may illustrate differences in beliefs and awareness between teachers at different intersections of experience. There is a growing body of literature considering the ways intersectionality shapes mathematics teachers' identities and, through that, beliefs and instructional practice (Brown & McNamara, 2011). Male mathematics teachers of color may hold differing beliefs and awareness than do female mathematics teachers of color or White male mathematics teachers. Future analyses that include interaction terms that cross gender, race, and other relevant characteristics and contexts may reveal important trends that could encourage researchers from multiple disciplines to collaborate and engage in collective interpretation.

Conclusion

We conclude with two important implications. First, it is imperative that mathematics education researchers strive to better understand potential influences on teachers' beliefs and awareness. Moreover, if teacher education programs aim to influence teachers' beliefs and awareness, programmatic mechanisms must be developed to support this endeavor. Simply giving teachers more mathematics or mathematics education courses may improve their mathematical and pedagogical knowledge, yet these courses will not necessarily influence teachers' beliefs and awareness. However, across both the upper elementary and middle grades, our findings indicate a negative association between the mathematical content and pedagogical knowledge that teachers draw on during instruction and beliefs supporting incremental mastery of skills. In what ways can and should mathematics teacher education programs influence teachers' beliefs about mathematics teaching and awareness of their students' mathematical dispositions?

Second, the analysis reported herein also identified potential influences on teachers' beliefs and awareness that appear to extend beyond the domain of teacher education. In this study, teachers' personal experiences, including race and gender, appeared to influence their beliefs about mathematics teaching and learning and their awareness of their students' mathematical dispositions. Teacher education programs, in their work to shape teachers' beliefs, must acknowledge, honor, and investigate teachers' personal experiences, including experiences associated with race and gender. Teacher candidates bring those experiences into teacher education programs and into the classrooms in which they teach. It is critical that mathematics teacher educators identify ways to incorporate discussions of the ways teacher race and gender might influence teachers' belief systems, yet still encourage changes in beliefs toward belief structures that promote effective mathematical learning environments.

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APPENDIX

Survey Items and Factor Loadings

TASSP Items and Loadings	
During mathematics class, students should be asked to solve problems and complete activities by relying on their own thinking without teachers modeling an approach.	.654
Students can figure out how to solve many mathematics problems without being told what to do.	.622
During mathematics class, I do not necessarily answer students' questions immediately but rather let them struggle and puzzle things out for themselves.	.619
Students learn mathematics best by working to solve accessible problems that entail a solution process that has not been demonstrated to them.	.615
To teach mathematics, first model the activity, then provide some practice and immediate feedback, and, finally, clarify what the assignment is and how it is to be completed.	-.488
During mathematics class, discussion should focus on students' ideas and approaches, no matter whether their answers are correct or incorrect.	.444
TMIM Items and Loadings	
Students learn mathematics best by paying attention when their teacher demonstrates what to do, by asking questions if they do not understand, and then by practicing.	.673
Mathematics skills are mastered incrementally, so instruction should only focus on one skill at a time, ordered by difficulty, and not move on until most students have mastered that skill.	.574
I like my students to master basic mathematical operations before they tackle complex problems.	.549
Learning mathematics requires a good memory because you must remember how to carry out procedures and, when solving an application problem, you have to remember which procedure to use.	.523
A lot of things in mathematics must simply be accepted as true and remembered.	.522

When planning mathematics lessons, teachers need to focus explicitly on rules and procedures.	.517
Students should be homogeneously grouped for instruction and assigned to a curriculum on the basis of their prior mathematical performance.	.482
TASMD Items and Loadings	
I learn about my students' perceptions of what "doing mathematics" means through explicitly asking them (e.g., students write about it, one-on-one discussions, group discussions).	.695
I learn about my students' perceptions of connections between mathematics and their everyday lives through explicitly asking them (e.g., students write about it, one-on-one discussions, group discussions).	.661
I learn about my students' perceptions of their mathematical ability through explicitly asking them (e.g., students write about it, one-on-one discussions, group discussions).	.628
For the majority of my students, I have a good sense of their motivations for wanting to succeed in mathematics.	.521
For the majority of my students, I have a good sense of whether or not they see how the mathematics we do in class connects to their everyday lives.	.509
In order to prepare students for assessments, when students are working on a problem in mathematics, I highlight more than one approach to solving that problem.	.459
I like to use mathematics problems that can be solved in many different ways.	.426
I have a good sense of what my unsuccessful students perceive as challenges to their mathematical performance.	.417