I FOUND SINUSOIDS IN MY GAS BILL

Imagine my delight when I discovered two sine waves right before my eyes

With the current emphasis on real-world mathematics applications, many recent trigonometry textbooks include examples of sinusoidal functions for which students are required to find a model. These examples typically refer to the height of a person above the ground as a function of length of time riding on a Ferris wheel or to the number of hours of daylight in a U.S. city as a function of the day of the year. Unfortunately, as real-world as these examples are, many students still reject them as artificial and contrived. Complaints of "I'll never do this in my adult life" and "Who cares how high the Ferris wheel rider is?" are common among the eleventh- and twelfth-grade students that I teach.

Given this uncomfortable situation, imagine my delight when, in the middle of teaching trigonometric modeling, I opened my gas bill and discovered two sine waves right before my eyes! See **figure 1.** The average-monthly-temperature function was an application of the sine wave that we had actually discussed in class, but here it was being used to demonstrate a relationship between outdoor temperature and natural gas consumption. The natural gas consumption was an application that we had not seen before.

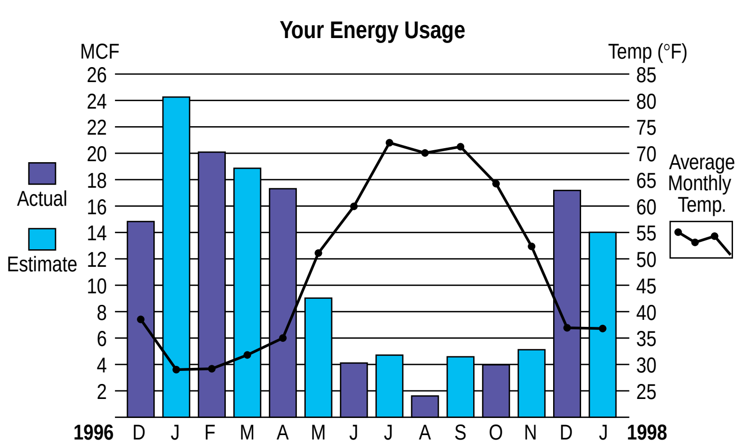

Fig. 1
Gas bill with two sinusoids
(Note that MCF is thousands of cubic feet}

As is usual with real-world data, the data do not perfectly fit the trigonometric model, but we can calculate a very good approximate model. Our class used as our general sine equation

$$\frac{y-k}{b} = \sin\left(\frac{x-h}{a}\right),$$

where $2\pi|a|$ is the period, $|b|$ is the amplitude, h is the horizontal shift, and k is the vertical shift. Hence, to find a model for these data, we need to calculate a, b, h, and k.

For the temperature graph, we let x be the number of the month of the year (1 = January, 2 = February, and so on), and we let y be the average monthly temperature. Creating a table of estimated ordered pairs that represent the temperature data, as shown in **figure 2,** is also helpful for this graphical analysis. We next calculate a. Working from January 1997 to December 1997, the sinusoid has a period of 12, since the number of months is twelve. Because $2(\pi)a$ = period = 12, $a = 12/2\pi$, or $6/\pi$.

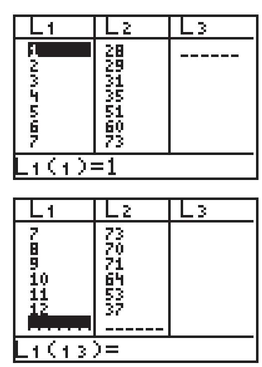

Fig. 2
Estimated ordered pairs: (month, temperature)

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Calculating k is a little more complicated. A straightforward way to find the vertical shift, however, is to find the mean of the maximum and the minimum. This approach makes sense because we are in effect finding the location of a horizontal midline, a kind of average functional value of the graph. We observe a maximum of about 73, which occurs in July; and a minimum of about 28, which occurs in January. Therefore, the vertical shift is k = (maximum + minimum)/2 = (73 + 28)/2, or 50.5.

Finding the amplitude, b, also makes use of the maximum and minimum. By calculating b = (maximum – minimum)/2, we find out how far, on average, the graph "sticks out" above and below the horizontal midline. Here, b = (73 – 28)/2, or 22.5.

Finally, to find the horizontal shift, h, we sketch on the graph the line $y = 50.5$. The "hill" of the sine curve begins to rise in May, when the temperature is about 51—close enough to 50.5 to reasonably use as h the value 5, which is the month number for May. Alternatively, the chart of estimated ordered pairs in **figure 2** reveals with the point (5, 51) that May can be a "starting time" for our sine curve.

Therefore, with $a = 6/\pi$, $k = 50.5$, $b = 22.5$, and $h = 5$, an appropriate equation for the graph would be

$$\frac{y - 50.5}{22.5} = \sin\left(\frac{x - 5}{\frac{6}{\pi}}\right).$$

Reproducing the gas-company graph with a statistical plot on a graphing calculator and then superimposing the graph of the model reveals that the model is indeed a good fit with the original data. See **figure 3.** The fit can also be confirmed by using the model to produce a table of values, as shown in **figure 4,** and comparing the resulting table with the list of estimated ordered pairs for temperature or with the temperature graph.

Analyzing the gas-use graph in a similar way is a little less elegant, probably because some of the points on the original graph are estimated and because gas use is not a function of only the temperature during a month. For example, May had almost twice the gas use of November, even though the average temperature was almost the same. Such factors as number of visitors in the home or amount of time spent at home can significantly affect use.

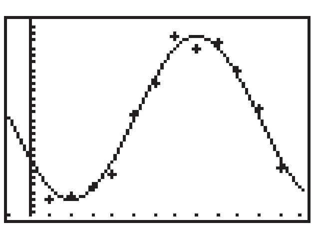

Fig. 3
Estimated model of month-temperature function, with scatterplot of (month, temperature) points from **figure 1**

X	Y1	
0	39.25	
1	31.014	
2	28	
3	31.014	
4	39.25	
5	50.5	
6	61.75	

X=0

X	Y1	
7	69.986	
8	73	
9	69.986	
10	61.75	
11	50.5	
12	39.25	
13	31.014	

X=13

Fig. 4
Table of (month, temperature) values produced by estimated model of month-temperature function

The model is a good fit with the original data

Nevertheless, looking directly at the graph and again listing estimated ordered pairs for points on the estimated-gas-use bar graph, as shown in **figure 5,** may be helpful. Then, again by working from January 1997 to December 1997, the maximum use is about 24.2 MCF (thousands of cubic feet) and the minimum is about 1.8 MCF, with a period of twelve months. The previous calculation techniques reveal that compared with $y = \sin x$, the graph has been vertically shifted 13 units and has an amplitude of 11.2 units. In addition, the graph is horizontally shifted to start approximately between April and May—say, month 4.5, since the mean MCF for the two months is about 13. If we start at this point, however, the graph also requires a reflection of the present function about $y = 13$ to produce the graph. An equation for this graph would be

$$\frac{y - 13}{11.2} = -\sin\left(\frac{x - 4.5}{\frac{6}{\pi}}\right).$$

L1	L2	L3
1	24.2	------
2	20	
3	18.8	
4	17.5	
5	9.2	
6	4.2	
7	4.8	

L1(1)=1

L1	L2	L3
7	4.8	
8	1.8	
9	4.8	
10	4	
11	5	
12	17.5	

L1(13)=

Fig. 5
Estimated ordered pairs: (month, gas use in MCF)

Using the sine function is a little awkward here because of the need for the reflection. A cosine curve with period 12, vertical shift 13, amplitude 11.2, and horizontal shift 1—because the maximum, which is a starting point for the cosine curve, appears in January—would probably be a more natural choice and would have as its equation

$$\frac{y-13}{11.2} = \cos\left(\frac{x-1}{\frac{6}{\pi}}\right).$$

This graph is not identical to the sine-based curve because of the "eyeballing" techniques used to determine the horizontal shift. See **figure 6.** In general, however, any sinusoid can be modeled as an offspring of either $y = \sin x$ or $y = \cos x$ because of the relationship $\sin(x + \pi/2) = \cos x$. Also, the reader should notice that neither graph demonstrates the same beautiful fit as the temperature graph, but each comes reasonably close to the observed data.

The gas-bill graphs clearly illustrate that sinusoidal curves are useful and meaningful in an everyday context

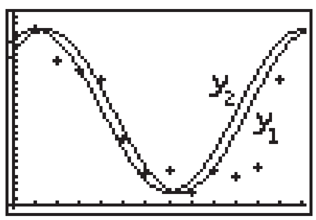

Fig. 6

$$\frac{y_1-13}{11.2} = -\sin\left(\frac{x-4.5}{\frac{6}{\pi}}\right)$$

is not identical to

$$\frac{y_2-13}{11.2} = \cos\left(\frac{x-1}{\frac{6}{\pi}}\right)$$

as a model of the (month, gas use) data.

This article would not be complete without noting that the TI-83 graphing calculator, for example, can automatically use regression techniques to fit a sinusoid to data. **Figures 7** and **8** illustrate the TI-83 versions of best-fit graphs for the temperature and gas-use data, respectively. Clearly, these graphs are more accurate and faster than those created by techniques described previously in this article. In particular, the rather crude "eyeballing" technique for finding the horizontal shift of the gas-use graph by hand does not produce the same horizontal shift observed in the TI-83 version. Otherwise, however, the equations produced by the TI-83 rather closely resemble those produced by less precise techniques. Deriving equations by hand by finding *a, b, h,* and *k,* though, gives students—whether in a whole-class, small-group, or individual setting—a chance to really create the model and develop some understanding of the relationship between data points and the equations representing them. The attractive aspect of the gas-bill graphs is that they clearly illustrate that sinusoidal curves are useful and meaningful in an everyday context.

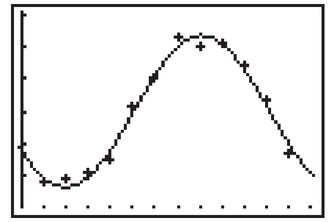

Fig. 7
TI-83 best-fit graph for (month, temperature) data:
$y = 23.44633922 \sin(0.5164431541x - 2.597127962) + 49.9408353.$

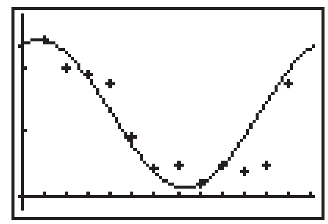

Fig. 8
TI-83 best-fit graph for (month, gas-use) data:
$y = 11.35628586 \sin(0.4760116037x + 1.2196451099) + 12.68535547.$

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