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# Teaching Probability and Statistics through Game Shows

The mathematical branch of probability has its origins in games and gambling. And so it is not surprising that our most common examples for teaching probability in the classroom—coins, dice, and cards—come from this domain. But how else can we teach probability, once these basic examples have been exhausted? Television game shows provide an answer. What we present here is just one of the game shows we have used to introduce students to several aspects of statistics and probability and to spark their interest in these topics.

While game shows have declined in popularity since the early days of television, most teenagers have seen *The Price Is Right* at some point. In fact, college students make up a significant part of a typical audience for this game show. Shows such as these provide a familiar, game-related setting for students to investigate probability and statistics.

This department is designed to provide in reproducible formats activities for students in grades 7–12. The material may be reproduced by classroom teachers for use in their own classes. Readers who have developed successful classroom activities are encouraged to submit manuscripts, in a format similar to the "Activities" already published, to the journal editor for review. Of particular interest are activities focusing on the Council's curriculum standards, its expanded concept of basic skills, problem solving and applications, and the uses of calculators and computers. Please send submissions to "Activities," *Mathematics Teacher,* 1906 Association Drive, Reston, VA 20191; or send electronic submissions to mt@nctm.org.

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In this article, we present activities relating to the game "Let 'Em Roll" from *The Price Is Right*. Each of the activities has a different learning emphasis and level of difficulty; all would be appropriate for a high school AP statistics class. Students will need graphing calculators with a random number generator for these activities. We have found that the most effective and engaging way to introduce these lessons is to show a clip of the game itself, which usually runs about two to three minutes.

## "LET 'EM ROLL"

*Game description (condensed):* The contestant rolls five dice a maximum of three times in an attempt to win a new car. The five dice are exactly the same: Each has a car pictured on three sides and dollar amounts ($500, $1000, and $1500) on the other three sides. A contestant who rolls cars on all five dice wins the car. If a rolled die does not show a car, the contestant can either take the money shown on the die and leave the game or "freeze" the die or dice showing the car and roll the remaining dice, should the contestant have rolls remaining. If the contestant obtains five dice with cars by the end of three rolls, she or he wins the car; if not, she or he wins the total dollar amount shown on the dice on the last turn.

*Set-up:* Each student should have a graphing calculator with a random number generator. In this article we use the TI 82, TI 83, or TI 83 Plus calculator.

*Learning objectives:* Probability through simulation, independence, complement rule, binomial random variables, expected value

## PART 1: SIMULATION USING THE CALCULATOR

Let 1, 2, 3 represent the sides with CAR on them, 4, 5, 6 the money. In **sheet 1,** students will first consider the chance of winning the car with just one roll of the five dice. After the students have repeated the simulation ten times, they answer questions about the results.

The answer to the last question is the ratio of the previous two answers (# of cars won/# of trials). From this simple exercise, students learn an easy way to simulate the game and are reminded how simulation can be used to estimate the probability of an event.

Next, expand the simulation. Students will "roll" all five dice three times, but the focus here is whether each die *eventually* yields CAR (see **sheet 2**). Ultimately, we want to estimate two quantities: (1) the probability a die, in three tosses, eventually says CAR, and (2) the probability the contestant wins the car.

## PART 2: MATHEMATICAL THEORY

In **sheets 3(a)** and **(b),** students will use their understanding of basic probability rules to find the mathematically correct answers to the probability questions on sheets 1 and 2. The connection is critical: Students should notice that their simulation answers on sheets 1 and 2 are close to, but not identical to, the mathematical answers on sheets 3(a) and (b).

Questions 1–3 on sheet 3(a) are straightforward. Question 2 simply uses independence of successive die rolls, while question 3 uses the complement rule. Question 4, while not totally critical to the story, references the binomial distribution (with $n = 5$ and $p = 7/8$, based on the earlier questions). The student can then answer question 5 two ways: using independence of the dice directly, or applying the binomial probability formula.

Now we expand our horizons to consider not only probability but also expected payoff.

Not surprisingly, the expected payoff from rolling the last die one more time far exceeds \$1500. But the purpose of questions 6–10 is to lead the students through the process of "assembling" a discrete random variable and then interpreting its expected value.

Finally, you can offer the following challenge question to your students, which reflects a commonly seen situation on *The Price Is Right:*

> Suppose you only have *three* cars after the first two rolls. Find the probability you'll win the car on the last roll (easy), and find the expected payoff based on re-rolling the last two dice (hard). How much money would you need to have showing on those remaining dice after the second roll not to risk it?

## SOLUTIONS, SHEETS 3(A) AND (B)

1. $3/6 = 1/2$ or .5
2. $(1/2)(1/2)(1/2) = 1/8$ or .125
3. $1 - 1/8 = 7/8$ or .875; alternatively, $(1/2) + (1/2)(1/2) + (1/2)(1/2)(1/2) = 7/8$ or .875
4. $x$ is a *binomial* random variable with $n = 5$ and $p = 7/8$
5. $P(x = 5) = (7/8)^5 = .5129$—a better than 50 percent chance of winning
6. $P(\text{car}) = 3/6 = 1/2$. There is no right or wrong answer to whether we should "risk it."
7. P(\$500) = P(\$1000) = P(\$1500) = 1/6; P(car) = 1/2, as above
8. See **table 1** for the answer.
9. $E(y)=500(1/6) + 1{,}000(1/6) + 1{,}500(1/6) + 15{,}000(1/2) = \$8{,}000$
10. Absolutely!

**TABLE 1**

**Answer to Question 8**

| $y$ | 500 | 1,000 | 1,500 | 15,000 |
|---|---|---|---|---|
| $p(y)$ | 1/6 | 1/6 | 1/6 | 1/2 |

## OTHER GAME SHOW ACTIVITIES

We have developed activity packets for other *The Price Is Right* games as well as other game shows. These packets and their solutions are available for free at statweb.calpoly.edu/carltonm/gameshows.

We wish to thank Brad Francini for granting us permission to use his description of "Let 'Em Roll" (gscentral.net/pricing.htm). ∞

Activity sheets follow.

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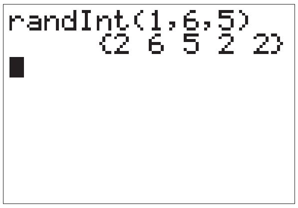
# First Simulation

Sheet 1

Use the calculator to simulate one throw of the 5 dice. Type the function randInt(1,6,5), [ENTER].

Example




For this trial, we have CAR on the first die, money on the second and third dice, and CAR on the fourth and fifth dice. So we did not win the car.

Repeat this simulation 10 times. (There is no need to type everything in again; just press [ENTER] each time.)

1. How many times did you win a car?

2. Now pool the class results.
   # of trials (# students × 10)

   # of cars won

   $P$(winning a car with one toss of all 5 dice)

*From the April 2005 issue of* MATHEMATICS teacher

# Second Simulation

Sheet 2(a)

Use the calculator to simulate three throws of the 5 dice. Type the function randInt(1,6,5), [ENTER], [ENTER], [ENTER].

Example

```
randInt(1,6,5)
     {6 6 1 1 5}
     {6 4 2 1 6}
     {2 3 1 2 6}
```

For this example, we have one of the CAR dice in positions 3 and 4 on the first throw, in positions 1 and 2 on the third throw, but not at all in position 5. So, we did not win the car—but we were close!

Play the game 10 times.
Each time, record the number of CARs you have after each roll.
NOTE: Since you keep each CAR that you previously rolled, this number should never decrease.

| | Number of dice that say CAR | | |
|---|---|---|---|
| Game # | . . . after 1st roll | . . . after 2nd roll | . . . after 3rd roll |
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |

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# Second Simulation (continued)



Use the data in the completed chart on sheet 2(a) to answer the questions.

1. What proportion of your dice *eventually* said CAR?

2. Now pool the class results.
   # of trials (# students $\times$ 10)

   # of dice that eventually said CAR

   $P$(a die eventually says car)

3. How many times did you win the car?

4. Now pool the class results.
   # of trials (# students $\times$ 10)

   # of cars won

   $P$(winning a car with three throws of the 5 dice)

# Mathematical Theory

Sheet 3(a)

Use your experiences from the simulations to answer the following questions. Explain your answers.

1. If you roll one die 1 time, what is the probability you roll a car?

2. If you roll one die 3 times, what is the probability you *never* roll a car?

3. What is the probability that, in 3 rolls of one die, you will eventually roll a car?

4. Let $x$ represent the number of dice (out of the 5) in "Let 'Em Roll" that eventually show a car after 3 rolls. What is the probability distribution of $x$?

5. What is the probability that all 5 dice eventually show a car? (That is, what is the chance of winning "Let 'Em Roll"?)

# **Mathematical Theory** (continued) Sheet 3(b)

Suppose you have rolled the dice twice, and you have four cars showing. The fifth die shows $1,500. Should you keep the $1,500 or roll that last die?

6. What is the *probability* you will win the car if you roll the last die one more time? Based on this result alone, would you roll the die?

Consider the problem from the *expected winnings* perspective.

7. Suppose you roll that last die. What are the probabilities of rolling $500? $1,000? $1,500? A car?

8. Suppose the car is worth $15,000. Let $y$ equal your winnings after the last roll. Based on the previous question, write down the probability distribution of $y$.

9. Find the expected value of $y$.

10. Compare this expected value to the $1,500 you have in hand (assuming you don't roll the last die). Based on expected payoff, would you roll the last die?