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# Biology as a Source for Algebra Equations: The Heart

At the high school level, physics is often the area of science looked to when teachers seek applications and connections with mathematics. However, biology also can provide opportunities to represent relationships and solve problems using simple algebra. This article developed from work in an inquiry-oriented, interdisciplinary high school course that integrated first-year algebra with an introductory environmental biology/anatomy and physiology course. Lessons and activities were derived by identifying areas where mathematics and biology content intersect (Horak 1997, 2000; Horak and Horak 1999). Team-teaching the courses in a two-hour block allowed for seamless study of the two areas, which deepened students' understanding and appreciation of both subjects.

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## HEART

Direct variation (and thus the linear equation) appears while studying the heart. The relationship among the heart cardiac output ($C$), the heart stroke volume ($V$), and the heart rate ($R$) can be expressed as $C = V \cdot R$ (Des Jardins 2002; Katz 2001). Each variable in this relationship represents a rate. For this equation, $V$ represents the volume of blood pumped from the heart **per stroke,** $R$ represents the number of heartbeats **per unit of time,** and $C$ represents the amount of blood pumped by the heart through the body **per unit of time.** In biology the terms *stroke* and *beat* are used in specific situations, but they may generally be thought of interchangeably in this context. This equation tells us that the output of the heart in a given amount of time is related directly to the volume of each stroke (beat) and the rate at which the heart is beating. Thus, the relationship is a direct variation among the rates.

In the "Understanding Your Heart" activity (**sheet 1**), students use this equation to model their own cardiac output. To graph the equation on the coordinate axis in question 3, students must graph an equation with a slope of 85. This slope is very steep, so determining the scale on the $C$-axis proved to be a valuable exercise for students. Discussions arose about whether they needed to begin the scale at zero and, if so, what the increments could be. If students use a graphing calculator to graph the equation, this task is worthwhile for choosing the appropriate viewing window using the context of the problem. The problems on the activity sheet ask students to determine their cardiac output and then to use that value to determine how long it takes their total volume of blood to circulate once through their bodies. They then compare these results with those for an infant.

Most students are aware that the heart of an athlete beats more slowly than that of a person of the same age who is not an athlete. Information about athletes' hearts is available online. Some useful sites are available on the Internet (Charles Sturt University 2001; Seiler 1996). When we asked our students what it means to be "athletically fit," they responded with a variety of characteristics. Among them were "have more muscles," "are strong," "are buff," and "aren't fat." Eventually they identified the notion that the heart of an athlete beats more slowly as he or she becomes more physically fit. Without much guidance, they usually determined the reason: An athlete's heart pumps more efficiently and effectively, and thus his or her stroke volume is greater. In the last exercise of the activity, students use the equation $C = V \cdot R$ and their own cardiac output to examine this relationship. If $C$ is constant and $V$ increases, what happens to $R$? Students make a table of values, so they can see that as $V$ increases, $R$ decreases. Graphing this equation using their table of values gave students an example of an equation that is not linear. We did not go into detail about this function's being hyperbolic, but students were able to make generalizations about the shape of such a graph.

## RESCUE BREATHING

In doing rescue breathing on an adult, a rescuer initially gives two breaths before beginning a cycle of ten breaths per minute (National Safety Council 2001). The number of breaths a rescuer administers to a victim in a given number of minutes can be represented by the linear equation $n = 2 + 10t$, in which $t$ is the number of minutes during which rescue breathing is being administered and $n$ is the number of breaths administered after $t$ minutes.

This resembles the standard linear equation $y = mx + b$. This equation is most meaningful for modeling this situation, however, if the constant is written first in the equation, because it represents the initial two breaths, given before the normal cycle begins. This is a concrete example when $10t + 2$ means the same algebraically as $2 + 10t$, but the standard form does not describe as well the real-world situation. When this equation was graphed on a coordinate plane, our students saw that 2 is the $n$-intercept and 10 is the slope. What do these statements mean in this context? The intercept of 2 represents the two initial breaths (when $t = 0$), and the slope of 10 is the rate (number of breaths/number of minutes) at which the breaths are given.

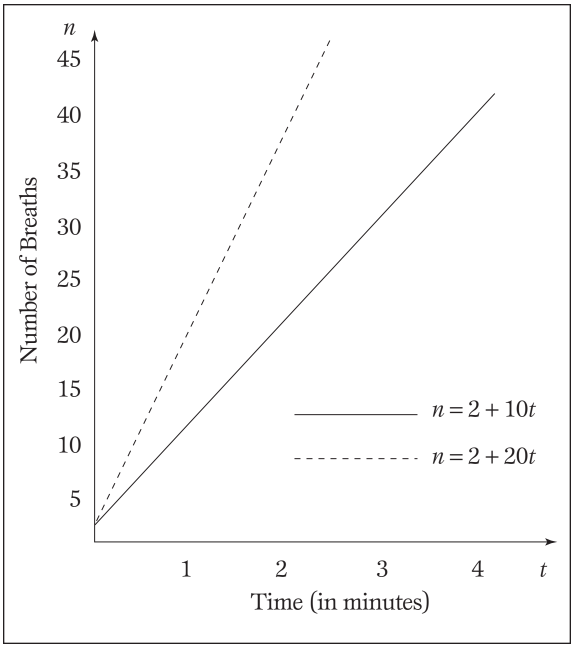


**Fig. 1** Graphs of number of breaths in rescue breathing

Similarly, in rescue breathing on an infant or child, after two initial breaths the rescuer begins a cycle of twenty breaths per minute (National Safety Council 2001). The number of breaths a rescuer administers to an infant in a given number of minutes can be represented by the linear equation $n = 2 + 20t$, in which, again, $t$ is the number of minutes during which rescue breathing is being administered and $n$ is the number of breaths administered after $t$ minutes.

Graphing this equation on the coordinate plane gives a line that has an $n$-intercept of 2 (the two initial breaths) and a slope of 20, the rate at which the breaths are given.

What generalizations have students made by comparing these two equations for rescue breathing? Students noticed that both equations have a constant of 2, since rescue breathing for both children and adults begins with two initial breaths. The $t$-coefficients, or slopes, are different because they represent the number of breaths a rescuer does per minute, i.e., they stand for real quantities, the **rates** of the rescue breathing (breaths per minute).

If these two equations are graphed on the same coordinate axis, as shown in **figure 1,** students can see that they both have the same intercept but that the line for the infant rescue breathing is much steeper than the one for adult rescue breathing. In the integrated algebra/biology course referred to earlier, students were trained in CPR and did rescue breathing as part of that training. They actually experienced the difference in the two rates. The slope of 20 meant doing twice as many breaths per minute as a slope of 10. The greater slope not only looked steeper but doing the rescue breathing made the slope even "feel" steeper for the students.

## CONCLUSION

Integrating high school mathematics with biology has the benefit of enriching students' learning of both. Mathematics helps students understand and model some biology concepts. Other linear relationships occur between the diameter of a twig of certain plant species and its length, the rate at which nerve impulses are conducted along a nerve fiber and the nerve diameter, and the food requirements of most animals and their size (Clow and Urquhart 1974). Other relationships are, of course, not linear. For example, the resistance of blood flowing through a blood vessel is inversely proportional to the fourth power of the radius of that blood vessel. This can be expressed as $R = 1/r^4$, where $R$ is the resistance of the blood flow and $r$ is the radius of the blood vessel. Embedding these examples into our normal teaching of mathematics requires little time taken from the usual content, but such examples enrich the content and exemplify algebra's use in the "real world."

## SOLUTIONS

1. Answers will vary.

2. Answers will vary.

3. $C = 85 \cdot R$. See **figure 2.**

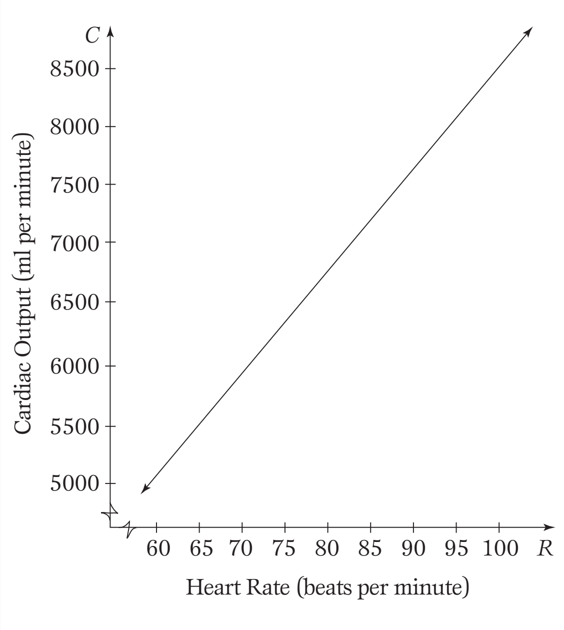


**Fig. 2** Answer to question 3

4. Cardiac output is 6375 ml if the heart rate is 75 beats per minute, 5100 ml if the heart rate is 60 beats per minute, and 8500 ml if the heart rate is 100 beats per minute.

5. Your heart rate increases when you exercise. As a result, your cardiac output increases. Your cardiac output would need to increase to get more oxygen to your muscles.

6. 5 liters = 5000 ml. Furthermore,

$$C = 85 \text{ ml per stroke} \cdot 70 \text{ beats per minute}$$
$$= 5950 \text{ ml per minute}$$
$$5000 \div 5950 \approx 0.84 \text{ minutes}$$

Thus, it takes your heart approximately 0.84 of a minute to circulate 5 liters of blood through your body.

7. $C = 25$ ml $\cdot$ 90 beats per minute = 2250 ml per minute

8. $2000 \div 2250 \approx 0.89$ minute. This length of time is approximately the same for an adolescent.

9. See **figure 3.**

| $C$ (your value) | $V$ | $R$ |
|---|---|---|
| 5950 | 70 | 85 |
| 5950 | 80 | ≈74 |
| 5950 | 90 | ≈66 |
| 5950 | 100 | ≈59 |
| 5950 | 110 | ≈54 |
| 5950 | 120 | ≈50 |

**Fig. 3** Answer to question 9

10. See **figure 4.** The graph of $C = V \cdot R$ is a hyperbola when $C$ is a constant. When $V$ gets large, $R$ gets small, and when $R$ gets large, $V$ gets small. No, the graph of the equation is not a line.

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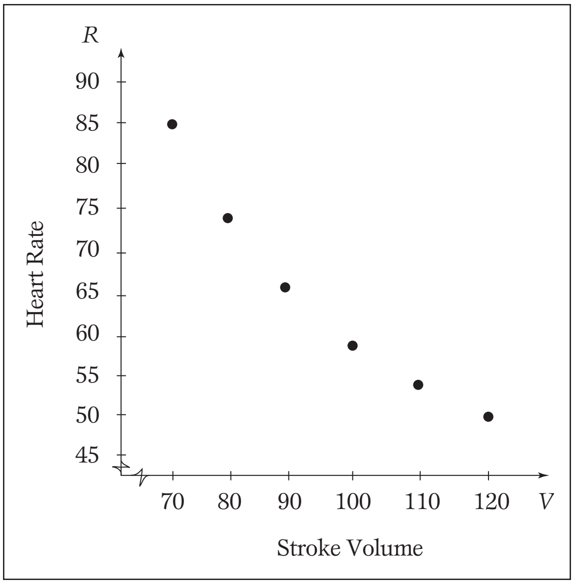


**Fig. 4** Answer to question 10


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Activity sheets follow.


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# Understanding Your Heart

Sheet 1

You are going to use the equation $C = V \cdot R$ to learn something about your own heart. In this equation, $C$ (cardiac output) represents the amount of blood pumped by the heart through a person's body per minute, $V$ (stroke volume) represents the volume of blood pumped from a person's heart per beat, and $R$ (heart rate) represents the number of times the person's heart beats in a minute. The average amount of blood pumped from a teenager's heart each time it beats is approximately 85 ml. Thus, you will use $V = 85$ ml during this activity.

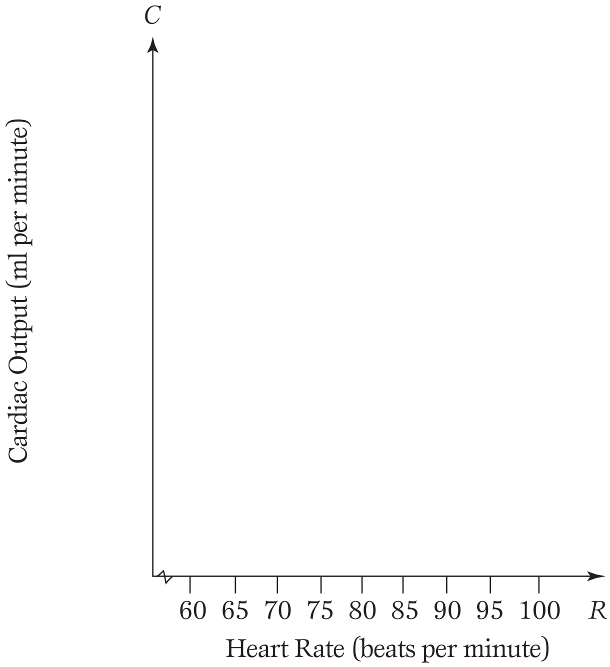
1. With a partner, determine the number of times your heart beats in a minute ($R$). To do this, sit quietly in a chair while your partner takes your pulse for 10 seconds. The best place for your partner to take your pulse is on your wrist. Your teacher will show you the exact location.

   Number of heart beats in 10 seconds = ______

   Number of heart beats in 1 minute = ______

2. Using the equation $C = V \cdot R$ and your heart rate (beats per minute) from question 1, compute the amount of blood your heart is pumping through your body per minute (your cardiac output) while you are sitting quietly.

3. Write an equation to describe the output per minute of a heart that has a $V = 85$ ml per beat. Your variables in this equation will be $C$ and $R$. Graph this equation on the coordinate axis. Values for $R$ have been provided for you on the $R$ axis, but you will need to determine the values to use on the $C$-axis.




4. What is your cardiac output if your heart rate is 75 beats per minute? ______
   60 beats per minute? ________
   100 beats per minute? _______

5. When you exercise, what happens to your heart rate ($R$)? As a result, what happens to your cardiac output ($C$)? Why do you think your cardiac output would need to change in this way when you exercise?

6. Suppose that you have approximately 5 liters of blood in your body. If your heart rate is 70 beats per minute, how long does it take for your heart to circulate 5 liters of blood through your body? You will need to use information from the graph in question 3 to get the answer to this problem.

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# Understanding Your Heart (continued) Sheet 1

7. A young child will have a smaller heart and, thus, a smaller value for $V$. Suppose $V = 25$ ml for a child, and suppose the child's heart rate ($R$) is 90 beats per minute. What is her cardiac output ($C$)?

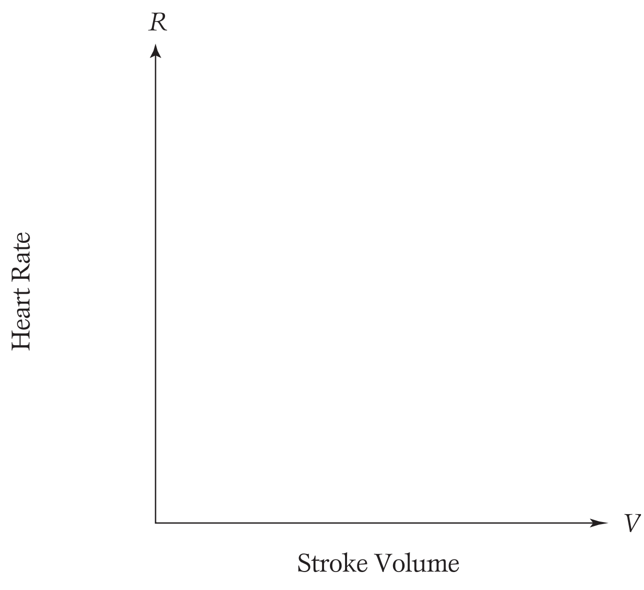
8. A child has approximately 2 liters of blood in his body. Using the cardiac output you calculated in question 7, how long does it take for the heart of a child to circulate this amount of blood through his body? How does this length of time for a child compare to the length of time for your blood to circulate through your body?

9. You are now going to use the cardiac output equation to model the effect exercising and becoming more physically fit has on your heart. In question 2 you computed your cardiac output while you are sitting quietly. This value will now be a constant for $C$ in the equation $C = V \cdot R$. Make a table for values of $V$ and $R$ that satisfy your equation. You should have at least 5 ordered pairs ($V$, $R$). Let the domain for $V$ be from 70 to 120 ml.

| $C$ (your value) | $V$ | $R$ |
|---|---|---|
| | | |

10. On the coordinate axis below, graph the ordered pairs ($V$, $R$) from your table of values in question 9. You will need to determine the scale of values for each axis.

    Describe the graph of the equation $C = V \cdot R$ where $C$ is constant and $V$ and $R$ are variables. What happens when $V$ gets large? When is $R$ large? Is the graph of the equation a line? Explain your answer.

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