

A FORMATIVE ASSESSMENT

Algebra students create their own funny faces onscreen after studying several parent functions and their transformations.

any American students begin their high school mathematics study with the algebra 1-geometry-algebra 2 sequence. After algebra 2, then, students with average or below-average mathematical ability face a dilemma in choosing their next mathematics course. College admissions counselors strongly encourage all students to take more mathematics in preparation for postsecondary study. For students to succeed in higher mathematics,





primary focus of the FST course is to allow students to learn mathematics conceptually through explorations and to examine mathematics in real-world contexts.

understanding the concept of functions is paramount. Accordingly, secondary school mathematics departments have found creative ways to provide more extensive treatment of functions in courses such as advanced algebra or functions, statistics, and trigonometry (FST).

At the high school where this activity was implemented, the mathematics department usually offers two sections of an FST course for juniors and seniors. (The course title comes from the University of Chicago School Mathematics Project's textbook Functions, Statistics, and Trigonometry [1998].) The major content differences between FST and a traditional precalculus course are that FST incorporates descriptive statistics and covers functions and trigonometry in less depth. Students who take an FST course in high school typically take an introduction to statistics, a precalculus, or a liberal arts mathematics course at the university level. A primary focus of the FST course is to provide students opportunities to learn mathematics conceptually through explorations and to examine mathematics in real-world contexts.

THE ACTIVITY

Two weeks into the academic year, I observed that my FST students, despite having had two years of algebra study, were unable to recall algebra facts or demonstrate procedural understanding. Moreover, they showed lower-than-expected motivation for learning mathematics. To help overcome these shortcomings, I designed a Funny Face Contest, a formative assessment activity on transforming functions.

At the beginning of this activity, I gave verbal directions specifying two criteria for students to use in creating a unique funny face. First, students, had to use at least one example from each parent function they had already studied: linear, quadratic, square root, absolute value, and "semicircle." Second, students were restricted to graphing at most ten functions but could plot points using the Stat Plot option (students were using TI-83 Plus graphing calculators, but this activity could be completed using any graphing calculator).

Most students came to my class knowing how to input functions into the graphing calculator. Students were also proficient in setting the Window to view particular portions of the functions. Nonetheless, I reviewed some important calculator features to aid their efforts:

- Choose ZStandard from the Zoom option. This action will provide a [-10, 10] by [-10, 10] window. Then, modify the window to provide the optimal view of your funny face.
- Turn on the axes. This setting should help you when transforming functions. Once you complete your funny face, be sure to turn off the axes.
- Be careful about restricting the domains of the functions you use. Doing so may make it easier for you to draw your funny face. (An example later in the article illustrates restricted domain.)

Having provided these criteria, I gave students the rest of the forty-five-minute class period to brainstorm for possibilities for creating their funny faces. Students were asked to finish the project for homework—one salient feature of formative assessments is that they should not be too time-consuming. During class, some students collaborated while others worked independently. Throughout the class, students sought assistance from me. Their inquiries ranged from "How do I restrict the domain of this function?" to "Is this what you want?" Answering these questions, both during class and afterward, provided a valuable opportunity to probe for student understanding and to correct any misunderstanding. At this stage, I emphasized that students had ownership of their creations and that possibilities were limited only by their effort and creativity.

STUDENTS' CREATIONS AND REFLECTIONS

On the following day, at the beginning of class, students turned in their funny faces. While I captured the screen shots, I asked students to reflect on how they created their funny faces and to write up their reflections.

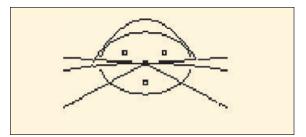


Fig. 1 Brandon's "Von Steubing" was voted the winner by one class.

Following are some samples of the funny faces created by my students and the students' reflections on their work:

- "After making two eyes and a nose with Stat Plot, I incorporated the other equations around the nose which was the center. I learned how to correctly place a certain shape from the equations by guessing and checking and I gained a better understanding of the equations." (See **fig. 1.**)
- "The functions I used for selected parts were decided by the shapes each function made. Like a semi-circle for a mouth and a V shape for the body." (See fig. 2.)
- "I began by experimenting with different formulas. I wanted to create a face that resembled something important to me. I chose to add whis-

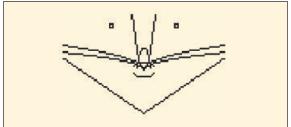


Fig. 2 For "Big Kitty," Rachel made use of semicircles and absolute value.



Fig. 3 Whiskered animals, like Jason's "Rufus," were popular choices.

kers so my face would resemble my kitty. The nose wound up being a circle with a parabola after seriously contemplating where I should put those two functions." (See fig. 3.)

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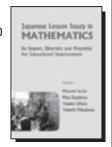
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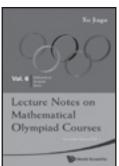


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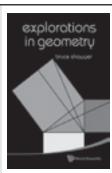


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the debriefing sessions represented a dialogue on significant mathematical concepts.

- "To make my funny face, I graphed a circle face. Then I played with the window in order to get the face to the left side of the screen. Once that was there, I played around with the different functions to make the body." (See **fig. 4.**)
- "I listed all types of functions I needed to use and checked them off as I used them. I experimented with different equations until I had them into an arrangement that looked like a face." (See **fig. 5.**)
- "I knew I could have hair coming off the side of the head by using $y = \sqrt{x}$ graphs and moving them up to the top of the head. To make a mouth, I used the semi-circle equation and lowered it down." (See fig. 6.)

After students had handed in their written reflections, I asked them to share their approaches, insights, and intuitions with classmates. Several students mentioned that at the onset they sketched their funny faces on paper. Others described how they transformed certain functions. At times I asked them to elaborate.

This debriefing session allowed students to learn from one another and to praise the creative

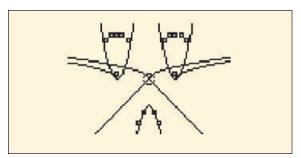


Fig. 4 Grayson used translations to create "Jimmy."

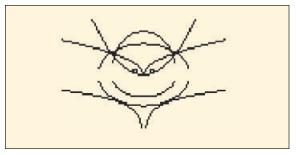


Fig. 5 Sam's "Wack-Teenage Mutant Ninja Turtle" was the unanimous choice in the other class.

aspects of the final products as well as the students' effort. The session also allowed me to guide further instruction (Ashline 2005). For instance, I felt that I needed to place more emphasis on the importance of using precise mathematical language. In short, these exchanges represented a dialogue on significant mathematical concepts.

ASSESSING THE PROJECT

To verify that students fulfilled the first requirement—using "at least one example of each parent function"—I captured their funny faces as well as the screen shots of Y = (equation editor) and L1-L2 (list editor). For example, the screen shots of Brandon's "Von Steubing" represent his funny face (see fig. 1) as well as six functions and four points (see figs. 7a and 7b).

Grading formative assessments can be difficult. The inquiry "Is this what you want?" indicates my lack of clarity on how I would grade the assignment. Initially, I had perceived that grading this twentypoint assignment, which represented about 2 to 3 percent of the quarter's grade, would not pose much difficulty. I was mistaken. All students expected to earn twenty points for their funny faces, yet some did not.

My verbal directions to use at least one example of each parent function were not sufficient. I should have provided the following criteria:

- 1. Did you use all five parent functions?
- 2. Did you provide the screen shots of the funny face, the equation editor, and the list editor?
- 3. Is the window configuration at a desired setting?

Teachers interested in implementing this activity can modify these requirements.

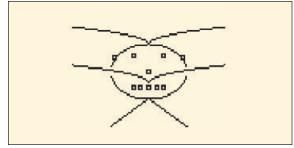


Fig. 6 Square root functions allowed Michael to create weird hair for his funny face, "Dan K."

In grading, I took off one point from the work of several students for not fulfilling the first criterion. Otherwise, I was quite liberal in giving all twenty points to most of the students. All students fulfilled the latter two criteria: capturing the relevant screen shots and providing the optimal window configurations.

JUDGING THE CONTEST ENTRIES

I had each student in one class choose the two best funny faces of the other class and vice versa. Students did not know that I would be using this form of peer judging. Further, I did not provide a specific rubric for judging. Instead, students were instructed to use individual criteria to deter-

In each class, a consensus emerged. Seventy percent of students in one class chose Brandon's "Von Steubing" (see fig. 1 and fig. 7) as the funniest face. In the other class, although many students liked Jason's "Rufus" (see fig. 3), Sam's "Wack-Teenage Mutant Ninja Turtle" (see **fig. 5**) received 100 percent endorsement. The creators of these two designs were declared the winners, and each received a school T-shirt as a prize.

mine which two faces were most creative.

CONCLUSION

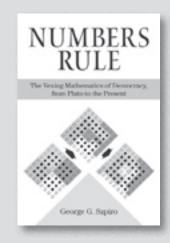
The Funny Face Contest aligns with the first recommendation of the President's Committee of Advisors on Science and Technology, Panel on Educational Technology (1997): "Focus on learning with technology" (p. 7). Jiang and McClintock (2000) further advocate that "we, as mathematics educators, should ... encourage and help our students to apply multiple approaches to mathematical problem solving and engage them in creative thinking" (p. 19). To attain this goal, curriculum developers such as Schwarz and Hershkowitz (1999) promote the "lever" of graphing calculators to learn mathematics through investigations.

Principles and Standards for School Mathematics (NCTM 2000) further affirms, "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). To implement the Funny Face Contest and make informed pedagogical decisions, mathematics teachers must become familiar with the advantages as well as the disadvantages of using graphing calculators in teaching mathematics (Garofalo et al. 2000; Hong,

Thomas, and Kiernan 2000).

Mathematics teachers should consider incorporating observations, projects, and student reflections in assessing students' mathematical understanding; as Principles and Standards notes, "such a collection of both informal and formal assessments can provide teachers ... with a more complete picture of student performance" (NCTM 2006, p. 4). Assessments do not necessarily mean giving students points for their work. Making mental notes about each student's work and the follow-up reflection allowed me to discern better his or her understanding.

Do the Math



Numbers Rule

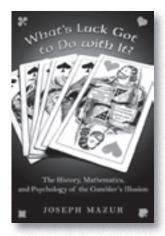
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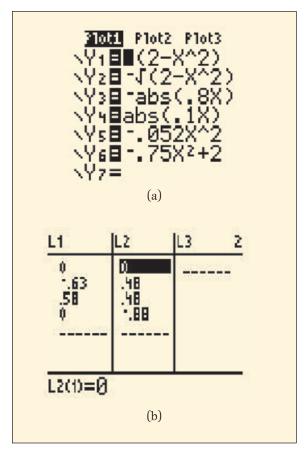


Fig. 7 The cursor hides the square root symbol in Brandon's Y_1 (a); his points are shown in (b).

Formative assessments afford occasions to gauge student understanding. This pedagogical tool can lend itself to providing more impromptu interactions between the teacher and the students. In contrast to having students solve routine textbook problems, creating funny faces engaged them "in a task for which the solution method is not known in advance" (NCTM 2000, p. 52). In fact, students were challenged to create something that did not result in one precise solution, an experience very different from their previous experience with mathematics.

Although the assignment was designated a contest, a spirit of student cooperation permeated the classroom. My students had fun and displayed great pride in their creations. As their teacher, I was particularly pleased to have witnessed much student motivation. Last, the experience reinforced my belief that formative assessments can provide meaningful teaching and learning opportunities.

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For more funny faces created by Yong Colen's students, go to the Mathematics Teacher Web site: www.nctm.org/mt.



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ematics history into mathematics education and developing challenging curricula for mathematically talented children.

Funny Face Contest: A Formative Assessment

Yong S. Colen

Here are some additional funny faces created by Yong Colen's students. Try to find all the functions that each student used to generate each funny face.

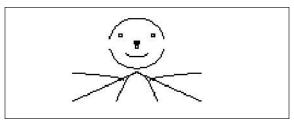


Fig. 1 Alex's "Allen Billoops"

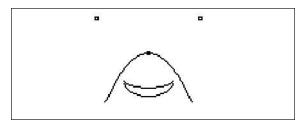


Fig. 4 Juli's "Yogi"

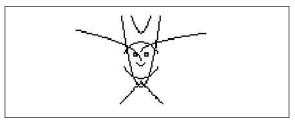


Fig. 2 Allie's "Samurai Molar Man All Dressed for Work"

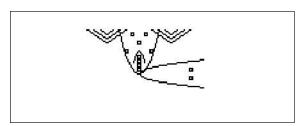


Fig. 5 Rachel's "Unhappy Clown-Man"

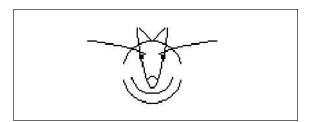


Fig. 3 Doug's "Rat Boy"