Connecting Algebra to Econo

Economics can be an avenue for teaching such algebra concepts as graphing curves, writing linear equations, solving systems of equations, and shifting graphs.

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Mastery of algebra has been considered a civil rights issue in a modern world that demands knowledge of mathematics (Moses and Cobb Jr. 2001). Algebra is often viewed as a gatekeeper class and is key to more advanced studies and career opportunities in many fields, especially STEM fields (science, technology, engineering, and mathematics). As Moses states, "In today's world, economic access and full citizenship depend crucially on math and science literacy" (Moses and Cobb Jr. 2001, p. 5).

However, what happens to those students who have not mastered fundamental algebra concepts and skills in ninth grade? What do you do when high school seniors are unable to solve a system of equations or tell you why algebra is useful? This

BACKGROUND: THINKSTOCK

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was the situation I faced as a first-year teacher in the Boston Public Schools, working with high school seniors, many of whom lacked understanding of fundamental algebra concepts As a teacher with a background in science and engineering, I understood the value and usefulness of mathematics and set out to teach a unit on algebra as it connected to another practical discipline—economics.

The unit lasted approximately three weeks and covered topics such as graphing, using and interpreting tables and graphs, writing equations for lines, solving systems of equations, and writing equations for shifted graphs. These lessons were taught to twelfth grade students with special needs in a substantially separate mathematics class that met five times a week for fifty-five minutes. The unit was designed for students who had previously struggled in mathematics, but the ideas presented here and the connections between algebra and economics can be used in any beginning algebra course.

WHY ECONOMICS?

Economics is a field that relies heavily on algebra, making it easy to find useful and concrete connections to the mathematics taught in schools. Further, economics is a discipline that is immediately relevant to the lives of students, and examples can be chosen that connect to student interests and experiences. Last, exposing students to a potential college major and incorporating college-level text into a unit will better prepare them for the academic opportunities and rigor of postsecondary education.

THE LAWS OF SUPPLY AND DEMAND

The first few lessons in this unit taught students the laws of supply and demand and how to represent those laws using tables and graphs. We began with a class discussion probing students' prior knowledge of economics. Many students, on the basis of their exposure to the news, associate the term *economics* with money, jobs, and unemployment. After discussing our experiences with the term, we defined *economics* as the study of how people choose to use limited resources for the production of goods and services (Tucker 2005).

After defining the term, the students observed the law of demand in action using a concrete example they could get excited about. Students were asked how many of my famous carrot-cake cupcakes they would buy at various prices. I started by polling them on how many cupcakes they would buy if I sold cupcakes at $1 apiece, then $2, and so on up to $10. By adding the quantity demanded by each student to get a total quantity demanded by the whole class, we were able to compare that with the process of summing individual demand to calculate market demand.

We then tabulated the results comparing price to (total) quantity demanded, called the *demand schedule*, and graphed the data to obtain the demand curve (see **fig. 1** for an example of a linear demand schedule and curve; see also the **sidebar**). During this process, we also reviewed the purpose of graphs, the vocabulary associated with graphing, the importance of labeling, and how to choose an appropriate scale. Because students were graphing data that they had generated, they were more invested in its presentation. When analyzing the graph and the table, students saw for themselves an inverse relationship between price and quantity

Are Supply and Demand Curves Really Straight Lines?

In this unit, we assumed that all supply-and-demand curves are linear. However, supply-and-demand curves are often nonlinear. Many introductory economics texts use linear curves, however, because they simplify calculations while maintaining the inverse and direct relationships in the laws of demand and supply, respectively (Rittenberg and Tregarthen 2009; Scott 1997). To understand why a demand curve might be nonlinear, consider the following scenarios:

Scenario A: A bakery decreases the price of cupcakes from $10 to $8. The quantity demanded then increases by 5 cupcakes.

Scenario B: A bakery decreases the price of cupcakes from $5 to $3. The quantity demanded then increases by 20 cupcakes.

In both scenarios, there is a $2 price reduction. So why would the change in quantity demanded be different?

The quantity demanded did not change proportionally because the price reduction took place at different price levels. In scenario A, the $2 drop occurred when prices were already high; few buyers would consider this decrease substantial. In scenario B, however, the $2 drop occurred at a lower price range; as a result, more people found the price decrease significant. In economics, the responsiveness of quantity demanded to price changes is called the price elasticity of demand. Similarly, the responsiveness of quantity supplied to price changes is called the price elasticity of supply (Tucker 2005).

Demand Schedule for Cupcakes

Price ($)	*Quantity Demanded (thousands)*
7	1
5	4
3	7
1	10

Demand Curve for Cupcakes

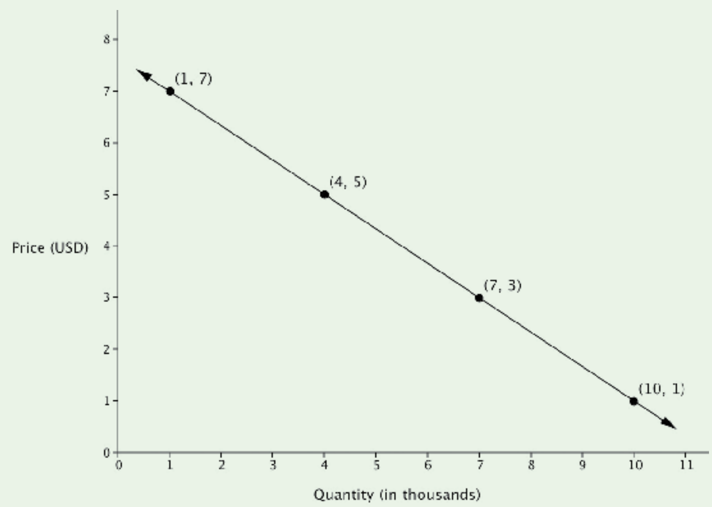


Fig. 1 The data in the demand schedule can be graphed as a demand curve.

demanded—in other words, the law of demand. After the law of demand was demonstrated, students were asked to graph demand curves given linear demand schedules and to interpret those graphs to determine how much of a given product would be demanded at various price levels and vice versa.

Once students understood the law of demand, we moved on to the law of supply. This law depends on the supplier's willingness to produce a given product; thus, it is conceptually more difficult for students to understand. Continuing the cupcake example, I created two new scenarios. In scenario 1, I was selling cupcakes to students and colleagues in my school. However, people were willing to spend only $1 apiece. In scenario 2, I had been invited to appear on *The Martha Stewart Show* to bake my cupcakes. People were now willing to spend as much as $10 a cupcake. I asked students in which scenario they thought I would spend my time and resources baking a large number of cupcakes. They all said scenario 2, and we discussed the direct relationship between price and quantity supplied—the law of supply. Once again, students were asked to graph supply curves given a supply schedule and interpret those graphs for various prices and quantities supplied.

CALCULATING MARKET EQUILIBRIUM

Once students had a solid understanding of supply and demand and how to graph these relationships from a table, they learned in the next few lessons how to model the laws using linear equations and how to use the mathematical models to calculate market equilibrium.

This part of the unit began by having students graph both supply and demand curves on the same set of axes. Students noticed that the two lines intersected, and they were asked a series of questions to help them interpret the graphs and the significance of the point of intersection. For prices above the point of intersection, students saw that the quantity demanded (Q_D) was less than the quantity supplied (Q_S); in other words, there was a surplus of product. For prices below the point of intersection, students saw that the quantity demanded was higher than the quantity supplied; in other words, there was a shortage of product.

It was only at the price level where the two lines crossed that the quantity demanded was equal to the quantity supplied and market equilibrium existed (see **fig. 2**). Having described the various market conditions, students were asked to think about how (and why) suppliers would react to the three scenarios presented in **figure 2**.

Students initially used only graphs to find the price and quantity at market equilibrium, but they quickly realized that an easier method for calculating the market equilibrium would be helpful. At this

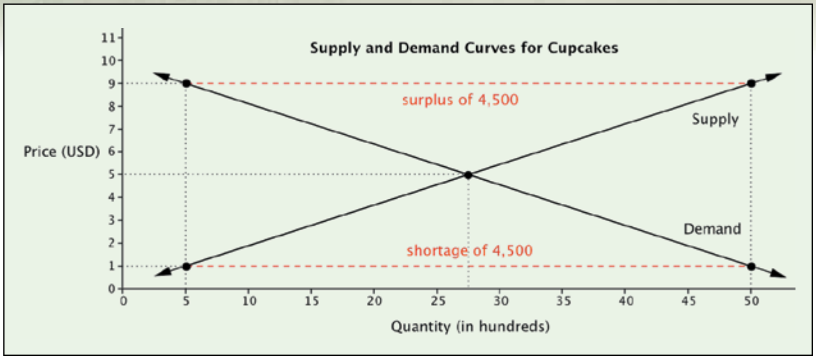


Fig. 2 When the price is $9 each, $Q_D < Q_S$, so there is a surplus of 4,500 cupcakes. When the price is $1 each, $Q_D > Q_S$, so there is a shortage of 4,500 cupcakes. When the price is $5 each, $Q_D = Q_S$, so there is market equilibrium.

Verbal Representation

When cupcakes are priced at $2 each, the supplier produces 200 cupcakes. However when cupcakes are priced at $8 each, the supplier produces 400 cupcakes.

Algebraic Representation

$$P - 2 = \frac{6}{200}\left(Q - 200\right)$$

where P = price and Q = quantity supplied

Tabular Representation

Price ($)	*Quantity Supplied*
2	200
8	400

Graphical Representation

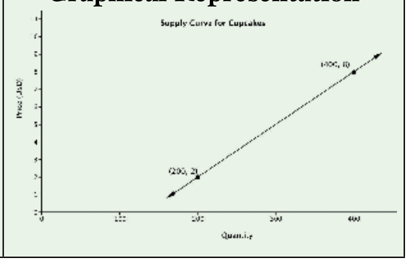


Fig. 3 Supply can be represented in multiple ways.

point, students were taught the point-slope form of a line as a modified version of the slope formula, which they already knew. Students were then asked to write equations for supply and demand curves given word problems, tables, and graphs. Thus, students had the opportunity to see verbal descriptions, tables, graphs, and equations as multiple representations of the same real-world scenario (see **fig. 3**). Once they were able to write equations for a supply or a demand curve, students could solve the linear equation and determine exact values for the quantity supplied or demanded at different prices.

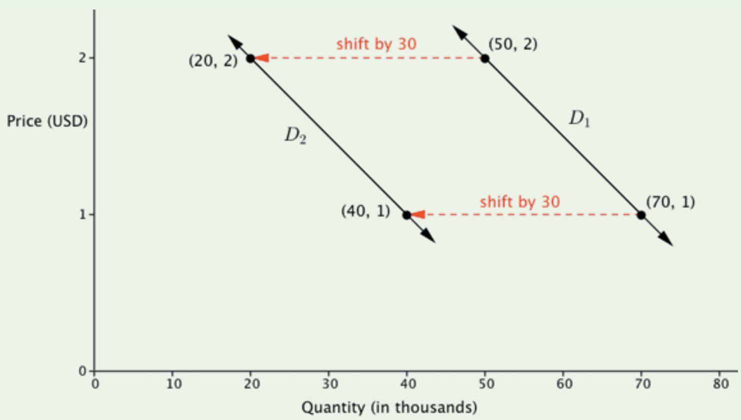
Although solving linear equations is useful for calculating the value of a variable, students also explored the limitations of algebraic models. Students found that even though linear equations represented lines that extended infinitely in both directions, some values of the domain and range, such as negative values for price or quantity supplied or quantity demanded, had no meaning in the real world. Therefore, although mathematical models have many benefits, such as describing real-world situations and providing a mathematical structure that can be manipulated and analyzed, students

Table 1 Nonprice Determinants of Demand and Supply

Nonprice Determinant of Demand	Relationship with Demand	Example
Number of buyers	Direct	An increase in the number of students with a cold will increase the demand for Kleenex.
Tastes and preferences	Direct	A decrease in the popularity of Silly Bandz will decrease the demand for Silly Bandz.
Income	Direct or Inverse	As a family's income increases, the demand for organic produce increases, and the demand for generic products decreases.
Nonprice Determinant of Supply	**Relationship with Supply**	**Example**
Number of sellers	Direct	An increase in the number of iPhone® app developers will increase the supply of apps.
Technology	Direct	With the invention of the cotton gin, there was an increase in the supply of cotton.
Resource prices	Inverse	A decrease in the price of silk will increase the supply of silk clothing.

Source: Tucker (2005)

Scenario: Consider the demand curve for Silly Bandz, D_1, while the toy is popular. However, after the fad passes, Silly Bandz now sell at 30,000 units less than before given any price level.



Prompt: Write the equation for both demand curves shown.

$$D_1:\ P-2=-\frac{1}{20}\left(Q-50\right)$$

$$D_2:\ P-2=-\frac{1}{20}\left(Q-20\right)$$

Question: What term could you add to the equation for D_1 to get the equation for D_2? How does this term relate to the way the graph shifted?

$$P-2=-\frac{1}{20}\left(Q-50+30\right)\rightarrow$$

$$P-2=-\frac{1}{20}\left(Q-20\right)$$

A +30 term was added to Q when the graph shifted 30 to the left.

Fig. 4 Students need to show the effect of nonprice determinants.

saw the importance of questioning when a model is viable and when it is not.

Finally, students were taught how to calculate the market equilibrium using a system of equations that they wrote for the supply and demand curves. Students solved by using both substitution and elimination, an alternative approach to solving graphically. The ease with which market equilibrium could be determined using equations, in certain situations, provided students with a concrete advantage for using algebraic models.

SHIFTS IN THE MARKET

Up to this point in the unit, only two variables had been discussed—price and quantity supplied or demanded. Price is a major factor in determining the quantity that will be supplied and demanded, but it is not the only factor. Economists describe a range of other factors that influence the supply and demand; these are known as nonprice determinants (see **table 1** for selected factors). Whereas increases and decreases in price cause movement on the curve, a change in a nonprice determinant will cause the entire demand or supply curve to shift. The reason for this is that a nonprice determinant affects how a product will be demanded or supplied at every price level; therefore, every point in a curve is affected.

For this set of lessons, students read and took notes on the section in *Microeconomics for Today* (Tucker 2005) describing nonprice determinants of supply and demand. As a class, we then discussed various scenarios in which these factors could come into play, and students made qualitative statements about how the supply or demand for a product would change (see examples in **table 1**).

Next, students were asked to think about the effect that nonprice determinants would have on the graphs of supply and demand—namely, that there is a shift in the curve because a third variable is introduced. By looking at supply and demand curves before and after a nonprice determinant changes, we analyzed how the quantity changed for a given price (see **fig. 4**).

Students also looked at a given quantity and saw how the price changed because of the nonprice determinant. Helping to identify the amount the graph shifts vertically (by looking at price change) or horizontally (by looking at quantity change) led us to discuss how the equation of the shifted line compared with the equation of the original line (see **fig. 4**).

Finally, students were asked to write their own equations for shifted supply and demand curves given (1) sufficient information to write an equation for the curve before the nonprice determinant changes and (2) a description of how the quantity supplied or demanded changed at every price level as a result of the nonprice determinant change. Alternatively, students were also given a description of how price changed at every quantity level.

REAL-WORLD APPLICATION: iPHONES THEN AND NOW

To conclude this unit, students worked on a final project (available at **www.nctm.org/mt**) that compared the iPhone® market in 2008 and 2010. In 2008, iPhones were one of the few smartphones available and were unmatched in the features and number of apps offered. By 2010, however, iPhones had increased competition; Android™ smartphones were introduced, and new BlackBerry® models were on the market.

Given supply and demand schedules for iPhones in 2008, students were asked to graph the supply and demand curves, write equations for those curves, and then use algebraic techniques to determine the market equilibrium (see **fig. 5**). Second, students were given information on how the demand for iPhones changed from 2008 to 2010. Using this information, students had to graph the new demand curve (see **fig. 5a**), write an equation for the demand in 2010, and calculate the market equilibrium for iPhones in 2010.

Students were also asked questions to help them understand why the market equilibrium of iPhones changed between 2008 and 2010. They were asked to identify the type of nonprice determinant of supply and demand that came into effect when other smartphones appeared on the market (i.e., the price of substitute goods) as well as to describe how that nonprice determinant influenced the supply and demand of iPhones.

To further understand the need for price changes based on changes in the market, students

A student's graph of supply and demand curves in 2008 as well as the new demand curve in 2010. Note: There is a small error in the demand curve for 2010. The point (0, 400) should be (0, 300).

A student's system of equations for supply and demand in 2008 and his or her calculation of market equilibrium.

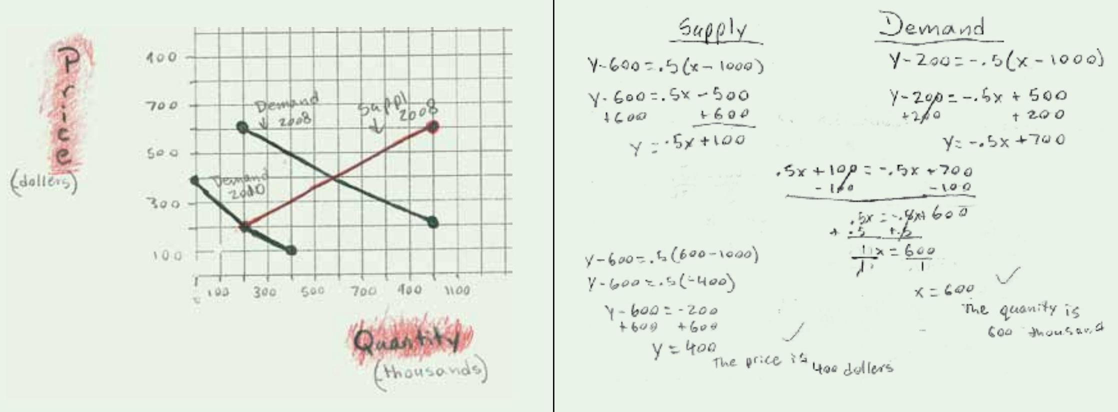


Fig. 5 Student work ties together several representations.

were asked to consider a hypothetical situation in which Apple® priced iPhones in 2010 the same way that they did in 2008. Using their algebraic models or graphical representations, students were able to see that the quantity of iPhones demanded would be zero in the scenario based on the demand in 2010. In fact, if we use the algebraic equation, the quantity demanded would be a negative value, which can be interpreted as no demand.

Finally, the project required students not only to consider how and why market equilibrium for a product changes but also to make an educated prediction as to how the market might change in the future. Students were asked to consider a hypothetical situation in which Apple strikes a deal with a microchip producer, thereby reducing the cost to produce iPhones. Students then needed to use their knowledge of economics and the mathematical models that they learned in class to predict how market equilibrium would change given these new conditions.

POSITIVE RESULTS

This unit combining algebra with economics was well received. Most students were engaged, and their algebra skills improved significantly. Students were also excited about being held to high expectations and challenged with college-level material; as one student commented, "[This is important] because it's getting you ready for college." Some even expressed interest in majoring in economics, asking questions about the field that went beyond the topics discussed in class.

Our mission as teachers is not only to teach the content but also to inspire our students by exposing them to new opportunities and ideas. One excellent way of accomplishing this is by showing the many ways in which mathematics can be connected to other disciplines and to the real world. These connections are important to make and will benefit students, regardless of their mathematical skills. I hope that the ideas presented here provide some assistance to teachers in the way they approach algebra instruction.

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