Mathematics Detective

DAVID B. SPANGLER

The Price *Isn't* Right

THE STORIES THAT YOU ARE ABOUT TO READ are true; only the names have been changed. In each scenario, employees made mathematical errors and charged customers incorrect amounts. So "come on down," and let us investigate what happened. Let us also do some mathematics to make sure that the price *is* right.

Mr. and Mrs. Lucky, who are senior citizens, ate dinner at a restaurant that displayed the sign shown in **figure 1.** Let us pick up the conversation at the end of their dinner.

Senior Citizen Special

Dinners 20% off

Fig. 1 Sign in a restaurant

Waiter. Is there anything else I can get you?

Mr. Lucky. No, thank you; I think everything is fine. All we need is our check.

Waiter. Well, let's see . . . there are two of you, so that means you get 40% off.

Mrs. Lucky. Excuse me, sir. I think we should get only 20% off the total bill.

Waiter. I think you get 40% off, but let me check with the manager.

The waiter goes out and returns with the manager.

Manager. Your waiter is correct. You're entitled to 40% off the total check.

Mrs. Lucky. Well, if that's the case, the next time my husband and I come here, we'll bring three of our friends so that we can all get our dinners free!

Mr. Inkblott went to Copies, Ink., to have 4800 photocopies made. After examining the sign **(fig. 2),** he decided to have 5001 copies made (even though he did not need that many). Later that day, he returned to Copies, Ink., to pick up his order. Let us pick up the conversation as Mr. Inkblott hands the clerk his credit card.

Copies, Ink.

If you order . . .	You pay . . .
up to 5000	7¢ each
5001 to 7500	4.6¢ each
more than 7500	3.5¢ each

Fig. 2 Price list for photocopies

Clerk. This is a good-sized order.

Mr. Inkblott. It is, and I made it even bigger because of your great prices here.

Clerk. The total comes to $2300.46.

Mr. Inkblott. What? That's impossible! There must be an error here!

Clerk. I'm sorry, but the cash register shows $2300.46.

Mr. Inkblott. Wait, let me try to explain your error. Let's suppose that you made me 5000 copies at 5¢ per page.

Clerk. But the price is 4.6¢ per page. Don't try to confuse me.

Prepared by DAVID B. SPANGLER, *david.spangler@aw.com. Spangler teaches at National-Louis University, Evanston, IL 60201. He is also an executive editor of mathematics for a textbook publishing company. He is always looking for ways to teach mathematics through engaging, real-world applications.*

Senior Citizen Questions

SUPPOSE THAT THE REGULAR PRICE FOR MRS. LUCKY'S dinner was $12.95 and that the regular price for Mr. Lucky's dinner was $14.50.

1. How much would Mrs. Lucky's dinner cost at 20 percent off?
2. How much would Mr. Lucky's dinner cost at 20 percent off?
3. What would be the total cost for the two dinners at 20 percent off each dinner?
4. What would be the total cost for the two dinners at 40 percent off the total of the regular prices?
5. Were Mr. and Mrs. Lucky *lucky,* or were the waiter and manager correct? Explain your reasoning.
6. Why do you think that Mrs. Lucky suggest to the waiter and manager that the next time they come to the restaurant they might bring three of their friends with them?

Copies, Ink. Questions

7. According to the sign, how much should 5001 copies cost?
8. What error was made in determining the total cost?
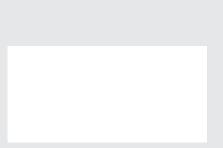9. Why did Mr. Inkblott attempt to explain the cost for 5000 at 5¢ per page?
10. How would you have tried to explain the error?
11. Why did Mr. Inkblott order 5001 copies instead of 4800 copies?
12. Would Mr. Inkblott have been even better off if he had ordered 7501 copies?
13. Although many types of stores offer discounts for large quantities, does the pricing structure at Copies, Ink., really make sense? From the point of view of Copies, Ink., how might the pricing structure for volume discounts be improved?

Here is a good chance for you to become a mathematics detective in your neighborhood. Find misleading or mathematically incorrect advertisements. Such advertisements may be found in newspapers, in direct-mail promotions, on television, in stores, and in other places. Also, be alert to situations in which employees at a place of business incorrectly use information given in store advertisements. Bring the advertisements and stories to class for discussion. Then send your favorites to this department. Copies or descriptions of the advertisements are sufficient if actual advertisements cannot be sent.

Send advertisements to this address:
  "Mathematics Detective," Journals Department
  National Council of Teachers of Mathematics
  1906 Association Drive
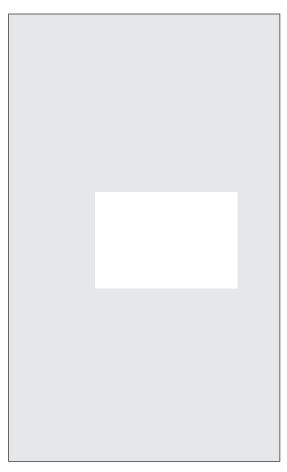 Reston, VA 20191-9988

(Answers on page 168)

1. $10.36
2. $11.60
3. $21.96
4. $16.47
5. Mr. and Mrs. Lucky were indeed *lucky*. They should have received a 20% discount for each of two dinners, or 20% off the sum of the two regular prices. As demonstrated by the distributive property, this result is *not* equal to 40% off the total of the two regular prices.

Amount of discount on each of the two dinners	Amount of discount on the total of the two dinners
$0.20($12.95) + 0.20($14.50)$ =	$0.20($12.95 + $14.50)$
$2.59 + $2.90	$= 0.20($27.45)$
$5.49	$= 5.49

The discounted price for the two dinners should have been $27.45 – $5.49, or $21.96. The waiter and manager wanted the couple to pay $27.45 – 0.40($27.45)$ = $27.45 – $10.98,

or $16.47. In general, if x and y are the regular prices of any two dinners, then

$$0.20x + 0.20y = 0.20(x + y).$$

6. The waiter and manager evidently figured that the discount that is applied to the total bill increases by 20% with each customer. Mrs. Lucky probably thought that by applying this incorrect reasoning to five dinners, the manager would realize the error, because on the basis of his incorrect logic, the total bill for five dinners would be reduced by 100%.
7. $230.05
8. The customer was charged 46¢ for each copy instead of 4.6¢.
9. Mr. Inkblott probably thought that he could help the clerk to mentally compute the cost of 5000 copies at 5¢ a page to be $250.
10. Answers will vary. Your explanation might be as follows: If 1 copy costs 5¢, then I should get 20 copies for $1. So I should get 100 copies for $5 and 1000 copies for $50. Thus, 5000 copies would cost $250. Because the actual price is only 4.6¢ per copy, I should pay less than $250. I certainly should not pay more than $2000!
11. The total cost for 5001 copies is actually less than the total cost for 4800 copies. The cost for 4800 copies at 7¢ a page is $336; the cost for 5001 copies at 4.6¢ per page is $230.05. Mr. Inkblott will save $105.95 by having an extra 201 copies made.
12. No. The cost for 7501 copies at $3.5¢ a page is $262.54. Because Mr. Inkblott does not need that many copies, he is better off going with the cheapest price, or $230.05 for 5001 copies.
13. Answers will vary. From the store's point of view, it really does not make sense to print more copies for less money. A pricing structure such as the one shown in **figure 3** might make more sense for volume discounts. With this structure, the discounts take effect incrementally and thus are not applied to all the copies. ▲

NUMBER OF COPIES	PRICE PER COPY
First 5000 copies	7¢ each
Next 2500 copies	4.6¢ each
All copies above 7500	3.5¢ each

Fig. 3 A pricing structure that reflects more realistic volume discounts