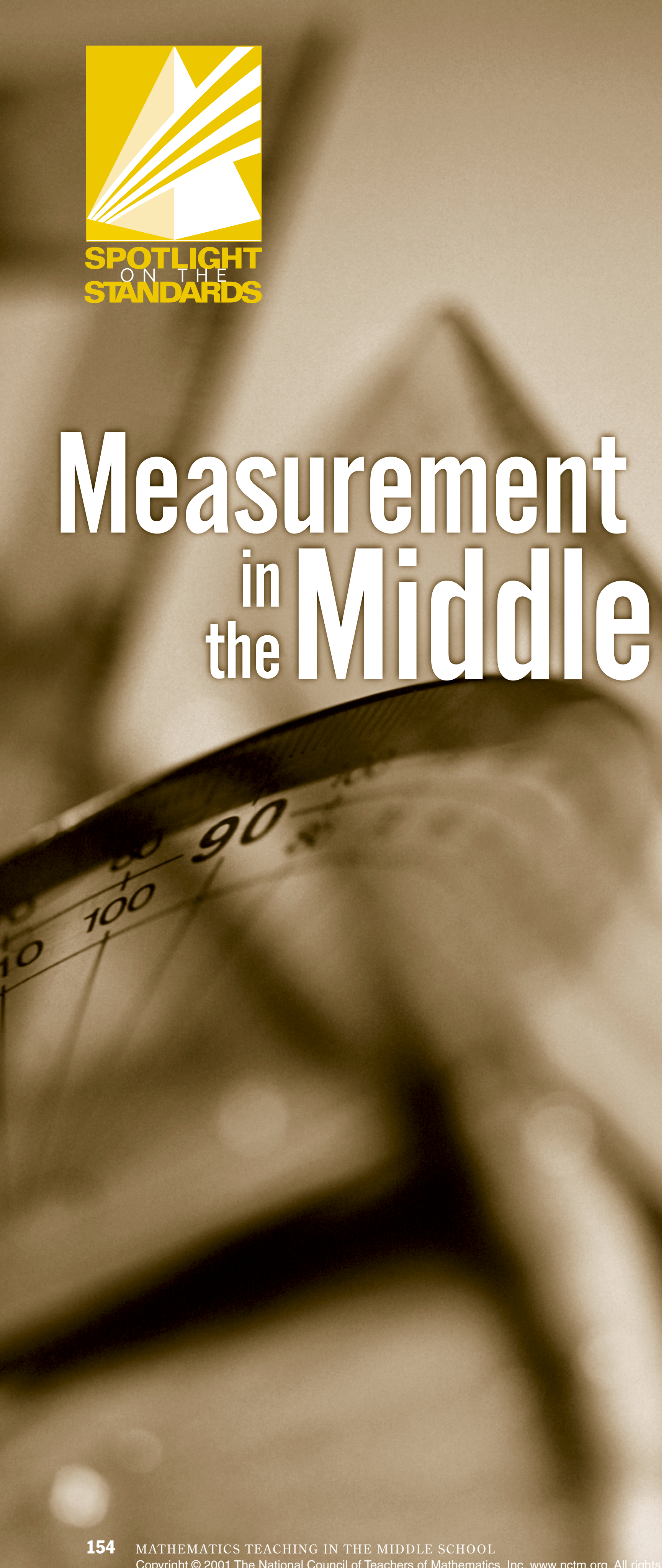


SPOTLIGHT ON THE STANDARDS

Measurement in the Middle Grades

ROBERT Q. BERRY III AND JOYCE WIGGINS

PEOPLE USE MEASUREMENT IDEAS everyday because the ideas arise out of the need to compare attributes of the real world and because humans are naturally inclined to make comparisons. Comparison is the basis for measurement. We make simple comparisons, such as that "the width of the desk is about an arms length," and complex comparisons expressed in terms of precise numerical measurements. Although measurement encompasses such topics as time, temperature, length, perimeter, and volume, this article specifically focuses on how middle school students learn about angle measurement.

Angle measurements are important in several careers. Drafters, architects, and contractors use angular measurements in the design and construction of homes. Airplane pilots must consider angle relationships in takeoffs and landings. Civil engineers use angle measurement to create buildings and bridges. Specific geometric angles are used by bicycle designers when designing mountain or road-racing bicycles. In skateboard design, to prevent the skateboard from breaking while in use, designers accurately measure and carve angles into plaster at various stress points.

ROBERT BERRY, *rberry@email.unc, is a doctoral student in mathematics education at the University of North Carolina at Chapel Hill, Chapel Hill, NC 27599. He is also the assistant director of the Center for Mathematics and Science Education at UNC-CH.* JOYCE WIGGINS, *jwiggins@chccs.k12.nc.us, is a Mathematics-Science Education Network Pre-College Program teacher at Guy B. Phillips Middle School in Chapel Hill, North Carolina.*

Edited by CAROL E. MALLOY, *University of North Carolina at Chapel Hill, Chapel Hill, NC 27599*

"Spotlight on the Standards" focuses on the grades 6–8 content and process standards found in NCTM's Principles and Standards for School Mathematics *(2000). The articles compare NCTM's* Curriculum and Evaluation Standards for School Mathematics, *published in 1989, with the* Principles and Standards *relating to the middle grades and suggest ways that teachers might incorporate Standards-based practices into their instruction.*

Whether using approximations or precise measurement, students must also develop an understanding of the processes of angle measurement.

Measurement Standards

CURRICULUM AND EVALUATION STANDARDS FOR School Mathematics (NCTM 1989) proposed that measurement for middle school students should include concrete experiences in which students use the processes of measurement to interact with their environment and actively explore the real world. *Principles and Standards for School Mathematics* (NCTM 2000) extends the 1989 measurement standard by calling for students to become proficient in selecting the appropriate size and type of unit for a given measurement situation. Students should also become proficient in selecting and using appropriate tools and formulas to determine measurements. Both the 1989 and 2000 standards state that students should have an understanding of angle measurement. The 2000 measurement standard, however, also states that students should understand, select, and use units of appropriate size and type to measure angles. That is, students must understand that angles have different measures, an attribute that is sometimes referred to as the *openness* of an angle. In addition, the 2000 measurement standard suggests that students should understand the mechanics of protractors, the tool used to measure the openness of angles.

Angle measurement is difficult for many middle-grade students for several reasons: they have no sense of angle size, they lack knowledge of angle attributes, and they do not understand the protractor as a measuring tool (Van de Walle 2001). To overcome these difficulties, students need opportunities to explore the openness of angles and to use the protractor as a tool to measure this attribute. In addition students need to explore and experience the relationship between the openness attribute and the protractor. To develop a repertoire of benchmark angles and decrease the possibility of developing misconceptions about protractors, students make comparisons among the openness of different angles.

Angle-Measurement Project

RECENTLY, WE WORKED WITH A CLASS OF SIXTH graders to help them understand angle measurement. They answered the following questions: What is a protractor? What purpose does it serve? How are protractors constructed? How do I use a protractor to measure angles? Although discussing these questions with a class is possible, having students construct a protractor and an angle model is a more powerful approach. These sixth-grade students constructed angle models, created protractors using nonstandard units of measure, measured and constructed angles using their protractors, and observed similarities between their protractors and a degree-unit protractor. They also used interactive geometry software to develop benchmarks for angles of common degree measures.

To explore the concept of angle, students constructed angle models by cutting two strips, each 2 centimeters by 15 centimeters, from poster board. The poster-board strips were attached at one end using a brass fastener to form an angle. By rotating one side of the angle models, the students focused on the openness attribute of angles. In addition, the students worked in groups to compare their models by ordering and classifying their angles according to openness. One group developed a classification system that led to a discussion of the terms *acute, right,* and *obtuse*. After exploring, sorting, and classifying the openness of their angle models, the students constructed protractors.

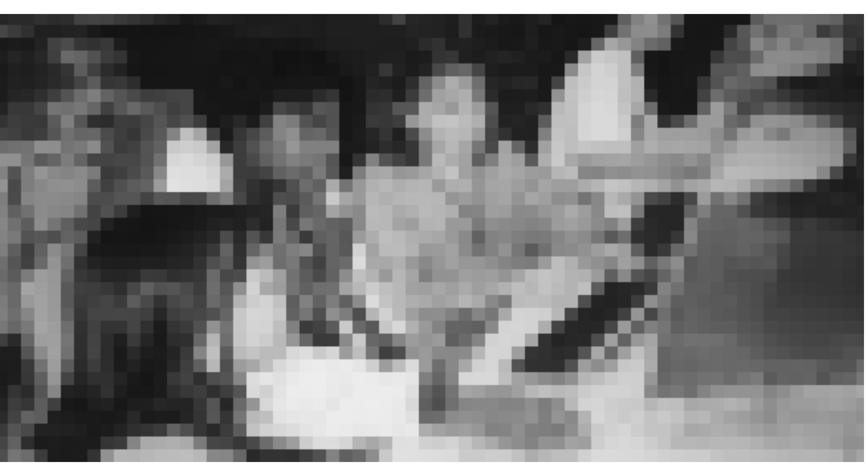
The students constructed nonstandard-unit protractors using waxed paper (see Van de Walle [2001] for instructions). The purpose of introducing nonstandard-unit protractors was to help the students develop the notion of *angular unit* and to help them recognize that a unit is simply an object that has a particular attribute. When they had finished making their protractors, the students labeled the center with a point. Jeffery commented, "Protractors are not a circle like this one; I think we should cut it in half." Melody said, "If we cut it in half, then we will have two protractors." These comments led us back to the angle models, and Jemma, another student, recognized that the protractor is circular because we rotate a side of an angle around like a circle.

After some discussion and demonstrations on how to use the nonstandard-unit protractors, the students practiced measuring their angle models and using *sections* as the informal unit of measure. The students used estimates of one-fourth, one-half, and three-fourths for angles that did not measure to the nearest whole section. After practicing, the students were placed in pairs to complete the "construct, guess, and measure" activity, which we created for sixth graders to estimate angle measures and develop benchmark angles. For this activity, one student in each pair used his or her angle model to show an angle; the other student guessed the measure of the angle, then used the nonstandard-unit protractor to measure it. As an extension, the students used their protractors to measure the interior angles of three-, four-, five-, and six-sided polygons, then found the sums of the measures. This activity explored the relationship between the number of sides of a polygon and the sum of its interior angles. The students also described the process for finding the sum of the angles for a 100-sided polygon and made a general argument for any polygon.

We concluded the period by having students compare their nonstandard protractors with a circular degree-unit protractor and a semicircular degree-unit protractor. Much of the discussion of the three types of protractors focused on why the degree-unit protractors had two sets of numbers. Noah explained, “The two numbers add up to be 180; I think it is because there are 180 degrees in a straight line.” To explore whether his conjecture was true, we constructed a 120-degree angle with an extended side and found the measure of the angle sharing the extended side. Sean suggested that using the semicircular degree-unit protractor was easier than using the circular protractor because “all you have to do is put the hole over the vertex and measure it, just like you do with the wax-paper protractor.” Wendy agreed but asked, “Which number do you use?” We examined Wendy’s question by measuring angles on the overhead projector using the semicircular degree-unit protractor. After measuring a few angles, Wendy answered her question by stating, “If the angle is less than a right angle, then use the small number; if the angle is greater than a right angle, then use the big number.”

Because a degree is such a small unit, students often have trouble using a standard protractor. The difficulties that students encountered with the small unit decreased after they had constructed and measured angles with their own nonstandard protractors and had compared their nonstandard protractors with a standard one. In addition, having students construct their own nonstandard protractors and angle models developed adequate mental structures for angle measurement by strengthening the relationship between the tool (protractor) and the measurable attribute (openness).

The second day of the lesson took place in the computer lab using interactive geometry software, such as The Geometer’s Sketchpad or Geometry Inventor. The students worked in pairs to do three activities using the software: (1) “construct, guess, and measure,” (2) “construct that angle,” and (3) “guess my angle.” These activities were designed to help students develop benchmark angles and to confirm their understanding of angle size. For “construct, guess, and measure,” pairs of students constructed eight angles, guessed the measure of the angles, then measured the angles to determine the difference between their guesses and the actual measurements. Many students asked, “What is a good guess?” In class discussion, we agreed that if the difference between the guess and the actual measure was 5 degrees or less, then the guess was a “good guess.” For “construct that angle,” the students constructed 30, 40, 60, 90, 120, 150, and 180 degree angles without using the measurement window. After constructing the various angles, the students measured them to see how close their constructions were to the actual measurements. For “guess my angle,” one student constructed an angle while another looked away from the monitor. The first student measured the angle, then hid the measurement. The second student had to guess the measure of the angle within 5 degrees.



Conclusion

MEASURING ENCOURAGES STUDENTS’ ACTIVE involvement in problem solving and mathematics discussions. According to *Principles and Standards*, measurement is central to the curriculum because of its power to help students see that mathematics is useful in everyday situations (NCTM 2000). The lessons described in this article are aligned with the recommendations of *Principles and Standards.* The students explored the openness attribute of angles and developed appropriate techniques for using protractors. They also increased their repertoire of benchmark angles, which will help them make reasonable angle estimates and check to see whether their measurements using a protractor are reasonable. Middle school students should use measurement concepts throughout the school year and should be able to integrate and make connections with the other content standards, as well as the process standards.

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