Stephanie S. Reilly

All Shades Are the Right Shade

Students spend a lot of time in algebra 1 solving linear equations and systems of linear equations. These subtopics of algebra can be problematic and difficult for students to grasp. After months of finding the solution to a linear equation (a line) and finding the solution to a system of linear equations (generally, a point), students struggle with understanding the solution to a linear inequality or a system of linear inequalities (a shaded region). Students might think, *Why do we shade at all, what does the shading mean, and why is an overlapping shaded region the solution in an inequalities systems graph?*

Unfortunately, it would be easy for systems of inequalities to become a long series of procedural steps that lack meaning. Think about what steps we often list for students as they work to solve a system of inequalities:

1. Graph the equation of the first inequality as if it had an equal sign by finding the x- and y-intercepts or by putting the equation into the $y = mx + b$ form.
2. Look at the inequality. Determine if the line should be dashed or solid. Erase parts of the line that you already drew if it should have been dashed. (Remind yourself to check the line *before* you draw it next time.)
3. Do a test case, and see which side of the line should be shaded.
4. Shade it lightly, since more shading will be required.
5. Repeat steps 1 through 4 for the second inequality.
6. Repeat for additional inequalities in your system.
7. Analyze your system graph to determine the overlapped shaded region.

Is it little wonder that students' minds have reverted to procedural mode after they have marched through all these steps? It is difficult to step back and think deeply about that overlapped shaded region.

This activity is designed to *introduce* systems of linear inequalities once students have an understanding of single linear inequalities. It is intended to let students discover for themselves that the solution of a system of inequalities can only be defined by—

1. graphing both inequalities; and
2. determining the overlapping shaded region.

After students become comfortable with the idea of a shaded region being

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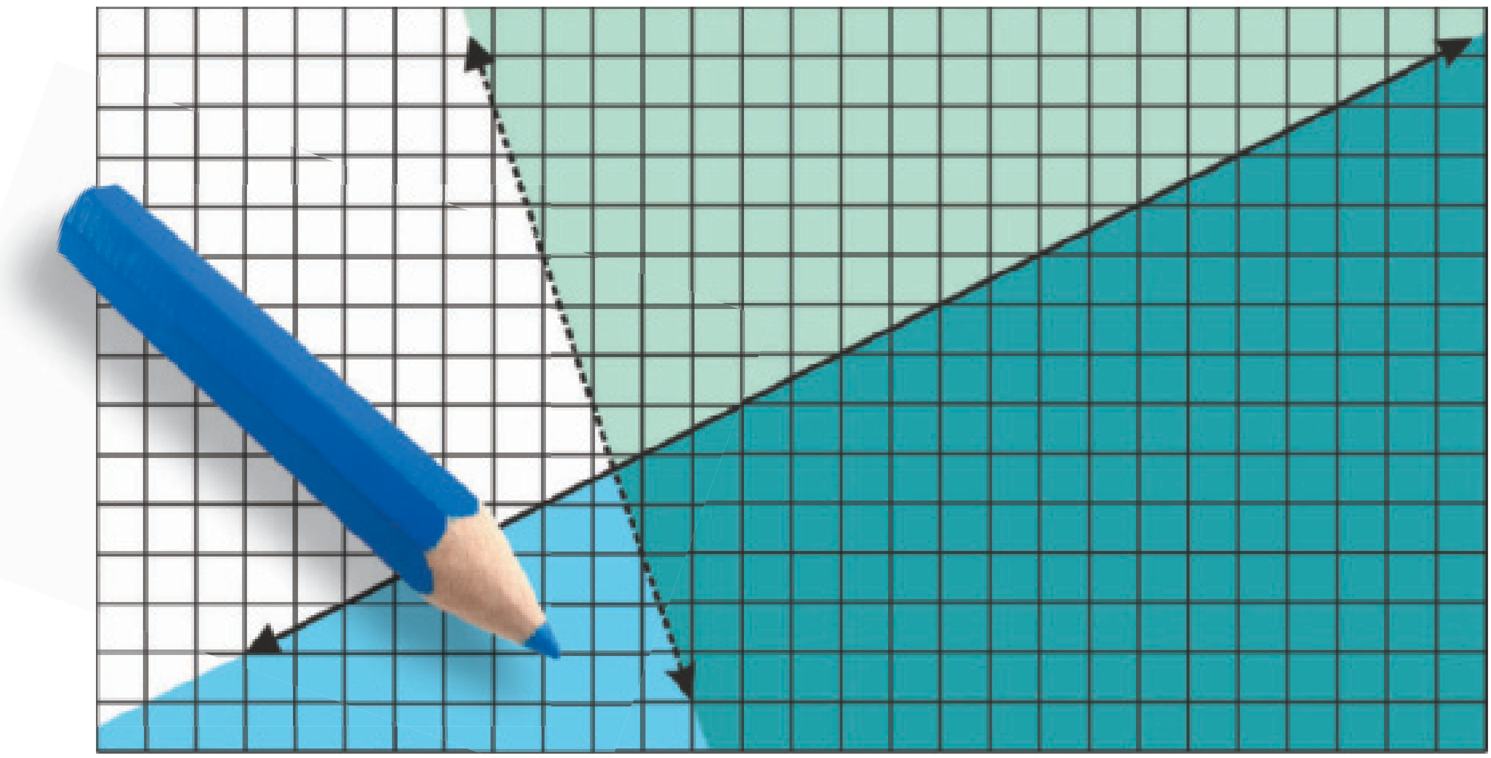

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a solution to a single linear inequality, they can then move into learning about solving two or more linear inequalities. Students will be able to learn *with understanding*, an NCTM tenet, when they construct their own meaning of a concept by reasoning about, proposing, and testing mathematical ideas and concepts. This activity involves both group and classroom discussions, which promote the making of connections among ideas and the reorganization of knowledge, all concepts promoted by NCTM.

I designed this activity for middle school algebra 1 students. It was implemented in classes averaging twenty students who were a mix of seventh graders and eighth graders. Heterogeneous groups were used for pairs and teams to allow for richer discussions. An optimal class size will be a combination of four (*pairs* of two students combine to form *teams* of four). However, pairs can be increased to three students, as needed, to accommodate differing class sizes.

To begin, students were paired with a partner; labeled team 1A, 1B, 2A, and so on; and given a team problem. These problems consisted of a contextual situation and instructions to write and graph an inequality related to this context. Team A and B problems are shown in **activity sheet 1**. Team A was instructed to write and graph the inequality related to the number of hours that Bobby *could* work without violating his mother's rules. Team B was instructed to write and graph the inequality related to the number of hours that Bobby *must* work to meet his expenses.

The key to this activity is graphing solutions on transparency sheets, using the same scale along each axis for the graphs, and working with light-colored dry-erase markers. Students enjoyed this novelty and did not anticipate being combined with the other half of their team. The pairs simply focused on writing and graphing their inequality. To ensure that the shading completely filled in the area (which helps students see the overlapped region later), I asked groups to color in their shading, as if they were in kindergarten.

Once the pairs completed their graphs, the pairs were matched to form teams (for example, team 1A joined team 1B), and the graphs were overlaid. Overlaying the transparencies instantly created a graph of the system of inequalities, to which the students must attach meaning. **Activity sheet 2** asked what the overlapped shaded area means in terms of their context. Until this point, the pairs had only focused on their half of the context's constraints and now had to expand their thinking to include the other constraint. Having never seen a system-of-inequalities graph before, the groups struggled to understand the overlapped region. Much discussion ensued.

Some pairs stubbornly clung to their constraint, seeing the problem only in terms of the one limitation. Others could not express how to satisfy both constraints simultaneously and could only suggest that the overlapped region represented “all possible combinations of x and y.” By repeating the phrase “overlapping shaded region” during group questioning, students began to realize that *both* inequalities are satisfied only within that overlapped shaded region. From that understanding, it was a smaller leap to define the overlapped region in terms of the context. Students reached this understanding at different times and with different depths of understanding. I was able to use these differences to promote group discussions by asking students who grasped the concept to explain it to their group.

Teams presented their system graph using an overhead projector (or document projector) and explained their context, the two inequalities, and the system graph. Teams defined the meaning of the overlapped shaded region in terms of their context (“the combination of hours worked as a math tutor and waiter so that Bobby makes enough money to pay for his cell phone while not violating his mother’s rules about the hours/week that he works”). The groups all had different contexts, and all were set within areas of interest for middle school students. (See **fig. 1** for a complete list of team problems.) After grappling with their own context and graph and listening to the different groups explain and illustrate their system graphs, the idea of multiple constraints and the overlapping solution region became clear.

Fig. 1 By dividing students into teams and having half of each team graph one relation of the two in each scenario, students work together and build meaning for the overlapping shaded solution to a system of linear inequalities.

Team 1: Bobby has two jobs. He makes $10 an hour as a math tutor and $6 an hour as a waiter. Bobby’s mother will not let him work more than 20 hours a week. Bobby needs to make at least $30 a week to pay for his cell phone and music downloads.

Team 2: Susie has a pet-sitting business. She charges $2 a day to watch cats and $3 a day to watch dogs. Susie cannot handle watching more than 6 pets on any one day because she spends so much time doing homework. However, she needs to make at least $6 a day to pay for her expenses.

Team 3: Joey owns a bakery. It takes 1 hour to make brownies and 3 hours to make a wedding cake. Joey makes $8 profit on the brownies and $16 profit on the cake. He can only work 12 hours a day and needs to make at least $48 a day to afford his car payment.

Team 4: Hannah has a DJ business. Sometimes she downloads an entire CD, sometimes just one song. She needs to make at least 12 downloads of CDs and songs a month to keep her collection current. CDs use 3 MB of memory; single songs require 2 MB. She has 36 MB on her iPod.

Team 5: Joe has two jobs each summer. He makes $12 an hour as a lifeguard and $6 an hour in the concession stand. The pool manager will not let any employee work more than 20 hours per week. Joe needs to make at least $36 a week to pay for weekly trips to the carnival.

BENEFITS

This activity was a wonderful alternative to direct instruction of systems of linear inequalities. It allowed the students to build their own understanding based on their prior knowledge of inequalities and their cultural experience within the real-life situations explored in the problems.

High-cognitive tasks such as this require students to use problem-solving skills and logical reasoning and organization to proceed to a solution (Stein et al. 2009). Had systems of linear inequalities been taught before this task, the cognitive demand on the students would have been significantly lowered; they would have merely executed the procedure already given to them. This activity brought students face to face with an appropriately challenging task. Scaffolding questions asked during the group work allowed students to assign meaning to the procedure.

Successfully navigating this task allows students to create their own understanding of a system-of-inequalities graph and gives them confidence to tackle other mathematical problems. Both of these outcomes are much more difficult to attain through direct instruction only.

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Stephanie S. Reilly, ssr17@pitt.edu, teaches high school in the Norwin School District, North Huntingdon, Pennsylvania. She is interested in how students learn mathematics with understanding and retention. Photo by Jane Reilly.

activity sheet 1

Name ______________________________

Read the problem below.

Team 1: Bobby has two jobs. He makes $10 an hour as a math tutor and $6 an hour as a waiter. Bobby's mother will not let him work more than 20 hours a week. Bobby needs to make at least $30 a week to pay for his cell phone and music downloads.

Let x = number of hours per week that he tutors
Let y = number of hours per week that he waits tables

Team A

1. Write an equality related to the number of hours that Bobby can work without violating his mother's rules.

2. Graph your inequality.

Team B

1. Write an inequality representing how much Bobby needs to work as a tutor and/or a waiter to make enough money.

2. Graph your inequality.

activity sheet 2

Inequalities Task Team Worksheet

Group members: ____________________ ____________________

____________________ ____________________

Work with the other half of your team (A and B). Lay your transparencies on top of one another. The shaded areas from the two graphs *overlap*. Answer the following questions about your system graph:

1. Write what your two variables represent.

x = ______________________________

y = ______________________________

2. Write the two inequalities.

Inequality (A): ______________________________

Inequality (B): ______________________________

3. What does the overlapped shaded area represents in terms of your context?

4. Find two ordered pairs that satisfy your system of inequalities (i.e., are within your overlapping solution area).

__________________ and __________________

5. Find two ordered pairs that do *not* satisfy your system of inequalities.

__________________ and __________________

Your group will present your context, system of inequalities representing the context, and graphical solution (showing your graph on the overhead projector) to the class. Explain what the solution area represents in terms of your context.