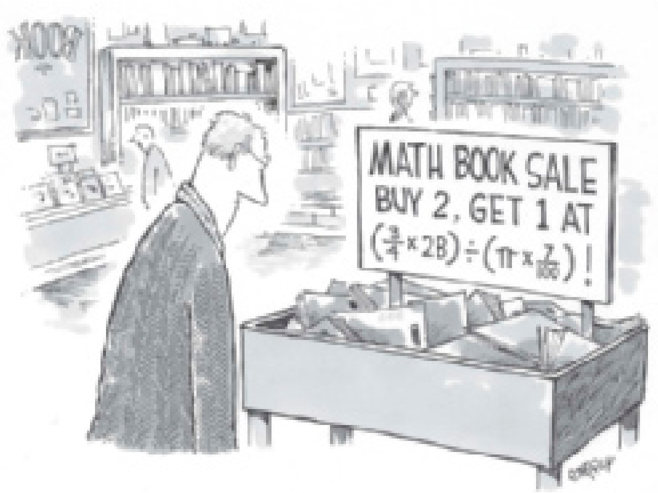
cartoon corner

Name ______________________________

SPEED BUMP by Dave Coverly



THE BOOK SALE

1. Suppose that the man in the cartoon buys 3 math books. Let *B* represent the regular price. The sale price for the third book (*P*) is given in the cartoon and shown below:

$$P = \left(\frac{3}{4}\times 2B\right)\div\left(\pi\times\frac{7}{100}\right)$$

a. If the regular price for each book is \$10, what is the sale price for the third book? Is this a good deal?

b. If the regular price for each book is \$18.95, what is the sale price for the third book? Is this a good deal?

c. Belini says that he can always get the sale price in the cartoon by using

$$P = \frac{150}{7\pi}B.$$

Is he correct? Why, or why not?

2. Is this sale ever a good deal? If so, describe when. If not, adapt the formula so that it is always a good deal. Justify that your new formula will always be a good deal.

3. Stores often hold a "buy one, get one," or BOGO, sale. The terms vary; some may offer the second item free, and others may offer the second item at half price. In the case of buy one, get one free, a person buying two \$10 books would pay \$10 instead of the regular price of \$20—a 50% discount. If the second item was half price, the person would pay \$10 + \$5 = \$15, which is a 25% discount on the list price of \$20 for the two books.

a. Extend the table at the bottom of the page to at least ten rows to show the *overall discounts* for the various sale scenarios listed.

b. What pattern do you notice about where the 25% discount occurs?

CHALLENGE

4. Create a table such as that below, but use the discounts of Free, 80%, 60%, 40%, and 20% in the columns. Look for, and define, patterns in this table *as you make each row.* Use what you learned in question 3 to predict how the pattern might appear in this table.

	Free	75% off	50% off	25% off
Buy 1, get the 2nd	$\frac{\$10}{\$20} = 50\%$	$\frac{\$7.50}{\$20} = 37.5\%$	$\frac{\$5}{\$20} = 25\%$	$\frac{\$2.50}{\$20} = 12.5\%$
Buy 2, get the 3rd	$\frac{\$10}{\$30} = 33.3\%$	$\frac{\$7.50}{\$30} = 25\%$		
Buy 3, get the 4th				
Buy 4, get the 5th				
…				
Buy *n*, get the (*n* + 1)st free				

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Edited by **Stephen P. Smith** and **Peggy House**, Northern Michigan University, Marquette. Classroom teachers interested in field-testing or submitting a cartoon should contact Stephen P. Smith, stepsmit@nmu.edu. The cartoons must include the date and the newspaper syndicate that holds the copyright. Submitted by **Sue McMillen**, Buffalo State College, mcmillse@buffalostate.edu.

SOLUTIONS

1a. The "sale" price would be \$68.24, which is more than the regular \$10.00 price of the book. (If students use the pi key on a calculator, they will obtain slightly different answers.)

$$\begin{aligned}P &= \left(\frac{3}{4}\times 2(10)\right)\div\left(\pi\times\frac{7}{100}\right)\\ &\approx \left(\frac{3}{4}\times 20\right)\div(3.14\times 0.07)\\ &= 15\div 0.2198\\ &\approx 68.24\end{aligned}$$

1b. The "sale" price would be \$129.32, which is more than the regular \$18.95 price of the book.

$$\begin{aligned}P &= \left(\frac{3}{4}\times 2(18.95)\right)\div\left(\pi\times\frac{7}{100}\right)\\ &\approx \left(\frac{3}{4}\times 37.90\right)\div(3.14\times 0.07)\\ &= 28.425\div 0.2198\\ &\approx 129.32\end{aligned}$$

1c. Belini is correct, since the expression in the cartoon is equivalent to Belini's expression. This can be shown with the following simplification.

$$\begin{aligned}P &= \left(\frac{3}{4}\times 2B\right)\div\left(\pi\times\frac{7}{100}\right)\\ &= \left(\frac{3}{2}B\right)\div\left(\frac{7\pi}{100}\right)\\ &= \left(\frac{3}{2}B\right)\times\left(\frac{100}{7\pi}\right)\\ &= \frac{3\cdot 100}{2\cdot 7\pi}B\\ &= \frac{150}{7\pi}B\end{aligned}$$

2. It will never be a good deal. Students may create a table to explore this answer, or they may solve it algebraically.

$$\begin{aligned}P &= \left(\frac{3}{4}\times 2B\right)\div\left(\pi\times\frac{7}{100}\right)\\ &\approx (1.5B)\div(3.14\times 0.07)\\ &\approx 6.82B\end{aligned}$$

On the basis of the equation, the sale price for the third book will always be 6.82 times as much as its regular price.

Answers will vary for adapting the formula to one that *does* provide a discount. One possible answer is

$$P=\left(\frac{3}{4}\times 2B\right)\times\left(\pi\times\frac{7}{100}\right).$$

This expression involves multiplying, rather than dividing, the two terms. This modification results in the third book always being priced as approximately $0.33B$.

3a. See **table 1**.

3b. Some patterns that students might have observed include that the first four entries in the "50% off" column also appear in the "Free" column, but they appear in every other row. The fourth row has no fractions. In the sixth row, the fractions are all sevenths; in the tenth row, the fractions are all elevenths. (By finding common

Table 1 Overall discounts are shown algebraically.

	Free	75% off	50% off	25% off
Buy 1, get the 2nd	\$10/\$20 = 50%	\$7.50/\$20 = 37.5%	**\$5/\$20 = 25%**	\$2.50/\$20 = 12 1/2%
Buy 2, get the 3rd	\$10/30 = 33 1/3%	**\$7.50/30 = 25%**	\$5/30 = 16 1/6%	\$2.50/30 = 8 1/3%
Buy 3, get the 4th	**\$10/40 = 25%**	\$7.50/40 = 18 3/4%	\$5/40 = 12 1/2%	\$2.50/40 = 6 1/4%
Buy 4, get the 5th	\$10/50 = 20%	\$7.50/50 = 15%	\$5/50 = 10%	\$2.50/50 = 5%
Buy 5, get the 6th	\$10/60 = 16 1/6%	\$7.50/60 = 12 1/2%	\$5/60 = 8 1/3%	\$2.50/60 = 4 1/6%
Buy 6, get the 7th	\$10/70 = 14 2/7%	\$7.50/70 = 10 5/7%	\$5/70 = 7 1/7%	\$2.50/70 = 3 4/7%
Buy 7, get the 8th	\$10/80 = 12 1/2%	\$7.50/80 = 9 3/8%	\$5/80 = 6 1/4%	\$2.50/80 = 3 1/8%
Buy 8, get the 9th	\$10/90 = 11 1/9%	\$7.50/90 = 8 1/3%	\$5/90 = 5 5/9%	\$2.50/90 = 2 7/9%
Buy 9, get the 10th	\$10/100 = 10%	\$7.50/100 = 7 1/2%	\$5/100 = 5%	\$2.50/100 = 2 1/2%
Buy 10, get the 11th	\$10/110 = 9 1/11%	\$7.50/110 = 6 9/11%	\$5/110 = 4 6/11%	\$2.50/110 = 2 3/11%
Buy n, get the (n + 1) free	$\frac{\$10}{10(n+1)}=\left[\frac{1}{(n+1)}\times 100\right]\%$	$\frac{\$7.50}{10(n+1)}=\left[\frac{3}{4(n+1)}\times 100\right]\%$	$\frac{\$5}{10(n+1)}=\left[\frac{1}{2(n+1)}\times 100\right]\%$	$\frac{\$2.50}{10(n+1)}=\left[\frac{1}{4(n+1)}\times 100\right]\%$

Table 2 Overall discounts are shown algebraically.

	Free	80% off	60% off	40% off	20% off
Buy 1, get the 2nd	\$10/\$20 = 50%	\$8.00/\$20 = 40%	\$6/\$20 = 30%	\$4.00/\$20 = 20%	\$2.00/\$20 = 10%
Buy 2, get the 3rd	\$10/30 = 33 1/3%	\$8.00/30 = 26 2/3%	\$6.00/30 = 20%	\$4.00/30 = 13 1/3%	\$2.00/30 = 6 2/3%
…					
Buy n, get the $(n + 1)$ free	$\frac{\$10}{10(n+1)} = \left[\frac{1}{(n+1)}\times 100\right]\%$	$\frac{\$8}{10(n+1)} = \left[\frac{4}{5(n+1)}\times 100\right]\%$	$\frac{\$6}{10(n+1)} = \left[\frac{3}{5(n+1)}\times 100\right]\%$	$\frac{\$4}{10(n+1)} = \left[\frac{2}{5(n+1)}\times 100\right]\%$	$\frac{\$2}{10(n+1)} = \left[\frac{1}{5(n+1)}\times 100\right]\%$

denominators, all rows could be similarly viewed. That is, the fifth row could be considered to have all fractions that are sixths.)

4. **Table 2** contains the first two rows and the last two rows of the challenge problem. Students should notice that many patterns—horizontal, vertical, and diagonal—are possible.

FIELD-TEST COMMENTS

I used this activity in two of my eighth-grade classes. One group of students in Math 8, a grade-level course, spanned an incredibly diverse range of achievement; the other group was an algebra/geometry 1 advanced class. I gave both groups an activity sheet I had prepared that provided some scaffolding and structure for their responses, because some Math 8 students have difficulty transferring their answers to questions to lined paper, and some advanced students often miss details and gloss over the requirements to explain their thinking.

Both groups needed some encouragement to manipulate the formula stated in the cartoon. I asked both groups whether or not the multiplication and division signs were those that I would expect eighth graders to use in their formulas. After prompting, most students were able to manipulate the formula independently to answer questions 1c and 2 accurately. Valuable discussions occurred among students about the order of operations and appropriate rounding strategies for problems involving money.

The vast majority of students in both groups were able to complete the first table correctly, and many were able to complete the nth row. Many students completed the challenge correctly and said it was easy to connect the patterns in the first table to those that appeared in the challenge table.

Deborah Regal Coller
Pathfinder (Middle) School
Pinckney, Michigan

I used this cartoon with five classes of eighth-grade advanced math students. Their regular math instruction uses a curriculum that integrates algebra and geometry, which closely aligns with an algebra 1 class.

The students were given this cartoon assignment as homework after we had a unit test on systems of equations. I added question 1c using \$100 as the regular price of a book to make sure that the students were processing the equation correctly. We discussed the cartoon in class, and students worked through the questions on their own. We did not use the challenge question.

We talked about the answers in class the next day. Students had obviously understood the process, because they answered most of them. All seemed to find the correct answers, but some struggled to find more than the obvious patterns on question 3. The ones who caught on to the patterns were much more excited about question 3 than questions 1 and 2.

This cartoon challenged my advanced students, and they enjoyed the part of the assignment that involved calculation. If I were to assign this task again, I would include more questions relating to finding a good deal (as seen in question 2), since some of the students thought more about that than I assumed they would.

Some had good, creative answers to question 2 and realized that they needed to make the formula a fraction of the cost of the book. It was a good exercise and made them think!

Ann Henley
River Trail Middle School,
John's Creek, Georgia

My sixth-grade and seventh-grade algebra 1 students completed this activity soon after their Presidents' Day holiday when many had been out shopping. It provided an opportunity to discuss consumer issues while brushing up on their percent skills.

We started the lesson by discussing a food product package that stated: "Now 20% More Shrimp." The students quickly realized that the package could now contain six shrimp, one more than the original five, and still boast a misleading 20 percent increase. With an admonition of caveat emptor, they dove into the activity.

Some students initially thought that the exclamation point in the cartoon was to be used as a factorial symbol when evaluating the expression. We discussed when to use the calculator's pi key versus the approximation 3.14. Beyond that, they were able to complete most of the exercises independently using a calculator. Students needed some teacher guidance when generalizing the patterns.

Mention *sale*, and the girls in the class become interested and are full of opinions. Being able to numerically justify a good deal is a skill that needs to be taught.

Pamela Haner
St. Catherine's School
Richmond, Virginia

OTHER IDEAS

- Students can graph the discounts. Start by asking them to graph the "Buy *x*, get 1 free" column. Discussion can follow exploring the idea of *approaching zero.*
- Ask students to use their math textbook and figure its price per page, depending on a cost of $25 or a cost you determine. They could also be asked to average the cost per word in their book by estimating the number of words on a page and then the number of words in the book.
- Prompt students to come up with their own sale-price schemes.