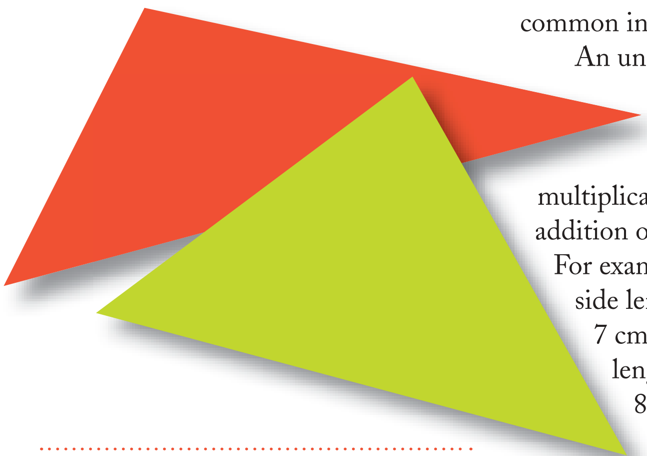
Similar Triangles

Within school mathematics, similarity is a topic that has many connections to real life. Similarity is related to scale drawings, such as those used in architecture, and models, including models of trains, cars, and boats. It is also used to solve problems that involve similar figures, which are common in middle-grades textbooks.



An understanding of similar figures requires proportional reasoning and the recognition that multiplication or division, instead of addition or subtraction, is necessary. For example, suppose a triangle has side lengths of 2 cm, 4 cm, and 7 cm. Multiplying these side lengths by 2 produces 4 cm, 8 cm, and 14 cm. Therefore, a triangle with these side lengths would be similar to the original. However, *adding* 2 to each original side length will produce a triangle that is 4 cm, 6 cm, and 9 cm. It is not similar to the original, although students often mistakenly think that it is.

This misconception is not exclusive to middle-grades students. In courses for preservice teachers, many students have difficulty understanding why addition and subtraction do not produce similar figures. These preservice teachers seem to believe that if the same operation is performed on each of the original side lengths, the resulting side lengths should form a triangle similar to the original, regardless of which arithmetic operation is performed. This misconception may be related to the rule that students learn in algebra: If an equation is balanced and both sides are subjected to the same operation, the equation will still be balanced.

After several semesters of attempting to explain, in abstract terms, why multiplication and division are necessary where similar figures are concerned, I realized that many preservice teachers, like middle-grades students, need visual and hands-on experiences rather than discussion alone. This activity was developed to help preservice teachers better understand similar figures. The same activity could be used with middle-grades students.

A GEOMETRY INVESTIGATION

The activity begins with the teacher asking students to compare the three sizes of triangles in a set of tangrams. If a class does not have access to tangrams, these manipulatives can be printed from the Internet and cut out of paper or card stock. As students

Edited by **Gwen Johnson**, University of North Texas in Dallas, and **James Dogbey**, Clemson University in South Carolina. This department's classroom-ready activities may be reproduced by teachers. Teachers are encouraged to submit manuscripts in a format similar to this department based on successful activities from their own classroom. Of particular interest are activities focusing on NCTM's Content and Process Standards and Curriculum Focal Points as well as problems with a historical foundation. Send submissions by accessing **mtms.msubmit.net**.

investigate tangram triangles, the teacher may want to briefly review types of triangles, such as *right* and *isosceles*. As students place the tangram triangles on top of each other, they should notice that the angles match up, indicating that the angles have the same measurements and that the triangles are *similar*. Students should be encouraged to describe the triangles in their own words; for example, they may say that the triangles "look the same."

Students then draw triangles on centimeter graph paper (which can also be printed from the Internet). However, be aware that photocopied centimeters are not always true to size because of a printer's variability. In **activity sheet 1**, students are instructed to draw a right triangle that has legs that measure 6 cm and 8 cm. If desired, students could use a ruler to measure the length of the hypotenuse.

The teacher could mention that 6, 8, 10 is a "Pythagorean triple." In other words, 6, 8, and 10 can be substituted into the Pythagorean theorem, since $6^2 + 8^2 = 10^2$. Students then add 3 centimeters to each leg and draw a right triangle that has legs of 9 cm and 11 cm. Students are instructed to cut these triangles out and try to match up their angles with the original 6-8-10 triangle. They will see that although the right angles of the triangles match up, the other two pairs of angles do not. Thus, the triangles are *not similar*. Students repeat the process with subtraction, multiplication, and division on each leg of the triangle. They should conclude that multiplying or dividing the side lengths of a triangle produces a similar triangle, but that adding to or subtracting from the side lengths do not.

A CONNECTION TO ALGEBRA

In the second part of this activity, students make connections between similar triangles and algebra. For

activity sheet 2, students should be familiar with function tables from elementary school, but a review may be helpful before beginning this part of the activity. Students are given a series of function tables in which the "In" values represent side lengths of an original triangle and the "Out" values represent the side lengths of a new triangle. They are asked to identify the rule that transforms the "In" values to the "Out" values.

Once the rule has been identified, students can determine whether the two triangles are similar. They should understand from the first part of the activity that function rules that involve addition or subtraction do not result in similar triangles; function rules based on multiplication or division do. Problem 6 on **activity sheet 2** may be challenging for students, since the rule is to multiply by 2.5. Teachers could give students a hint by suggesting division. For example, 20 ÷ 8 = 2.5.

CONCLUSION

One of the expectations described in *Principles and Standards for School Mathematics* (NCTM 2000) is that students in grades 6–8 should use ratios and proportions to solve problems that involve scale factors. However, data from the National Assessment of Educational Progress indicate that students find these problems to be difficult (Blume, Galindo, and Walcott 2007). Many students do not understand why addition or subtraction fails to produce similar figures. **Activity sheet 1** can convince students of this. **Activity sheet 2** can help students make connections to algebra as they realize that the ratio of the side lengths of similar triangles can be seen as an algebraic rule that defines a function.

REFERENCES

Blume, Glendon W., Enrique Galindo, and Crystal Walcott. "Performance in Measurement and Geometry from the Viewpoint of the *Principles and Standards for School Mathematics*." In *Results and Interpretations of the 2003 Mathematics Assessment of the National Assessment of Educational Progress,* edited by Peter Kloosterman and Frank K. Lester Jr., pp. 95–138. Reston, VA: National Council of Teachers of Mathematics, 2007.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

The solutions to **activity sheets 1** and **2** are appended to the online version of "Mathematical Explorations" at **www.nctm.org/mtms**.

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activity sheet 1

Name ______________________________

SIMILAR TRIANGLES

1. Examine the three sizes of triangles in your set of tangrams. Does the shape of the triangles appear to be the same? Would you say that the triangles are similar? Why, or why not?

2. Stack the triangles of different sizes on top of each other. Try to match up the angles. What can you say about the angles of similar figures?

3. Draw a right triangle that has legs of 6 cm and 8 cm on centimeter graph paper. We will call this drawing the *original triangle*.

4. Add 3 cm to each of the legs of the original triangle. The new legs measure _____ cm and _____ cm. Draw a right triangle with these legs on the centimeter graph paper.

5. Cut out both of the triangles, stack, and try to match up the angles the way you did with the tangram triangles. Check with several other students to see if their results agree with yours. Describe how well the angles match up.

6. Does *adding* a certain number to the side lengths of a triangle produce a similar triangle? Explain.

7. Remember that the original triangle has legs that measure 6 cm and 8 cm. Subtract 3 cm from each of these numbers. The new legs measure _____ cm and _____ cm. Draw a right triangle with these legs on the centimeter graph paper.

8. Cut out the triangle that has legs that measure 3 cm and 5 cm. Try to match this triangle with the original triangle. Check with several other students to see if their results agree with yours. Describe how well the angles match up.

9. Does *subtracting* a certain number from the side lengths of a triangle produce a similar triangle? Explain.

10. Remember that the original triangle has legs that measure 6 cm and 8 cm. Multiply each of these lengths by 2. The new legs measure _____ cm and _____ cm. Draw a right triangle with these side lengths on the centimeter graph paper.

11. Cut out this triangle. Try to match up the angles to the angles of the original triangle. Check with several other students to see if their results agree with yours. Describe how well the angles match up.

12. Does *multiplying* the side lengths of a triangle by a certain number produce a similar triangle? Explain.

13. Would *dividing* the side lengths of a triangle by a certain number produce a similar triangle? Explain. If you are not sure, try it.

14. What conclusions can you make about side lengths and similar triangles? What operations provide triangles that are similar to the original?

activity sheet 2

Name ______________________________

SIMILAR TRIANGLES AND FUNCTION RULES

For the following function tables, find the rule that transforms the original triangle into the new triangle. Then, state whether the triangles are similar. The first example has been completed for you.

1.

Original	7	9	10
New	11	13	14

Rule: Add 4
Similar? No

2.

Original	5	7	11
New	10	14	22

Rule: ______________
Similar? ______________

3.

Original	20	23	30
New	11	14	21

Rule: ______________
Similar? ______________

4.

Original	18	45	60
New	6	15	20

Rule: ______________
Similar? ______________

5.

Original	6	10	14
New	17	21	25

Rule: ______________
Similar? ______________

6.

Original	8	10	12
New	20	25	30

Rule: ______________
Similar? ______________

SOLUTIONS TO ACTIVITY SHEET 1

1. Tangrams should show similar triangles of the same shape but different sizes. Explanations will vary.
2. The corresponding angles are the same measure. Explanations will vary.
3. Drawings will vary.
4. 9 cm; 11 cm
5. The angles do not match up. Explanations will vary.
6. No, the triangles are not similar. Explanations will vary.
7. 3 cm; 5 cm
8. The angles do not match up. Explanations will vary.
9. No, the triangles are not similar. Explanations will vary.
10. 12 cm; 16 cm
11. The angles match up. Explanations will vary.
12. Yes, the triangles are similar. Explanations will vary.
13. Division does produce similar figures. Explanations will vary.
14. Multiplication and division of lengths of similar figures produce similar figures.

SOLUTIONS TO ACTIVITY SHEET 2

1. Add 4; no
2. Multiply by 2; yes
3. Subtract 9; no
4. Divide by 3; yes
5. Add 11; no
6. Multiply by 2.5; yes