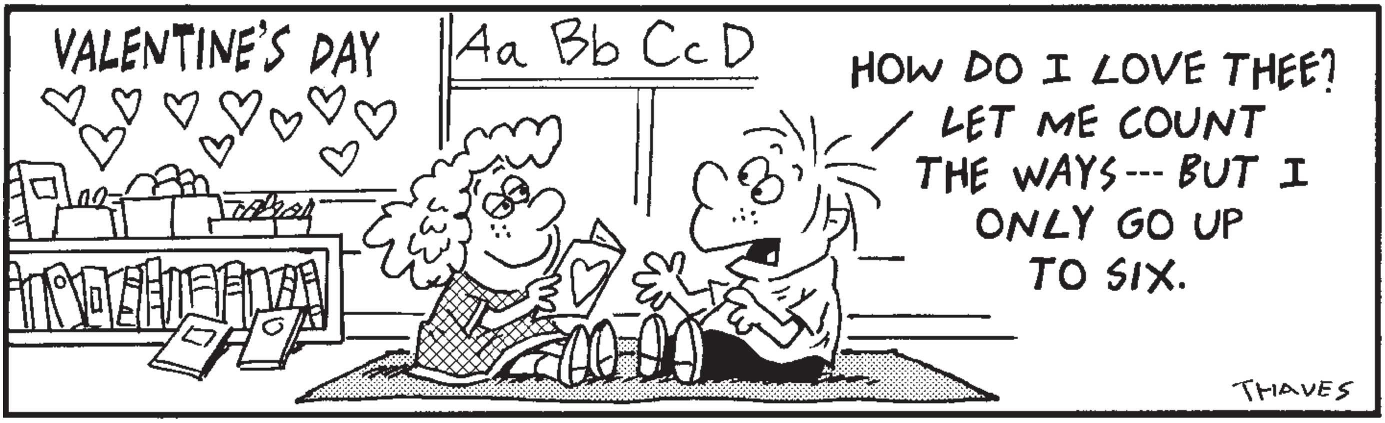
cartoon corner

Name ______________________________

FRANK AND ERNEST by Bob Thaves



LET ME COUNT THE WAYS!

1. When we read a number, we probably do not think about the base that it is in, because the value depends on the symbols used (such as 0, 1, 2, . . . , 9) and their placement. For 378, the 8 represents 8 units, or 8 ones. What do the 3 and 7 represent?

2. Our number system, called *base ten,* uses 9 symbols and a 0 to represent quantities and groups items in tens. The number 10 represents one group of 10 and no units, 100 represents ten 10s (or one hundred) and no units, and so on. The number 378 means $(3 \times 100) + (7 \times 10) + (8 \times 1)$. Write these numbers in expanded form:

a. 487 =
b. 1392 =
c. 1010 =
d. 999 =

3. Our base-ten number system probably evolved because we have ten fingers and early number systems relied heavily on finger counting. If we had only six fingers, we might now be using a base-six system (as in the cartoon) and six symbols: 1, 2, 3, 4, 5, and 0. For example, $35_{\text{base 6}}$ means $(3 \times 6) + (5 \times 1)$. Hence, $35_{\text{base 6}} = 23_{\text{base 10}}$. Expand these base-six numbers and find their base-ten equivalents:

a. $53_{\text{base 6}} =$
b. $11_{\text{base 6}} =$
c. $40_{\text{base 6}} =$

4. When you count your fingers in base ten, you write 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. To count in base six, you write 1, 2, 3, 4, 5, 10, 11, 12, 13, 14. You read the last five numbers as “one zero,” “one one,” “one two,” and so on. Continue counting in base six until you have also counted all your toes. Write and say the numbers that follow as $14_{\text{base 6}}$ (“one four”)$_{\text{base 6}}$. See the table below.

5. Expand these base-six numbers and find their base-ten equivalents. The first one is started for you.

a. $1512_{\text{base 6}}$
$= (1 \times 6^3) + (5 \times 6^2) + \ldots$
b. $1234_{\text{base 6}} =$
c. $555_{\text{base 6}} =$

6. Start with these base-ten numbers and convert them to base six:

a. 137 =
b. 575 =
c. 1244 =

In **base ten**, we group in 10s	10^n . . .	$10 \cdot 10 \cdot 10 = 10^3$ (or 1000)	$10 \cdot 10 = 10^2$ (or 100)	10 (or 10^1)	10^0 (or 1)
In **base six**, we group in 6s	6^n . . .	$6 \cdot 6 \cdot 6 = 6^3$ (or 216)	$6 \cdot 6 = 6^2$ (or 36)	6 (or 6^1)	6^0 (or 1)

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Edited by **Stephen P. Smith**, and **Peggy House**, Northern Michigan University, Marquette. Classroom teachers interested in field-testing or submitting a cartoon should contact **Stephen P. Smith**, stepsmit@nmu.edu. The cartoons must include the date and the newspaper syndicate that holds the copyright.

SOLUTIONS

1. The 3 refers to hundreds; the 7, to tens.

2. a. 487 = $(4 \times 100) + (8 \times 10) + (7 \times 1)$
b. 1392 = $(1 \times 1000) + (3 \times 100) + (9 \times 10) + (2 \times 1)$
c. 1010 = $(1 \times 1000) + (0 \times 100) + (1 \times 10) + (0 \times 1)$
d. 999 = $(9 \times 100) + (9 \times 10) + (9 \times 1)$

3. a. $53_{\text{base 6}} = (5 \times 6) + (3 \times 1) = 33_{\text{base 10}}$
b. $11_{\text{base 6}} = (1 \times 6) + (1 \times 1) = 7_{\text{base 10}}$
c. $40_{\text{base 6}} = (4 \times 6) + (0 \times 1) = 24_{\text{base 10}}$

4. To count toes (11 through 20 in base ten), the base-six counting sequence continues as follows: 15, 20, 21, 22, 23, 24, 25, 30, 31, 32.

5. a. $1512_{\text{base 6}} = (1 \times 216) + (5 \times 36) + (1 \times 6) + (2 \times 1) = 404_{\text{base 10}}$
b. $1234_{\text{base 6}} = (1 \times 216) + (2 \times 36) + (3 \times 6) + (4 \times 1) = 310_{\text{base 10}}$
c. $555_{\text{base 6}} = (5 \times 36) + (5 \times 6) + (5 \times 1) = 215_{\text{base 10}}$

Note: Call students' attention to the similarity between $999_{\text{base 10}}$ and $555_{\text{base 6}}$. In base ten, the number that follows 999 is 1000; in base six, the number that follows 555 is 1000.

6. a. 137 = $(3 \times 36) + (4 \times 6) + (5 \times 1) = 345_{\text{base 6}}$
b. 575 = $(2 \times 216) + (3 \times 36) + (5 \times 6) + (5 \times 1) = 2355_{\text{base 6}}$
c. 1244 = $(5 \times 216) + (4 \times 36) + (3 \times 6) + (2 \times 1) = 5432_{\text{base 6}}$

Help students develop a systematic approach by asking, "What is the largest grouping of powers of 6 contained in this number?" For example:

- What is the largest power of 6 contained in 575?
  (6^3, or 216)
- How many groupings of 216 are contained in 575?
  (2, and $575 - 2(216) = 143$)
- How many groupings of 6^2, or 36, are contained in 143?
  (3, and $143 - 3(36) = 35$)
- How many groupings of 6 are contained in 35?
  (5, and $35 - 5(6) = 5$)

Therefore, $575_{\text{base 10}} = 2355_{\text{base 6}}$.

FIELD-TEST COMMENTS

The students in my eighth-grade algebra class finish quizzes and tests at different times, so this cartoon was given to them to work on after a quiz. No explanations were given. Apparently, students (and their parents) spent a very frustrating evening trying to complete the assignment.

In going over the handout in class, I discovered that my students could read and mimic how to convert from base six to base ten and calculate the correct answers. The same students did not have the luxury of examples to follow for converting from base ten to base six and did not attempt problem 6. After going over the first example in question 6 together, the light bulbs went on, and my students were able to finish the last two problems quickly without my help.

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I worked on this cartoon with my eighth-grade students. My first group struggled a bit with some of the explanations and computations, so I revised my plans for the other groups. With the other groups, I did some teaching of the concepts before they worked on questions 3 and 4. It helped tremendously to discuss some of the examples as a class before they tried these two questions on their own.

Some of my students wondered aloud about the purpose of this exercise (especially some of my more advanced students). When I explained that it would help us better appreciate the base-ten system we use, most seemed satisfied. We also talked about how it is good to stretch our brains a little. For many of my students, this cartoon stretched their brains.

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OTHER IDEAS

- Computers rely on a binary, or base-two, numeration system using only two symbols, 0 and 1. Learn how to read and write binary numbers.
- Base sixteen, using the letters A–F for 10–15, occurs often in computer science. After becoming familiar with both base-two and base-sixteen numbers, students can work on problems that toggle between the two. *Hint:* Group the base-two numbers into four-digit clusters.
- Explore historical cultures that used other number systems. For example, the ancient Babylonians had a base-sixty system with which they could express all numbers using only two wedge-shaped symbols pressed into clay, and the Maya of Central America used a base-twenty number system.