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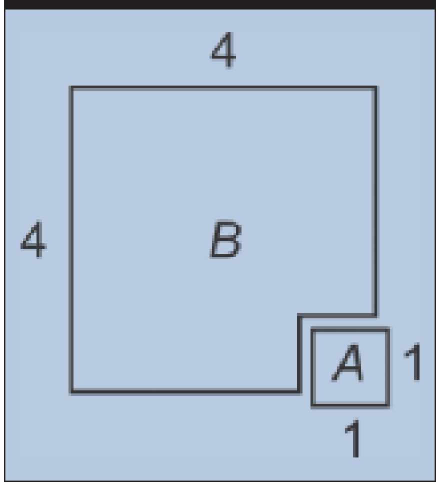
Let’s Cut a Square

Geometry is one of the oldest branches of mathematics. Many important mathematical ideas and theorems are rooted in geometry and can be visualized using geometric interpretations. Students learn about shapes, their properties, and the methods of calculating perimeters and areas of basic shapes early in their school years. Geometry can and should be used to help students visualize and better understand a broad range of mathematical concepts. In the following activity, students use the concepts of area and length to help them understand an important algebraic idea.

Students begin this activity with a 4 in. × 4 in. square piece of paper. Students know that the area of this square is 16 square inches. At this point, I remind them that we could fit sixteen 1 in. × 1 in. squares inside our square. Next, students cut a single 1-inch square from one corner. (See **fig. 1**.) Some students like to take the 1-inch square and repeatedly trace it inside the 4-inch square to verify that it will take exactly 16 of them to cover the entire original square.

Students can use geometric terminology as they discuss these two new shapes. Shape A is a *square* with area of 1 square inch. Shape B is a *concave hexagon* with area of 15 square inches.

Fig. 1 Students are asked to cut a 1 in. × 1 in. square from one corner of a 4 in. × 4 in. square.



Students automatically know that shape B is 15 square inches because it was originally 16 square inches, but 1 inch has been removed. It is important to draw students’ attention to the different possible ways that they can describe what they did using correct terminology.

Consider shape B. If we did not know its area, we could break it into rectangles to make the calculation easier. Ask students to divide shape B into two rectangles, as in **figure 2a**. We can then label the remaining rectangles shape C and shape D.

Students may notice that we can

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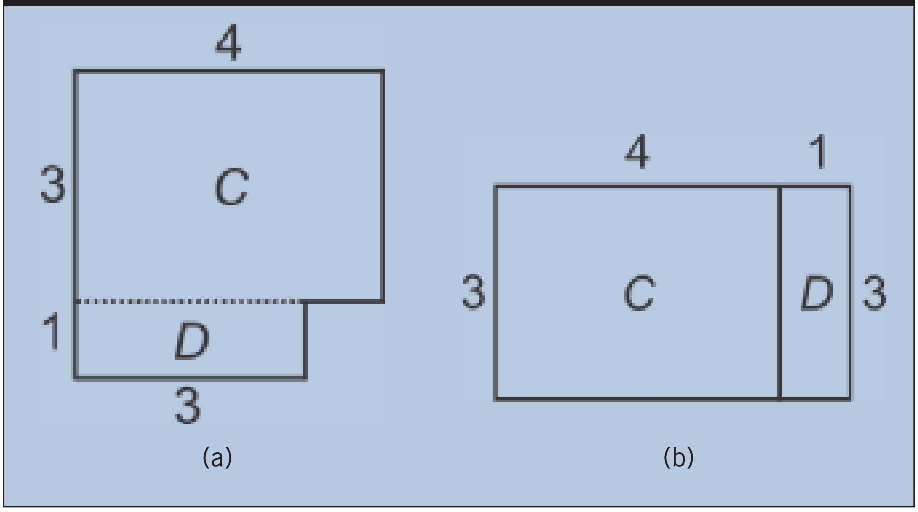
combine rectangles C and D into one big rectangle because the length of rectangle D is equal to the width of rectangle C. The lengths of the sides of both rectangles can be labeled. That way, students can more easily see which pairs of sides match when they form the new rectangle from the two rectangles and the resulting dimensions of that new rectangle. Using the rectangles in **figure 2a**, students can construct one rectangle whose dimensions are 3 in. × 5 in. (see **fig. 2b**).

Ask students to discuss a few questions about the process they have just completed:

- Is it a coincidence that the sides of the rectangle are 1 fewer and 1 more than the sides of the original square?
- Is this construction connected to the fact that the cutout square is 1 in. × 1 in.?

Fig. 2 After cutting a square from one corner, the remaining figure can be divided into two rectangles and rearranged to form a single rectangle.



- If we could cut out a square other than 1 in. × 1 in., could we rearrange the remaining part into a rectangle whose sides can be expressed using sides of the squares?

Usually I ask students to work in small groups on two squares that are different from the 4 in. × 4 in. square with 1 in. × 1 in. cut out that we discussed as a class. Different groups choosing different numbers

Table 1 With different squares and cutouts, patterns emerge among the dimensions of the figures created by cutting and rearranging.

Side of the Large (Original) Square	Side of the Small (Cutout) Square	Rectangle *C*		Rectangle *D*		Length of the Final Rectangle	Width of the Final Rectangle
		Length	Width	Length	Width		
4	1	4	3	3	1	5	3
4	2	4	2	2	2	6	2
5	2	5	3	3	2	7	3
6	2	6	4	4	2	8	4
6	3	6	3	3	3	9	3

for their squares will produce a variety of examples for discussion. After every group has completed its investigation, numerical data can be collected (see **table 1**). From these data, students can analyze the experiments they completed and formulate these observations:

- The length of rectangle *C* is equal to the length of the side of the large square.
- The width of rectangle *C* is equal to the difference of the lengths of the sides of the large and small squares.
- The length of rectangle *D* is equal to the width of rectangle *C*.
- The width of rectangle *D* is equal to the length of the side of the small square.
- The length of the final rectangle formed from rectangles *C* and *D* is equal to the sum of the lengths of the sides of the large and small squares.
- The width of the final rectangle formed from rectangles *C* and *D* is equal to the width of rectangle *C*. This width is also equal to the difference of the lengths of the sides of the large and small squares.

The "final rectangle" is the remaining part of the large square after the small square was cut out. The area of the "final rectangle" is the difference of the areas of the two squares. Therefore, after performing this investigation, students "discover" the algebraic formula $a^2 - b^2 = (a - b)(a + b)$.

This activity can be used in the fifth and sixth grades as an example of finding the areas of complex shapes by dissecting them into simple shapes. In this case, the result of the investigation can be formally stated as this:

> The difference of the areas of two squares is equal to the area of the rectangle whose length is equal to the sum of the lengths of the sides of the squares and whose width is equal to the difference of the lengths of the sides of two squares.

Schedule this helpful activity in introductory algebra courses before the formula $a^2 - b^2 = (a - b)(a + b)$ is introduced. This exploration will help students see connections between algebra and geometry and gives them a geometric illustration of an algebraic formula. It also makes the given formula more meaningful to students.

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