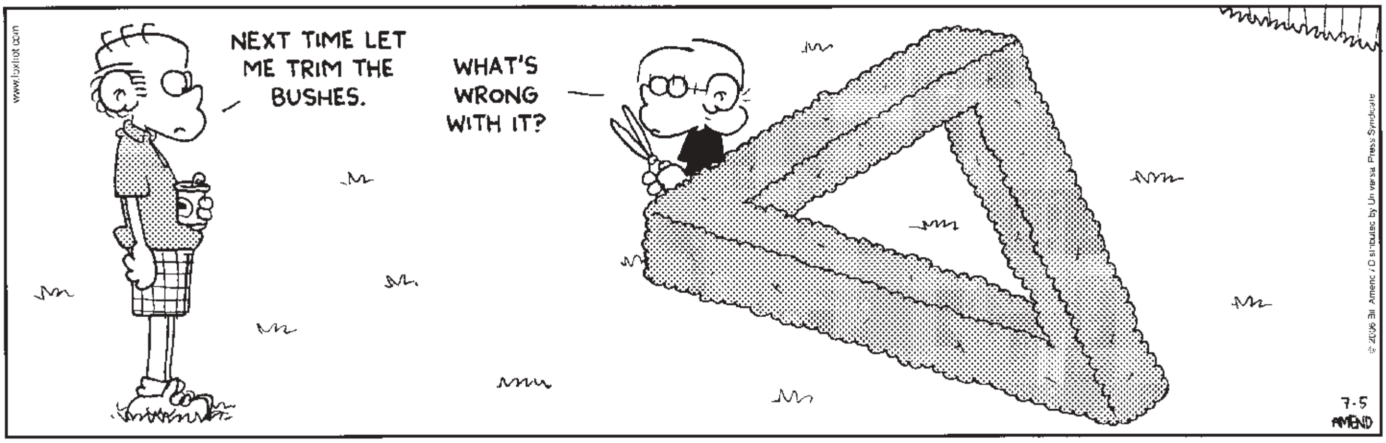
cartoon corner

Name ________________________________

FOXTROT by Bill Amend



TWISTED MATHEMATICS

The hedge in the comic is shaped like a *Penrose triangle*. It is an example of an object that cannot exist in three-dimensional space. It is similar to a *Möbius strip,* which can be formed from a strip of paper. Cut three 2-inch-wide lengthwise strips from a blank piece of paper (preferably 14-inch legal-sized paper).

1. Using one strip of paper, tape the two narrow ends of one strip together to form a paper ring.

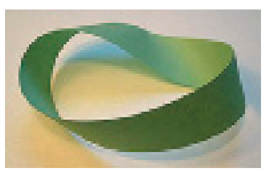
a. How many edges does the paper ring have? How many surfaces?

b. Using a felt-tip marker, start at the tape and draw a line along the center of the strip until you reach the starting point. Describe the result.

2. On a second strip of paper, draw arrows on the narrow edges pointing in opposite directions (see the diagram below). Tape the edges together (requires a half twist) so that the arrows point in the same direction when the ends are joined. You have made a Möbius strip.

a. Use the felt-tip marker and draw a line along the center of the strip until you reach the starting point. Compare this result with the ring above.

b. How many surfaces does the Möbius strip have?



c. Starting at the seam, shade the edge of the strip with a marker and continue until you reach the starting point. How many edges does the Möbius strip have?

d. If a bug starts at the seam of your Möbius strip and crawls along the shaded line you colored until it arrives back at the starting point, how far will it have traveled?

3. Cut your strip along the center line that you drew, and describe the result.

4. Take the result from the previous cut, and cut this new object down the center. What happens?

5. Make a new Möbius strip with the third paper strip, and shade a wide stripe along its center. The stripe should be 1/3 the width of the strip. Cut the strip along one edge of your colored stripe. What happens?

from the March 2011 issue of mathematics teaching in the MIDDLE SCHOOL

Edited by **Stephen P. Smith**, and **Peggy House**, Northern Michigan University, Marquette. Classroom teachers interested in field-testing or submitting a cartoon should contact **Stephen P. Smith**, stepsmit@nmu.edu. The cartoons must include the date and the newspaper syndicate that holds the copyright.

SOLUTIONS

1. a. The ring has two edges. Students will probably describe the edges as the "top" and the "bottom." The ring has two surfaces. Students will probably describe them as the "inside" and the "outside."

b. The drawn line will appear on one side of the paper only. This should help students recognize that there are two distinct surfaces, one marked and one unmarked. To mark both surfaces, one must lift the marker from the paper and move it to the other side.

2. a. The drawn line will appear on the entire strip, including both sides of the original paper, which occurred without lifting the marker from the paper.

b. The Möbius strip has only one surface, as demonstrated by the continuous line discussed above.

c. Make sure the students mark the edge of the paper and that the ink soaks through. If they mark a small X as the starting point, the entire strip will be "edge colored" when they return to the starting point, even though they did not lift the marker. The Möbius strip has one edge only.

d. The distance the bug will have traveled will be twice the length of the original paper strip before the ends were taped (adjusted a bit for whatever overlap occurred in taping the ends).

3. The Möbius strip cut down the center stays in one piece. It is twice as long and half as wide as the original, but it has two full twists in it. It is not a Möbius strip, which you can show by making a line along its center. When the line returns to the starting point, it will be on only one side of the paper, showing that the result of cutting the Möbius strip down the center produces an object with two surfaces, not one.

4. Cutting the new strip (from step 3) down its center produces two strips that are wound around each other, each having two full twists.

5. The Möbius strip cut 1/3 the distance from the edge produces two strips: a narrower Möbius strip that is 1/3 the width and the same length as the original (it will be the center 1/3 of the original that the students previously shaded), and a longer, thin strip with two full twists in it. The longer strip is 1/3 the width and twice the length of the original strip; it consists of what were originally the uncolored outer edges of the original.

(Suggestion: It is convenient to use centimeter grid paper to make the strip for task 5. Cut the strip 3 cm wide and have the students shade the center 1/3 of the strip. They can then cut along the grid line, making it easier to know where 1/3 the distance from the edge is located.)

FIELD-TEST COMMENTS

At first, my students did not know what to think; I did not do an introductory lesson. We had just completed our study of decimals and were polishing off the study of fractions. Also, we had not done any type of activity that involved measuring and cutting paper, much less taping it together, twisting it, and so on. I discovered that my students could all use more practice with scissors and with gross motor skills in our mathematics class.

On the day we began this lesson, we examined the cartoon and read the first paragraph aloud together. Then we dug right in—passing out the paper, rulers, markers, scissors, and so on. The students had trouble "seeing" the Möbius strips. They kept thinking they were making mistakes putting their ends together, since the paper rings would not lay flat on the tables because of the twists in the papers. They "got it" when they traced the midsections of the rings with markers and cut them into pieces. They *loved* it!

They wanted to keep going with this math project. Several decided to use the spare paper I had available for those who needed to remake their strips. When all the paper was gone, they used their own notebook paper. They wanted to experiment to see what would happen if you had 1, 1 1/2, 2, 2 1/2, 3, 3 1/2, 4, and so on, twists in the paper strips. A few students even made T charts to record their findings. Some of their Möbius strips are still on a shelf in our classroom—the students want to continue to experiment with them and are not willing to let go of them just yet.

My next class assignment was to create a "Guess My Mathematician" riddle for students' math pen pals. A few students chose August Möbius and Robert Penrose to use as subjects for their riddles because of this particular lesson. What an impact!

Tina Gay
K. E. Taylor Elementary School, Lawrenceville, Georgia

I used the activity with seventh graders and honors eighth graders in prealgebra and algebra. In both classes, the lesson was an enrichment activity. I did a little prepping before I gave the activity to my students. I cut the strips of paper in advance and listed the supplies they would need on the board: 1 marker, 3 strips of paper, tape, and scissors. I also retyped the questions onto a student-friendly handout with fewer words and more room for them to write their answers.

I walked my seventh-grade students through each step of the process. I had to physically help some of them twist their paper to make the Möbius strip. They worked with partners to answer the questions before we discussed each one as a class. My honors eighth-grade students worked in groups of three and at their own pace. The activity took most of one 42-minute class period.

Some students will try to cut perpendicularly into the Möbius strip before cutting along the line, so make sure that you specifically tell them not to do this. They should fold the paper and make a small cut in the middle first.

Katie A. Hendrickson
Athens Middle School, Athens, Ohio

OTHER IDEAS

- Allow your students to experiment to see what would happen if they had 1, 1 1/2, 2, 2 1/2, 3, 3 1/2, 4, and so on, twists in their paper strips.
- Ask students to create T charts to record their findings after making the different numbers of twists to try to detect a pattern.
- Ask the students to predict what would happen if a paper link had 20, 50, or 100 twists.
- Can they think of a practical use for a ring that contains more twists than the simple Möbius strip?
- If your school recognizes the "100th Day of School" each year, have your students create interlocking Möbius strips to hang in your classroom or in the hallway to "illustrate" the number 100.

Tina Gay
K. E. Taylor Elementary School, Lawrenceville, Georgia

NCTM 2011

INDIANAPOLIS, IN
APRIL 13–16, 2011

Annual Meeting & Exposition

Geometry: Constructing and Transforming Perspectives

Rev up your professional skills and keep your knowledge and career on track.

NCTM's Annual Meeting offers numerous opportunities to help you move in the right direction. There's something for everyone—whether you're a classroom teacher, administrator, teacher educator, preservice teacher, or math specialist. Attendees will:

- **Discover** new strategies to use in the classroom
- **Network**, network, network
- **Assess** current teaching methods and find what works best for them
- **Evaluate** new products and technology
- **Collaborate** with colleagues and gain new insights

You'll speed back to the classroom excited to share what you've learned!

NCTM
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
(800) 235-7566 | WWW.NCTM.ORG

Visit **www.nctm.org/meetings** for up-to-date information.