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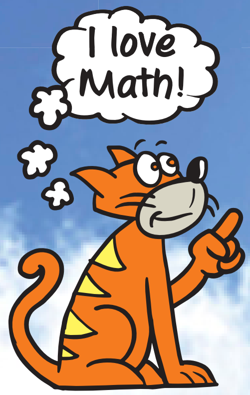

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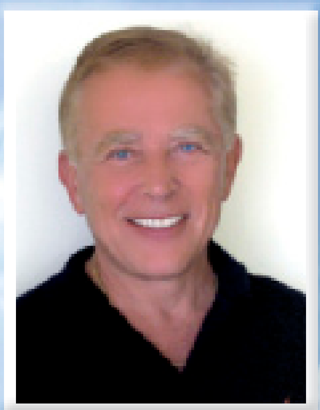

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BIG IDEAS LEARNING®

Amy B. Ellis and Robert Ely

Different Approaches to the Mystery Table

A small group of eighth-grade students had been working on rectangle problems that focused on the quantities of area and perimeter. The students could readily develop numerical patterns, but it was unclear whether they were connecting their patterns to the relationships that were found among the side lengths, the area, and the perimeter. Their teacher, who was part of a mathematics-education research team, gave students the problem in **figure 1** to check their understanding.

The teacher purposefully chose values in the columns that could refer to either area or perimeter. The students were familiar with multiplicative patterns that could represent area. However, it was also possible to use these same values to draw long, skinny rectangles with appropriate side lengths to represent perimeter. The teacher's goal in introducing an open-ended problem was to see whether the students connected the data to the quantities of area and perimeter and how these connections might support the development of algebraic expressions and functions.

Three students used different mathematical reasoning and strategies, but all found the same solution for the mystery rectangle with side length 30. As you read, try to figure out each strategy from the student's work.

Fig. 1 This problem requires students to connect number patterns to an understanding of dimensional analysis.

Here is a mystery table of values for a rectangle. Is this table's second heading asking for the *area* or the *perimeter*?

Side Length	?
7	63
11	143
16	288
25	675
30	?

Edward's Strategy

Edward said that the mystery column represented perimeter, but he could not fully explain why. (See his work in **fig. 2**.) He described his strategy:

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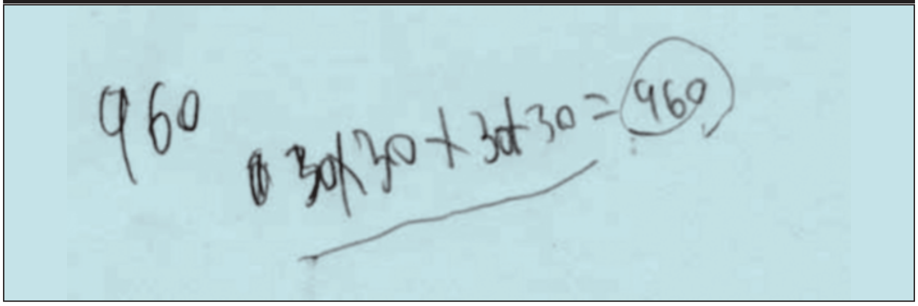
Fig. 2 The three students solved the problem, but their strategies were different. Edward wrote this solution.



> All you had to do is do 30 times 30 plus 30 plus 30, I mean, 7 times 7, 49, and then 7 plus 7 is 14, would be 14 plus 49 is 63. And then, I just did that for all of them.

For each row in the table, Edward calculated $(s \times s) + s + s$, where s is the rectangle's side length. Does his strategy work for all the entries? Why? Does Edward's strategy connect to the quantities in the problem, and does it help him decide whether the mystery column represents area or perimeter?

Marko's Strategy

Marko decided that the mystery column represented area. He explained:

> The whole 7 times 9 equals 63 and the 11 times 13 equals 143. They're, they have like a distance of 2 in between each number. And 16 times 18 is 288, so 30 times 32 is 960.

Marko noticed that each side length could be multiplied by a factor to get the number in the second column and that this factor is 2 units larger than the side length. To get the answer, he calculated $s \times (s + 2)$ each time. Does Marko's strategy work? Why? Is it connected to the quantities in the problem, and does it help Marko think about the rectangle's area?

Amal's Strategy

Amal decided that the mystery column represented area, but he was bothered by the fact that the values in the first column were "not going up by a fixed amount." In a conversation with his teacher, his thinking is revealed:

Amal: What I found was, I knew the first one was 7 times 9 equals 63, I saw that. And then, I figured out all of them, and 11 times 13 equals 143, and 16 times 18 equals 288. And then 25 times 27 equals 675.
Amal: And then what I figured, like 11, the difference in between 7 times 9 and 11 times 13, the difference in between the 9 and the 13 is 4, and then the difference between the two numbers, 7 and 11 is 4, and then the same thing with 18 and 13, the difference is 5, and then the difference in between those two numbers is 5.
Teacher: Okay.
Amal: And then the same thing for the 16 and the 25. The difference in between the numbers was 9. And the difference in between 18 and 27 is 9. So I, for 30, what I did, I added 5 more on the 27, so I'd multiply it by 32 instead, since you multiplied that by 27.
Teacher: And then, how'd you get the 32?
Amal: Because 27 plus 5.

As Amal spoke, the teacher drew the differences that Amal had calculated (see **fig. 3**). Amal introduced a middle column of multiplicative factors and then looked for vertical patterns in his three columns. Even though he saw no pattern in the first two columns, he noticed that whenever the first column went up by k, the second column went up by k, as well. What does this strategy indicate about Amal's understanding? Is it connected to the quantities of perimeter or area?

Fig. 3 As Amal explained his strategy, the teacher wrote the numbers he verbalized at right and left.

	Side		???
	7 • 9 =		63
4		4	
	11 • 13 =		143
5		5	
	16 • 18 =		288
9		9	
	25 • 27 =		675
5		5	
	30 • 32 =		?

MAKING SENSE OF THE THREE STRATEGIES

All three strategies generated the same answer, which is reasonable if the mystery column represented area. To promote sense making, a teacher should encourage students to understand the origin of each strategy and why it makes sense.

In this problem, the context was the relationship between the rectangle's known side length and its area. For instance, Edward could easily have extended his horizontal $(s \times s) + s + s$ strategy correctly to any other side length, but he treated it as a numerical pattern in the table without connecting this pattern to the concept of area. When the teacher asked Edward to explain why his strategy made sense, he struggled to provide an explanation and said, "I don't know. It just works!"

Amal struggled in a similar manner when trying to explain why his strategy worked. The teacher decided to help Edward and Amal make sense of their solutions by comparing them to Marko's work. Marko connected his numerical strategy to the idea of length times width.

The teacher hoped that by comparing the different strategies, the students would see that the pattern $(s \times s) + s + s$ was equivalent to $s(s + 2)$. She also hoped that they would begin to think about this product as a meaningful way of representing the area of a rectangle with one side being two units longer than the other (one correct interpretation of the problem). As a starting point, the teacher drew **figure 4** on the board and asked how the picture could be used to explain Edward's strategy:

Amal: So say it was 7 by 7, and then it was like it is, like that. [He covers up the rows to make the 7×7 rectangle.] And inside here would be like 49.

Teacher: Oh, could you just shade those for us so we could see?

Amal: Like that. [See **fig. 5**.]

Teacher: Ah, so there's the 49, the 7 times 7. Okay.

Amal: And then, plus 7 would be another row right here, and plus another 7 would be like that. [He touches the second and first rows.]

Fig. 4 The teacher drew a 9×7 table to illustrate the problem.

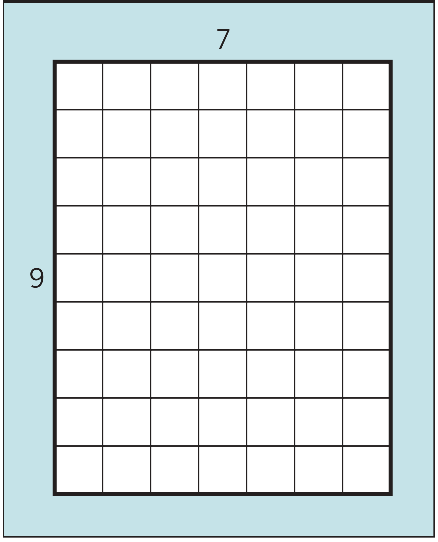
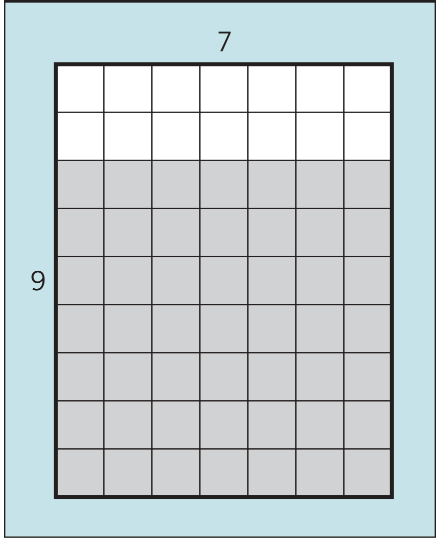


Fig. 5 Amal's shading connected both Edward's and Marko's reasoning.



Amal figured out how Edward's and Marko's strategies were related, but he also understood how both strategies connected to the idea of area. To do so, Amal treated multiplication as a method of counting a two-dimensional array and treated addition as a way of placing extra rows in the array. He could then see his own strategy in terms of multiplying length by width.

The teacher then asked Marko how he could connect his strategy to what Amal had demonstrated. Marko explained that the additional two rows are represented by $(s + 2)$ when multiplying $s(s + 2)$. The teacher followed up by eliciting students' ideas about how to transform $(s \times s) + s + s$ into $s(s + 2)$. Once the students simplified the expression, they could see how Edward's strategy represented a way to calculate area.

ENCOURAGING REASONING HABITS

Reflecting on a solution to a problem and seeking and using connections (Martin et al. 2009) are reasoning habits that can help students make sense of their strategies. These habits of mind help students understand *why* solutions work and *how* they connect to the situation at hand.

Once the students began to think about their solutions in terms of length, width, and area, they were able to understand their own strategies better and make connections across the three solutions. They could then make sense of expressions such as $(s \times s) + s + s$ and understand how that expression and $s(s + 2)$ represented two different ways of calculating the area of a particular type of rectangle. The teacher's willingness to look beyond the correct answers to foster reflection on the solutions helped the students make sense of their numerical strategies.

REFERENCES

Martin, W. Gary, John Carter, Susan Forster, Roger Howe, Gary Kader, Henry Kepner, and Patricia Valdez. *Focus in High School Mathematics: Reasoning and Sense Making.* Reston, VA: National Council of Teachers of Mathematics, 2009.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

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