palette of problems

David Rock and Mary K. Porter

1. Camille planted a vegetable garden, 1/2 of which was in tomatoes. She planted 1/4 of the ground remaining in broccoli. She then planted lettuce in 1/2 of the rest. The amount that remained was planted in cauliflower. What percent of her garden was planted in cauliflower?

2. A person blinks an average of 25 times per minute when awake. Last year (not a leap year), Katelyn averaged 9 hours, 45 minutes of sleep each day. How many times did Katelyn blink last year?

3. There are 13 two-digit multiples of 7. Find the sum of all two-digit multiples of 7 whose digits sum to a prime number.

4. Grace's math teacher gives her a large piece of paper that is 0.01 mm thick. Grace knows that folding the paper doubles its thickness. For example, after two folds, the total thickness is 0.04 mm thick. Assuming that this is possible, how many times would Grace have to fold this paper to make its thickness equal to the distance from Earth to the moon? *Note:* The average distance of Earth to the moon is 384,400 km.

5. Carly and Cheryl bought six items for $207. One item cost $1. All other prices were different prime numbers (in whole dollars only), and each digit (1 through 9) was used once and only once. The sum of the digits of one item totaled 7, and the price of one pair of pants was $67. What was the cost of the most expensive item?

6. In the sum

$$8 + 88 + 888 + \cdots + 888{,}888{,}888{,}888{,}888{,}888{,}888,$$

what are the last two digits (in the tens and units places)?

7. A total of 232 people were asked which of three desserts they like best: cookies, pie, or ice cream. Of those surveyed, 40% more people liked cookies than liked ice cream. Twice as many people liked ice cream as liked pie. How many people liked cookies best?

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8. Mila took a trip and drove the first 20 miles at a speed of 50 miles per hour (mph). If she wants her average speed for the entire trip to be 40 mph, how fast should she drive for the last 70 miles of her trip? Round your answer (in mph) to the nearest tenth.

9. A rectangular prism is 18 inches tall and has a square base whose length is 9 inches. Find the surface area of the prism.

10. Oliver plans to cover the top and bottom of the rectangular prism described in question 9 with material A, which costs 2¢ per square inch. He then plans to cover the other four faces of the prism with material B, which costs 1.5¢ per square inch. How much money will Oliver spend on the materials to cover the rectangular prism?

11. A rectangular prism, with the same dimensions as in questions 9 and 10, is constructed of metal. This prism is melted down into a solid, right circular cylinder. It is 2 feet tall and has the same volume as the prism. What is the radius of the base of this circular cylinder? Round your answer to the nearest tenth of an inch.

12. Sage's pet rabbit has 15% less fur than his cat, and his dog has 20% more fur than his cat. To the nearest tenth of a percent, what percent more fur does the dog have than the rabbit?

For questions 13–15, use the following definition of ♣:

$$A \clubsuit B \text{ means } B - 2A + 3$$

13. Find the value of the following:

a. $2 \clubsuit 6$
b. $-1 \clubsuit -1$
c. $1/4 \clubsuit 1/2$

14. Find all values of x such that $x \clubsuit 9 = 0$.

15. Find all values of C for which $5 \clubsuit C = C \clubsuit 5$, or explain why no such number C exists.

The solutions are appended to the online version of the "Palette of Problems" at **www.nctm.org/mtms**.

solutions to palette

(Continued from pages 534–35)

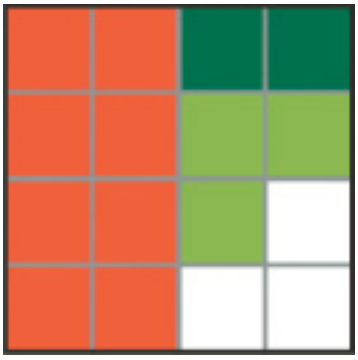
(Alternative approaches to those suggested here are encouraged.)

ANSWERS

1. 18.75%
2. 7,801,875
3. 308
4. 45 or 46 folds
5. $89
6. 68
7. 112 people
8. Approximately 37.8 mph
9. 810 in.^{2}
10. $12.96
11. Approximately 4.4 inches
12. Approximately 41.2% more fur
13. a. 5; **b.** 4; **c.** 3
14. 6
15. 5

SOLUTIONS

1. If you draw the garden as a 4×4 square, draw a line down the middle, indicating that half is for tomatoes. The broccoli takes 1/4 of what is left, or 1/4 of 1/2, or $1/4 \times 1/2 = 1/8$ of the original plot. The remaining area at this point is 3/8 of the original plot. The lettuce takes 1/2 of that remaining 3/8, or $1/2 \times 3/8 = 3/16$. Therefore, cauliflower takes up 3/16 of the original plot; 3/16 = 0.1875, or 18.75%.



2. There are 24 hours in each day, but Katelyn sleeps for 9.75 of those hours. She is therefore awake for 14.25 hr., or $14.25 \times 60 = 855$ min. Multiplying that result by 365 days in the year and 25 times per minute, she blinked 7,801,875 times last year.

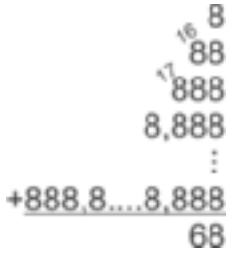
3. The two-digit multiples of 7 are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, and 98. The numbers whose digits sum to a prime number are 14, 21, 49, 56, 70, and 98; the sum of these numbers is 308.

4. The moon is at its closest (Perigee) to Earth at 363,300 km and is at its farthest (Apogee) at 405,500 km. The average distance of Earth to the moon is 384,400 km. You can use the formula $(0.01)2^n$, where n is the number of folds to determine the thickness of the paper. The answer must be at least 45 and could be 46, depending on the distance of Earth to the moon. With trial and error, we find:

- For 45 folds, the thickness will be about $(0.01)2^{45}$, or 351,843,720,900 mm, or about 351,843 km.
- For 46 folds, the thickness will be about $(0.01)2^{46}$, or 703,687,441,800 mm, or about 703,687 km.

5. Since the total is $207 and one item costs $1 and another $67, $207 – $67 – $1 = $139. Remember, you must use the digits 1–9. Using 67 and 1 leaves 2, 3, 4, 5, 8, and 9. You have four items remaining using these digits, and all must be prime. Also, the sum of the digits of one item totals 7. Therefore, this item will either be $25, $52, $34, or $43. The number 43 is the only prime, so $43 must be the cost of another item. You now have 1, 43, and 67 for a total of 111, and three items remain. Since 207 – 111 = 96 and the digits 2, 5, 8, and 9 remain, one item has two digits and the other two have one digit. Using a 2, only 2 and 29 are prime. But if 29 is used, the remaining digits will not allow prime numbers. Therefore, the cost of one item is $2. Using 5, you can create 5, 58, 85, 59, or 95. Only 5 and 59 are prime. But if 59 is used, 8 would remain, and 8 is not prime. Therefore, the cost of another item is $5, and the final item is $89. The items cost $1, $2, $5, $43, $67, $89, with the most expensive item being $89.

6. This is a sum of 21 numbers. The sum of the units digits is 8(21) = 168, so the ones digit is 8 regrouping 16 tens. The sum of the tens digits is 8(20) + 16 (from ones) = 160 + 16 = 176, so the tens digit is 6 regrouping 17 hundreds.



7. Let c, p, and i represent the number of people in the survey who preferred cookies, pie, and ice cream, respectively. Since 40% more people liked cookies than liked ice cream, we can write $c = 1.4i$. Also, twice as many people liked ice cream as liked pie, so $i = 2p$, or $p = (1/2)i$. Since 232 people answered the survey, $c + p + i = 232$. In this equation, if we replace c and p with expressions involving i, we can write and solve:

$$1.4i + 0.5i + i = 232$$
$$2.9i = 232$$
$$i = \frac{232}{2.9}$$
$$i = 80$$

Since $c = 1.4i$, then $c = (1.4)(80) = 112$, so 112 people in the survey liked cookies best.

8. Manipulate the formula $d = rt$, making it $d/r = t$, isolating t, the total time for her trip. Since she traveled 20 miles at 50 mph, she spent 2/5 hour on the first part of the trip. Since she wants to average 40 mph on a trip of length 70 miles, her total time will be 20 + 70 miles at 40 mph, or 9/4 hr. This makes her time for the second part of the trip 9/4 – 2/5 = 37/20 hr. Use $d/r = t$ again to obtain $70/r = 37/20$, which makes $r \approx 37.8$.

9. The area of the prism's base is (9 in.)(9 in.) = 81 in.^{2}, which is also the area of the top of the prism. Each of the four faces has this area,

$$(9 \text{ in.})(18 \text{ in.}) = 162 \text{ in.}^2,$$

so the total area of these four faces is (4)(162) = 648 in.^{2}. So the total area for the prism is

$$81 \text{ in.}^2 + 81 \text{ in.}^2 + 648 \text{ in.}^2 = 810 \text{ in.}^2.$$

10. From the solution to question 9, we know that the top and the bottom of the prism are each 81 in.^{2}. The cost to cover the top and base of the prism with material A is (2¢ per square in.) × (81 + 81)in.^{2} = (2)(162)¢ = 324¢, or \$3.24. From the previous question, we know that the total area of the four side faces is 648 in.^{2}, so the cost of covering these four faces with material B is

$$(1.5¢ \text{ per square in.})(648 \text{ in.}^2) = 972¢,$$

or \$9.72. Thus, Oliver will spend \$3.24 + \$9.72 = \$12.96 on the materials to cover the rectangular prism.

11. From the solution to the question 9, the area of the prism's base is 81 in.^{2}, and the prism's height is 18 inches. The volume of the prism is (81)(18) = 1458 cubic inches. Let r be the radius of the base of the right circular cylinder, in inches. Then πr^2 in.^{2} is the area of this base, and the volume of the cylinder (which is 2 feet, or 24 inches, tall) is $24\pi r^2$ in.^{3}. This must be the same as the volume of the rectangular prism, which is 1458 cubic inches, so we write and solve:

$$24\pi r^2 = 1458$$
$$r^2 = \frac{1458}{24\pi}$$
$$r^2 = \frac{60.75}{\pi}$$
$$r = \sqrt{\frac{60.75}{\pi}}$$
$$\approx 4.39742 \text{ in.}$$

Thus, the radius of the base of the cylinder is approximately 4.4 inches.

12. Let r, c, and d represent the amount of fur of Sage's rabbit, cat, and dog, respectively. Since his rabbit has 15% less fur than his cat, $r = 1c - 0.15c$, so $r = 0.85c$, or $c = r/0.85$. Since Sage's dog has 20% more fur than his cat, $d = 1c + 0.20c$, so $d = 1.2c$, or $c = d/1.2$. We set these two values of c equal to each other as

$$\frac{r}{0.85} = \frac{d}{1.2}.$$

Multiplying both sides of this last equation by 1.2, we see that

$$d = \frac{1.2}{0.85} r$$
$$d = (1.411764\ldots)r,$$

so d is approximately 41.2% more than r. Sage's dog has approximately 41.2% more fur than his rabbit.

13. a. $2 \clubsuit 6 = (6) - 2(2) + 3$
$= 6 - 4 + 3 = 5$

b. $-1 \clubsuit -1 = (-1) - 2(-1) + 3$
$= -1 + 2 + 3 = 4$

c. $1/4 \clubsuit 1/2 = (1/2) - 2(1/4) + 3$
$= 1/2 - 1/2 + 3 = 3$

14. According to the definition, $x \clubsuit 9 = 9 - 2(x) + 3 = 0$, so we solve for x as follows:

$$9 - 2x + 3 = 0$$
$$-2x + 12 = 0$$
$$-2x = -12$$
$$x = 6$$

15. According to the definition,

$$5 \clubsuit C = C - (2)(5) + 3,$$

so $5 \clubsuit C = C - 7$. Using the definition again, we find that

$$C \clubsuit 5 = 5 - (2)(C) + 3,$$

so $C \clubsuit 5 = 8 - 2C$. Then $5 \clubsuit C = C \clubsuit 5$ is true when $C - 7 = 8 - 2C$, or $3C = 15$, which means that $C = 5$.