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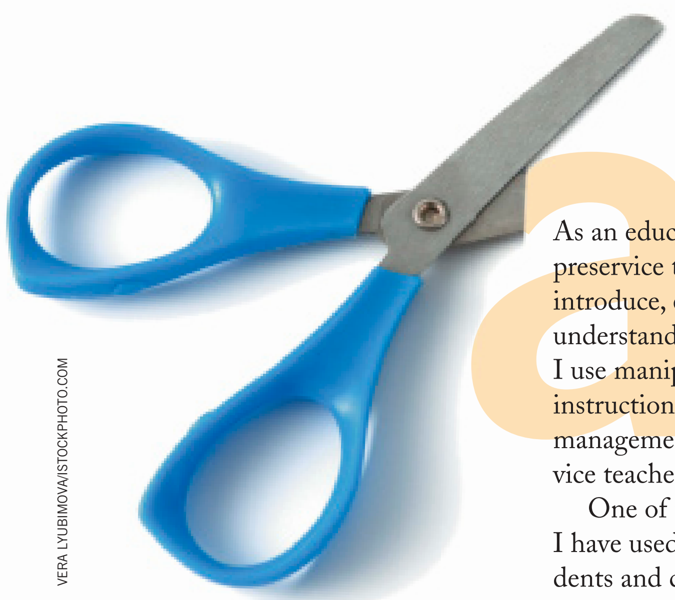
A Different Perspective on the Multiplication Chart



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As an educator of middle-level preservice teachers, I look for ways to introduce, explore, and assess students' understandings of number concepts. I use manipulatives to model both instructional strategies and classroom management issues for these preservice teachers.

One of the simplest manipulatives I have used with middle school students and currently use with preservice teachers is what I call a *multiples mat*, which is a 12 × 12 times table that has been laminated and cut into horizontal rows. We use it in math class several times during the year.

My multiples-mat work was inspired by a letter to the editor in this journal titled "Fraction Strips" (Horne 1994). The author of the letter had used fraction strips to show the addition and subtraction of fractions. However, the concepts that can be taught with this manipulative are not limited to fractions. While using both paper and craft-stick versions of the manipulative, I found that several other numerical concepts could be explored by my sixth-grade students. The "aha!" moments that preservice teachers experience as we have reviewed and explored numerous common number concepts are similar to those I have experienced with my sixth-grade students.

PATTERNS ON MULTIPLES MATS

After handing out individual envelopes containing the multiples mats, I lead students through a short series of lessons to review mathematical vocabulary (see **activity sheet 1**). We initially explore patterns within specific rows: Numbers in row 2 end in an even number, row 5 alternates with a 5 or 0 in the last-digit place, and numbers in row 10 end in zero. I also ask questions that make students think about the digits located in the units place, the tens place, and so on. In **figure 1**, for example, middle school students easily recognize that the units digit in row 5 alternates between 5 and 0. Some students will also comment about the tens digit starting with nothing (or 0), then 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, and so on. Verbalizing understanding of simple patterns is an essential step that will eventually lead to creating rules using symbolic notation.

Asking students to look at other rows allows them to review their multiplication tables while investigating patterns. This is beneficial for students who cannot automatically recall their multiplication facts. As students share their discoveries, I informally assess their fact fluency and mental flexibility.

Eventually, students construct a complete times table. I encourage them to look for diagonal and vertical patterns. Some students seem to see patterns easily, whereas others need guidance. Using a ruler or long strip of paper helps students block off part of the mat. I ask them to write their observations using as much mathematical vocabulary as possible.

It will become obvious if some students have limited knowledge generalizing about numbers. For example, some students have not experienced what happens when multiplying any number by an even number (the result is always even). Some may also not know that multiplying any number by an odd number results in an alternating pattern of odd and even numbers (see **fig. 2**).

When studying this number topic, an excellent resource to read to students is *Among the Odds and Evens: A Tale of Adventure* (Turner 2001). This story addresses simple concepts about numbers that should have been explored in the lower elementary grades. Furthermore, the patterns found in the multiples of 2, 5, and 10 lead to discussions about divisibility patterns. I encourage students to make generalizations and conjectures as they arise from discussions about

Fig. 1 The strip containing the multiples of 5 has patterns that students can see quickly.

5	10	15	20	25	30	35	40	45	50	55	60

Fig. 2 By examining an odd row (or column), students can see that multiplying any number by an odd number results in an alternating pattern of odd and even numbers.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	50	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

the patterns they see in the multiples mats. Some of their ideas can become very sophisticated.

Students are often intrigued by the diagonal patterns they discover. Their explanations vary, such as describing the pattern that appears when following the numbers from the top-left square (1) diagonally down to the bottom-right square (144). The number pattern is 1, 4, 9, 16, and so on (see **fig. 3**). Some students, depending on the grade level and prior explorations, may recognize this pattern as 1^2, 2^2, 3^2, 4^2, and so on. If not, this perfect visual allows them to see square numbers of the form n^2. I have always had students see the pattern of 1, 4, 9, 16 as +3, +5, +7. As one transitions to functions, the preference is for a rule that is applicable in all situations. With sixth graders, however, I initially just want them to gain flexibility in considering how to describe the number patterns.

Fig. 3 The numbers that appear diagonally from left to right are square numbers.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	50	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Fig. 4 Numbers in diagonals from right to left produce a palindrome.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	50	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The diagonal patterns moving in the opposite direction from top right (12) to bottom left (12) differ slightly, depending on whether the diagonal has an odd or even set of numbers (see **fig. 4**). The numbers appear to be a palindrome, such as 5, 8, 9, 8, 5. Students may describe the differences in the numbers as being odd; as having a pattern of +3, +1, –1, –3; or by relating that a number goes up until it

Fig. 5 By lining up rows, students can see that the sums and differences produce a sequence of multiples.

4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
9	18	27	36	45	54	63	72	81	90	99	108

(a) Addition

9	18	27	36	45	54	63	72	81	90	99	108
5	10	15	20	25	30	35	40	45	50	55	60
4	8	12	16	20	24	28	32	36	40	44	48

(b) Subtraction

Fig. 6 A multiples mat can be used to create other multiplication strips. In this case, the multiples of 19 are generated from the 9 strip and the 10 strip.

9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	50	70	80	90	100	110	120
19	38	57	76	95	114	133	152	171	190	209	228

Fig. 7 Fractions, ratios, or proportional relationships can be seen in both vertical and horizontal formats by pairing two rows or columns.

5	6
10	12
15	18
20	24
25	30
30	36
35	42
40	48
45	54
50	50
55	66
60	72

5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72

hits a middle number, then it reverses or goes back down. In an even set of numbers, the middle two numbers are the same, such as with 8, 14, 18, 20, 20, 18, 14, 8. Again, students may describe some sort of growth pattern that is related to odd numbers.

ADDING, SUBTRACTING, AND EXTENDING TABLES

One surprising aspect for my students has been the demonstration of other number operations using the multiples mats. I ask students to pull out the 4, 5, and 9 strips and describe the patterns in these strips (see **fig. 5a**). Some students will find 4 + 5 = 9, 8 + 10 = 18, 12 + 15 = 27, and so on. I feign surprise and tell them to continue looking for patterns, since the manipulative is *not* an addition table. They then tell me that this manipulative could help them with subtraction facts.

Sure enough, several students move the strips (see **fig. 5b**) and discover that 9 – 5 = 4 or 9 – 4 = 5. Middle-level preservice and in-service teachers have also been surprised by this discovery. The ensuing discussion is very interesting. Students start validating which three strips can be added or subtracted and discover that multiple pairs of strips make up the same third strip, such as the 1 and 6, 2 and 5, and 3 and 4—all equaling the 7 strip.

This discovery provides an opportunity to discuss multiples, the distributive property, and order of operations. Using the students' prior discoveries of 4 + 5 = 9, 8 + 10 = 18, and 12 + 15 = 27, one can rewrite the problems to demonstrate that 3(4) + 3(5) = 3(9) or 3(4 + 5) = 27. I emphasize that the number outside the parentheses, in this example the 3, represents a specific location on the strip. The notation of 3(4) is the third number from the left on the 4 strip, or 12, which is a multiple of 4.

Fig. 8 Shading common multiples simplifies adding and subtracting fractions. In this example, the fractions 5/6 and 3/8 are being added.

5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
3	6	9	12	15	18	21	24	27	30	33	36
8	16	24	32	40	48	56	64	72	80	88	96

If one uses the same location on each strip, the same multiple, then you can add or subtract with the strips. I also show students subtraction using the distributive property, such as

$$3(9) - 3(5) = 3(4) \text{ or } 3(9 - 5) = 3(4).$$

Some students will create models, such as that in **figure 6**. This leads to further conjectures as to whether any two strips can be added to produce a third and possibly unknown strip. The times table for 19 is a favorite of mine to discuss with students. After examining the pattern, many students memorize at least 19 multiples of 10. Most students quickly see that the units place value is decreasing by one and that the tens, or the number formed by the hundreds and tens, is increasing by two.

FRACTIONS, RATIOS, AND PROPORTIONAL REASONING

Other number concepts can be explored using a multiples mat. For example, two strips can demonstrate multiple sets of equivalent fractions, ratios, or proportions (see **fig. 7**).

Students can also use the strips to add and subtract fractions. For example, when adding 5/6 and 3/8, students would use the 5 and 6 strips and the 3 and 8 strips (see **fig. 8**). They then look for the denominators that are similar, such as

$$\frac{20}{24}+\frac{9}{24}=\frac{29}{24}=1\frac{5}{24},$$
$$\frac{40}{48}+\frac{18}{48}=\frac{58}{48}=1\frac{10}{48}=1\frac{5}{24}, \text{ or}$$
$$\frac{60}{72}+\frac{27}{72}=\frac{87}{72}=1\frac{15}{72}=1\frac{5}{24}.$$

Fig. 9 A multiples mat can be used to solve ratios in the Roses problem, which involves multiples of 4, 5 and 6.

Red	5	10	15	20	25	30	35	40	45	50	55	60
Pink	4	8	12	16	20	24	28	32	36	40	44	48
Yellow	6	12	18	24	30	36	42	48	54	60	66	72
Total	15	30	45	60	75	90	105	120	135	150	165	180

If subtracting the two fractions, the problem solutions would become

$$\frac{20}{24}-\frac{9}{24}=\frac{11}{24},$$
$$\frac{40}{48}-\frac{18}{48}=\frac{22}{48}=\frac{11}{24}, \text{ or}$$
$$\frac{60}{72}-\frac{27}{72}=\frac{33}{72}=\frac{11}{24}.$$

I have found that when students write out the solutions for *all* the denominators that the two fractions have in common, they better understand the importance of the lowest common denominator. They also get additional practice in simplifying fractions. Since a student can use a multiples mat to find common denominators, the manipulative can also be used to compare and order fractions.

Students can also use this manipulative to work out solutions to ratio or proportion problems. See the Roses problem, for example:

> A florist sells roses in only three colors: red, pink, and yellow. The roses are bought in the ratio of 5 to 4 to 6. If the store has received a shipment of 360 roses, how many are pink?

Students can pull the 5, 4, and 6 strip and add the total number of roses until 360 is found (see **fig. 9**). In this instance, the student could use the column that totals to 180, then double the number or roses. This would mean the florist received 120 red, 96 pink, and 144 yellow roses, resulting in a final answer of 96 pink roses.

FUNCTIONS

A multiples mat can also be used to generate lists of numbers from which students create simple rules. However, there are some limitations as to the functions that can be created. The vertical orientation becomes preferable for such problems. Most students will not have difficulty with simple relationships, such as $f(x) = 3x$, which can be demonstrated with the 1 and 3 strips. However, some relationships can be demonstrated that help students gain an understanding of the concept of a constant (see **fig. 10**). Slightly offset the strips that model $f(x) = 3x + 3$ and $f(x) = 3x - 3$. Although the strips demonstrate only simple functional relationships, such rudimentary explorations are important for middle-level students.

CONCLUDING THOUGHTS

Incorporating multiples mats during the teaching or review of number concepts is relatively simple, because middle-level students are familiar with multiplication charts and often use them regularly when calculating. I have found this manipulative to be especially helpful when working with special needs students and those who struggle to memorize math facts. Feel free to share this manipulative with your special education teachers, instructional aides, parents, or colleagues.

Fig. 10 The functions $f(x) = 3x + 3$ and $f(x) = 3x - 3$ are verified by shifting the 3 strip up or down alongside the 1 strip.

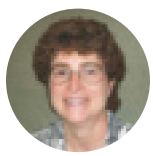
	3		1	0
1	6		2	3
2	9		3	6
3	12		4	9
4	15		5	12
5	18		6	15
6	21		7	18
7	24		8	21
8	27		9	24
9	30		10	27
10	33		11	30
11	36		12	33

REFERENCES

Horne, Loretta A. "Fraction Strips." *Mathematics Teaching in the Middle School* 1 (November–December 1994): 244.

Turner, Priscilla. *Among the Odds and Evens: A Tale of Adventure.* Illus. by Whitney Turner. New York: Scholastic Books, 2001.

Ed. note: Since the solutions to the activity sheets are open-ended and based on student choices, no separate solutions section is provided online.



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activity sheet 1

Name ______________________________

PATTERNS

Construct a multiples mat using your entire set of number strips. Then use the strips to help you explore number concepts. Shade the grids below. Record your observations.

1. Put the entire multiples mat together. How would you explain what you see to a classmate who had been absent for today's activity?

2. Look for patterns that appear horizontally and vertically. Color at least one horizontal pattern and one vertical pattern on the grids below.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	50	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	50	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

3. Describe your horizontal pattern.

4. Describe your vertical pattern.

activity sheet 1 (continued)

Name ______________________________

5. Look for patterns that appear on the multiples mat diagonally from top left to bottom right and from top right to bottom left. Color at least one pattern from each direction on the grids below.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	50	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	50	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

6. Describe the top-left to bottom-right pattern.

7. Describe the top-right to bottom-left pattern.

8. Align the 4, 5, and 9 strips. What relationships do you see?

9. How could you use the multiples mat to help you write the times table for 19?