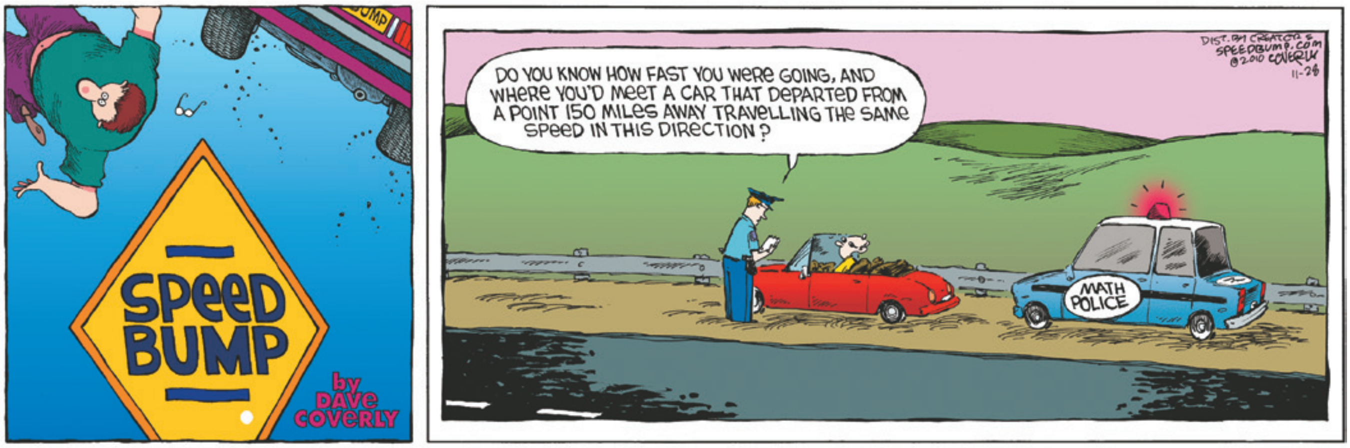
cartoon corner

Name ______________________________

SPEED BUMP by Dave Coverly



THE MATH POLICE

For the questions below, assume that the driver stopped by the police is Car 1 and is traveling from Algebraville. Assume that the other car is Car 2, which leaves from Calculicity and is driving toward Car 1. Both drivers leave at the same time and travel at steady speeds.

1. Complete the first three rows of the table at the bottom of the page. Assume that the cars are traveling at 25 miles per hour (mph).

2. Use the table to determine formulas for "distance cars are from starting city" and "time until cars meet" for two cities that are multiples of 150 miles apart (that is, 150*n* miles apart). It may be helpful to fill in rows 4 and 5, as well.

3. Suppose that the two cities are 300 miles apart and that Car 1 is traveling at 50 mph and that Car 2 is traveling at 25 mph.

a. How far will each car be from the starting city when they meet?

b. When will they meet?

4. Suppose that the two cities are 300 miles apart and that Car 1 is traveling at 60 mph and that Car 2 is traveling at 30 mph.

a. How far will each car be from the starting city when they meet?

b. When will they meet?

CHALLENGE

5. Suppose that the two cities are 300 miles apart and that both cars are traveling at 30 mph but Car 1 leaves Algebraville at noon and Car 2 leaves Calculicity at 1:00 p.m.

a. How far will each car be from the starting city when they meet?

b. When will they meet?

	Distance between Cities (in Miles)	Distance Cars Are from Starting City When They Meet (in Miles)	Time until Cars Meet (in Hours)
1	150		
2	300		
3	450		
4			
5			
n	150*n*		

from the March 2012 issue of mathematics teaching in the MIDDLE SCHOOL

Edited by **Stephen P. Smith**, Northern Michigan University, Marquette, and **Peggy House**, Northern Michigan University, Marquette. Classroom teachers interested in field-testing or submitting a cartoon should contact **Stephen P. Smith**, stepsmit@nmu.edu. The cartoons must include the date and the newspaper syndicate that holds the copyright.

SOLUTIONS

For all problems, but especially 3, 4, and 5, students may find it helpful to sketch the situation.

1. Row 1: Since Car 1 and Car 2 are traveling at the same rate, they will meet exactly halfway between the two cities, or 75 miles from each starting city. Since they are traveling 75 miles at 25 mph, they will meet in

$$\frac{75 \text{ mi.}}{25 \frac{\text{mi.}}{\text{hr.}}} = 3 \text{ hr.}$$

Students might also calculate "how many hours" × 25 mph = 75 miles.

Row 2: As in row 1, the cars will meet halfway, or 150 miles from each starting city. Using the same approach as for row 1, the two cars will meet in 6 hours.
Row 3: The cars will meet halfway, or 225 miles from the start. Using the same approach as row 1, the cars will meet in 9 hours.

2. See the table below.

3. a. In 1 hour, the combined distance will be 50 mi. + 25 mi. = 75 mi. In 2 hours, 150 miles; in 3 hours, 225 miles; and in 4 hours, 300 miles. Thus, they will meet in 4 hours.

b. Car 1 will travel 4 hr. × 50 mi./hr. = 200 mi. Car 2 will travel 4 hr. × 25 mi./hr. = 100 mi.

4. a. This problem does not result in a whole number of hours traveled. In 1 hour, the combined distance will be 60 mi. + 30 mi. = 90 mi. In 2 hours, the two cars will travel 180 miles; in 3 hours, 270 miles. So far, Car 1 has traveled 180 miles and Car 2, 90 miles. They have 30 miles yet to cover. Students may use several different approaches (some incorrect) to solve this problem. One way to think about it is that they have 1/3 of 90 miles left, so it will take them 1/3 hour, or 20 minutes, to cover the last 30 miles. Therefore, it will take 3 hours and 20 minutes for the cars to meet.

b. In 3 hours and 20 minutes, or 3 1/3 hours, Car 1 will travel 3 1/3 × 60 mi./hr. = 200 mi. Car 2 will travel 3 1/3 × 30 mi./hr. = 100 mi. Some students may quickly solve this problem if they realize that Car 1's rate is twice Car 2's rate and that Car 1 is traveling twice as far.

CHALLENGE

5. After 5 hours, Car 1 will travel 5 hr. × 30 mph = 150 mi. Car 2 will travel 4 hr. × 30 mph = 120 mi. Combined, they will travel 150 + 120 = 270 mi. Thus, they will still be 30 miles apart. When each travels another 15 miles, or 1/2 hour, they will meet. Car 1 will travel 165 miles over 5 1/2 hours. Car 2 will travel 135 miles over 4 1/2 hours, so they will meet at 5:30 p.m.

Distance between Cities (in Miles)	Distance Cars Are from Starting City When They Meet (in Miles)	Time until Cars Meet (in Hours)
150	75	3
300	150	6
450	225	9
600	300	12
750	375	15
$150n$	$\frac{150n}{2}$ (i.e., half the distance between the cities)	$\frac{\frac{150n \text{ mi.}}{2}}{25 \frac{\text{mi.}}{\text{hr.}}} = 3n \text{ hr.}$ (i.e., the distance each car travels divided by the rate at which they travel)

FIELD-TEST COMMENTS

If you ask adults what they remember from algebra, they will groan and talk about those dreaded "Two trains leave the station at the same time" word problems. They most likely jumped into the problem by writing an equation to model the situation and then solved it without understanding the underlying concept. This cartoon lesson provided a great way to build that concept without having to write the equation.

I worked through this activity with seventh-grade and eighth-grade algebra classes. The eighth-grade students had already completed some similar word problems, so they were able to work individually and then share their methods with the class. Initially, a few did not understand what the "150*n*" represented in the chart. After looking at the relationship between the numbers in the first two columns of the table, they were able to continue.

As the students discussed their answers, most of them realized that they could set up basic equations to model the scenarios. For example, given the relationship of the speeds in question 3, if x is the number of miles traveled by Car 2, then Car 1 is $2x$, because it traveled twice as far. If $2x + x = 300$, then $x = 100$, and $2x = 200$ for a total of 300 miles.

I worked through each question with the seventh-grade class to develop the concept. We drew a color-coded diagram that remained on the board throughout the discussion. Being able to see the relationships among the rate, the time, and the distance traveled was helpful. Students quickly caught on to the concept and were able to articulate their answers to the questions.

Being able to reference this activity when distance problems occur in the textbook should help them make sense of algebraic representations.

Pamela Haner,
St. Catherine's School,
Richmond, Virginia

Wow! This lesson naturally stimulated my Math 8 and Algebra/Geometry 1 students to incorporate multiple representations in the process of solving the problems presented. It also gave them a context so that they could communicate through different entry points.

After I introduced the lesson, students were encouraged to work in pairs to solve the problem and then share their results in groups of four. At the conclusion, they reflected in writing about interacting with the problem; analyzing task components; attempting various strategies and results, whether correct or incorrect; and comparing their proposed solutions with the correct solutions.

The table discussed in question 1 encouraged students to complete missing entries and explore the patterns that emerged. Given the tabular format, several of my students chose to extend the table for question 2 and create tabular representations of the scenarios presented in the other items. Others relied on various graphic or pictorial models. Yet another subset of students generated equations to represent the various scenarios described by each item.

Many middle school science curricula include work with formulas relating distance, rate, and time. This lesson offered opportunities for authentic, deep, and cross-curricular connections between math and science for middle school students.

A few of my students pondered questions about how nonconstant rates of speed would impact the calculations.

Terrific extension possibilities to basic calculus concepts for middle school are embedded within this one lesson!

Deborah Regal Coller,
Pathfinder (Middle) School,
Pinckney, Michigan

I paired my fifth-grade students to tackle this activity, which we completed in mid-September. They had not worked with distance formulas at that point in the school year, so working in pairs proved to be a valuable resource in that they learned that two heads are better than one.

Since we had not worked in pairs often, it was relatively new for the students. They held quiet math discussions with their partners to determine the best way to attempt to solve each problem. Some pairs found two distinctly different ways to solve the problem and learned the art of compromise; others bounced ideas off each other until something struck a chord and they decided to proceed accordingly.

Students found it helpful to draw representations as a problem-solving strategy. I did not suggest it for this activity—the students made that decision. They were also free to select and try any of the problem-solving strategies they were familiar with to complete this project.

When the assignments were turned in, several groups mentioned how fun "problem solving in pairs" (as I had called it) was and asked when they could do another assignment like that. I asked them if they were referring to "problem solving in pairs" or "cartoon math"; they promptly said, "Both!"

For an extension activity, I want to ask students to create their own distance word problems and write them down on individual index cards (with their calculated answers on the back). We can then use the index cards throughout the remainder of the school year, especially when reviewing for statewide testing in the spring and benchmark testing at the end of each quarter.

Using their own problems should provide for some interesting class experiences for each student. Seeing other students solve their self-created math problems should pique students' interest in math.

Tina Gay,
K. E. Taylor Elementary School,
Lawrenceville, Georgia

OTHER IDEAS

The difficulty of distance = rate × time problems may be increased by creating distance and rate combinations resulting in fractions of hours (e.g., the cars may be 60 miles apart and one car may be traveling 40 mph and the other car may be traveling 25 mph). Likewise, the problem could include lunch stops of unequal duration or other complicating factors. Changing known data is another possibility (e.g., if one knows the rates the cars are traveling and how long they are traveling, students could be asked to find the distance traveled).