Proportional and the Visually

MYBAITSHOP/VEER

Reasoning Impaired

A hands-on activity helps promote sight-impaired students' understanding of relative and comparative sizes.

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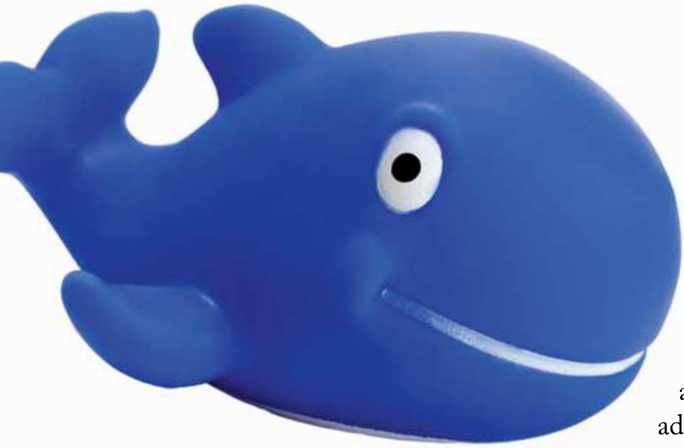
Proportional reasoning is an important aspect of formal thinking that is acquired during the developmental years that approximate the middle years of schooling. Students who fail to acquire sound proportional reasoning often experience difficulties in subjects that require quantitative thinking, such as science, technology, engineering, and mathematics. These students may also have difficulty with many real-life skills, such as cooking, reading a map, and scaling an object. As a result, teachers need to deliberately target proportional reasoning concepts regularly and over time to ensure that maximum opportunities for study are offered for their students. Lamon (1993) suggested that proportional reasoning might be the most commonly applied mathematics in the real world and thus extremely important in our daily lives.

As part of a research project investigating the enhancement of numeracy (mathematical literacy) through developing middle-grades students' proportional reasoning, the authors were invited to work with a class of twelve visually impaired students. The impairment of these ten-year-old to thirteen-year-old students ranged from limited vision to no sight. Visual impairment can result from damage to the eye, the vision pathway to the brain, or the visual center of the brain. The impairment can be present at birth or occur later in life from disease or injury. It may be constant or may change over time. These factors influence the way that visually impaired students learn and the ways in which they are taught. These factors must be considered when developing proportional reasoning experiences for this class of students. A person can think proportionally when he or she understands the covariation that is inherent in the multiplicative relationship between two quantities (Boyer, Levine, and Huttenlocher 2008). This concept is targeted with this visually impaired population through the use of hands-on modeling-clay activities.



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VISUALLY IMPAIRED MATH LEARNERS

Many factors determine the ability of visually impaired students to learn mathematics. The cause of the impairment, the age of its onset, the skill of teachers, and even the students' geographic location in the world and subsequent access to assisted learning technologies combine to determine the level of students' mathematics learning success. For these reasons, no one-size-fits-all formula is possible for both teaching visually impaired students and their learning of mathematics (Klingenberg, Fosse, and Augestad 2012). Klingenberg and his colleagues emphasized the importance of a teacher knowing the cause of a student's visual impairment.

Since many aspects of mathematics require visualizing concepts, visually impaired students can be at a disadvantage (Klingenberg, Fosse, and Augestad 2012). However, the consensus is that visually impaired students with no additional learning difficulties, given opportunities of direct instruction and practice with quality tactile materials, should be able to achieve at grade level in mathematics (Rosenblum and Herzberg 2011).

Currently, many visually impaired students have access to learning through the use of Braille (Nemeth Code for Braille Mathematics) and assisted learning technologies, although Osterhaus (2006) also emphasized additional creative techniques and strategies. Visually impaired students can access their learning through alternatives, such as large print, audio, and tactile formats. They may also use adaptive technologies, such as voice recognition, magnification, and low vision devices. In addition, it is very important for this group of students to be physically active, as this assists the development of spatial understanding and mathematical thinking (Klingenberg, Fosse, and Augestad 2012). The notion of using movement and hands-on activities was the cornerstone of our thinking as we prepared our lesson for this unique class.

FACTORS CONSIDERED IN DIFFERENTIATING INSTRUCTION

Although we had no specific expertise in working with visually impaired students, we were supported and guided by their teacher who was highly trained in the field. She informed us that the students had strong speaking and listening skills and enjoyed hands-on and physical activities. Some students were able to work independently, whereas others required high levels of support from peers, the teacher, or the teacher assistant.

Lesson planning was mutually cooperative: We supplied the theoretical background of proportional reasoning, and the classroom teacher provided input into possibilities for instruction. We had to consider the learning needs and strengths of the students to devise an effective way of engaging them in activities to develop aspects of proportional reasoning.

Our model lesson incorporated a mixture of teaching strategies, including discussion, outdoor activity, hands-on materials, and questioning, to emphasize some important aspects of proportional reasoning, such as fractional, multiplicative, and relative thinking. These concepts were then targeted through an activity involving a linear scale. When describing the lessons below, we refer to ourselves as *instructors*.

AN ACTIVITY OF SCALE

Earlier findings of the research project suggested that middle-years students have considerable difficulties with aspects of proportional reasoning involving scale. Therefore, the practical manipulation of a one-dimensional scale of length was designed for the lesson.

As visitors to the class, we had to devise a context that was suitable for use with visually impaired students, appropriate to the lesson goals, and inherently engaging. The students live in a part of Australia that is close to the water; they are familiar with many large sea creatures, such as whales, sharks, and seals. The coastline in the vicinity is renowned for shark attacks. Therefore, activities were chosen to investigate and compare the relative lengths of large sea creatures. The four phases of the lesson are described below.

Phase 1

(5 min.) Students were engaged in a discussion about the height of one of the instructors who is 2 m (approximately 6 feet, 8 inches) tall. Some of the questions asked were these:

1. Is the instructor taller than you?
2. Is there something in the room about the same height as the instructor?

Students established, either through limited vision or from voice direction, that the instructor was much taller than they were and was about the same height as the doorway. The teacher informed us that the students had previously measured the length and height of classroom objects using benchmarking (e.g., body lengths, hand widths) as well as measuring tools (e.g., metersticks). Because students determined that the instructor was approximately 2 m tall and was about the same height as the doorway, this 2 m measurement was used as a benchmark throughout the lesson.

Phase 2

(5 min.) Questions focused on student knowledge of animals that were taller or longer than the instructor. Responses included a giraffe, an elephant, and a whale. The discussion was then guided to the length of sea creatures. For example, students were asked how long were the largest species of whale, shark, or dolphin. Students knew some of the answers but needed guidance for others. They were then asked to remember the creatures' lengths and to sequence them in order from shortest to longest. Next, the students were led to think multiplicatively and compare the creatures' lengths. For example, how many times longer is the humpback whale than the dolphin, or how many times longer is the great white shark than the instructor? **Table 1** summarizes the data from the discussion.

Table 1 Data on the lengths of various sea creatures were discussed and ordered, from longest to shortest, to complete comparisons.

Sea Creature	Approximate Maximum Length
Humpback whale	16 m
Giant squid	10 m
Great white shark	6 m
Dolphin	4 m
Walrus	3 m
Juvenile leopard seal	1 m

Fig. 1 A plastic model of a walrus contained tactile elements that assisted the visually impaired students.

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Phase 3

(15 min.) The class moved outside into a covered area where students could safely perform some length estimates of the creatures we had discussed. Students estimated the length of the humpback whale by stepping out a measurement of approximately 16 m. Some students had enough vision to do this safely by themselves; others either worked together or were assisted. Students' estimates of distance were generally quite reasonable. Using a meterstick, two students counted and measured the exact distance, which allowed the class to compare their estimates with the correct 16 m length. Using the meterstick instead of a trundle wheel initially seemed awkward, but the students handled it well and were able to count the meters together as they measured.

The activity was repeated to represent creatures of different lengths. We regularly asked such questions about the relative length of the creatures:

- Which creature is twice as long as the walrus?
- Which creature is half the length of the humpback whale?

The students generally showed considerable memory prowess in recalling the lengths of the creatures we had discussed in class. They were also confident about their responses to our questions, requiring multiplicative and fractional thinking.

Phase 4

(45 min.) Students returned to tables placed in a shared indoor working space. They worked in pairs with modeling clay and a plastic model of a sea creature discussed earlier (see **fig. 1**).

Students were then asked to make a model of the instructor who was 2 m tall, so that it was in correct proportional length to the plastic sea creature model they had been given. If a student had a model dolphin (4 m long in real life), then the model of the instructor had to be half the length of the model dolphin. If the student had a juvenile leopard seal (1 m long in real life), then the model of the instructor would have to be twice as long as the plastic sea creature

Fig. 2 After tactile measurement, students often had to vary the length of the human model in proportion to the plastic sea creature.

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Fig. 3 Scaling up or down was an example of a *growth problem* (Lamon 1993).

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Fig. 4 The model instructor (2 m) was scaled up or down in linear proportion to the length of the giant squid model (10 m).

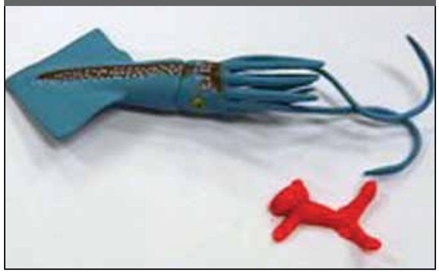

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Fig. 5 More scaling was necessary when analyzing a juvenile leopard seal (1 m).

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(see **figs. 2** and **3**). These examples are what Lamon (1993) termed *growth problems*. These problems express a relationship between two continuous quantities, such as height or length, and involve scaling up or down.

While the student pairs shared the plastic model, they each had to create their own clay scale model of the instructor. During the activity, most students were able to complete three scale-model lengths of the instructor. During this phase, we, along with the teacher, consistently asked students to verbalize information about the scales of the models that they were making. For example, they were asked how many times longer is the whale than the instructor, and what fraction of the whale's length is the instructor? Moving from solving proportional problems through using manipulatives, such as models, to discussing numbers and calculations is an important trajectory in developing students' proportional reasoning.

STUDENT RESPONSES

Phases 1 and 2

Students responded enthusiastically to this lesson sequence. They enjoyed the topic and the discussions and were challenged by estimating the lengths of the different creatures and comparing them with the height of the instructor. This activity engaged strong fractional, multiplicative, and relative thinking, which are important aspects of proportional reasoning. The questions and responses began by a simple absolute comparison of which creatures mentioned, including the instructor, were longer or taller or shorter. One student mentioned that although the instructor was tall, when compared with a giraffe, he was not tall. This student was engaging in relative thinking.

Multiplicative thinking began to emerge as the students were asked to say how many times longer or taller one creature was when compared with another or what fraction of the length of one creature was another creature. This was also challenging for the students, as they had to listen to and remember the measurements of the various creatures discussed.

Phase 3

The estimating, pacing, and measuring activity in the covered area outside the classroom reinforced these same concepts. The students thoroughly enjoyed estimating and checking the actual lengths of the sea creatures. They found some of the lengths quite astounding, for example, the 10 m giant squid. Students had varying abilities to visualize these lengths, but the physicality of stepping out the lengths assisted all the students.

Phase 4

The final scale-modeling phase was quite challenging although generally very well done. With limited vision, the students were able to feel the varying lengths of the creatures and their scale-model instructor. Often students would have to resize their model instructor because they would count and check how many times their model would lie alongside the plastic sea creature. If the model was not constructed accurately to linear scale, they would sometimes start again or perhaps simply lengthen or shorten the legs of their model and then retest it for accuracy (see **figs. 4** and **5**).

The students' determination to accurately make their models and the discussions surrounding the manipulation of linear scale were very rich and reflected sound understanding of the underlying aspects of proportional reasoning. One student commented that it was much harder to get the scaling correct when working with the humpback whale because it was difficult to estimate the length of the instructor model, being one-eighth the length of the humpback. He found that it was easier to work with the relatively smaller-length sea creatures to initially estimate and then manipulate into the correct linear scale. As the students worked in pairs but made their own models, they helped each other and compared their models. This collaborative approach elicited further discussion about the scaling and proportions of their respective models.

FURTHER CONSIDERATIONS

In the absence of a 2 m tall benchmark, alternate benchmarks could easily be used for this activity. The benchmark length and the relative size of the creatures and objects to be compared can be manipulated to make the task easier or harder, thus differentiating the instruction to suit the students.

Language and mathematical expression are important. The focus in this lesson sequence was on one-dimensional scale, so constant reference was made to length or height (one dimensional). A word such as *bigger* should be used with caution in this context because it implies consideration of other dimensions. For example, the humpback whale may grow to be eight times the length of a 2 m instructor, but it would be hundreds of times larger in terms of volume.

BENEFITS FOR THE BROADER COMMUNITY OF LEARNERS

Although this lesson sequence was specifically designed for students with visual impairment, our consensus, and that of the teacher, was that it could be replicated for many other diverse learners. This activity's use of multiple senses and the opportunities it created for intense discussion about proportional elements such as linear scale and fractional, multiplicative, and relative thinking would make it an appealing and rewarding activity for most learners. This emphasis on hands-on experiences engaging component aspects of proportional reasoning is an important step in the conceptual development of this type of thinking.

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