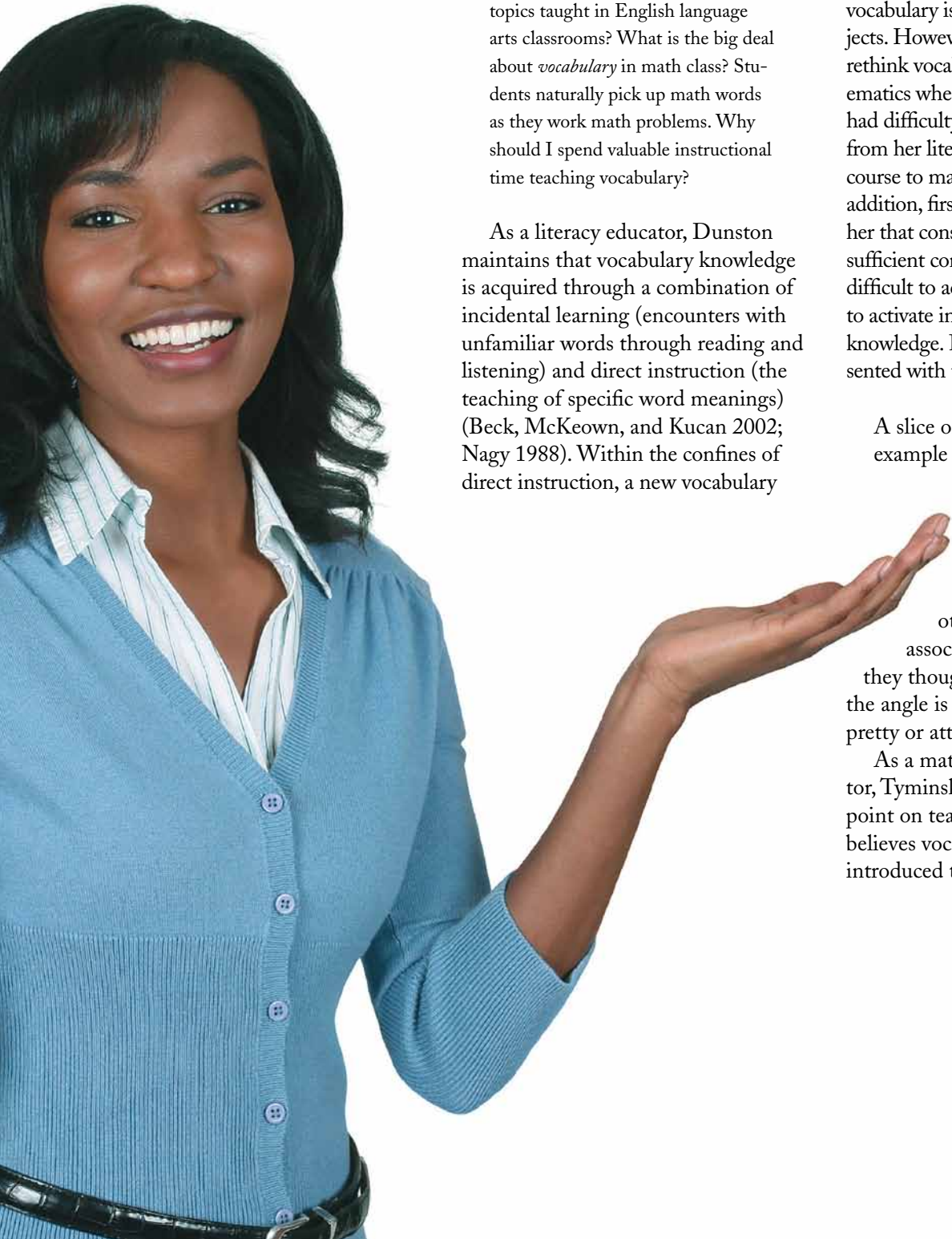


# What's the Big Deal about Vocabulary?

Pamela J. Dunston and Andrew M. Tyminski

*Techniques for teaching mathematics terminology allow adolescents to expand their abstract reasoning ability and move beyond operations into problem solving.*

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Patrick was an in-service, middle school mathematics teacher enrolled in a literacy education graduate course. When the professor required that he teach vocabulary within a mathematics lesson, he expressed his frustration, as follows:

I'm a math teacher! I don't do vocabulary. Isn't that in the realm of topics taught in English language arts classrooms? What is the big deal about *vocabulary* in math class? Students naturally pick up math words as they work math problems. Why should I spend valuable instructional time teaching vocabulary?

As a literacy educator, Dunston maintains that vocabulary knowledge is acquired through a combination of incidental learning (encounters with unfamiliar words through reading and listening) and direct instruction (the teaching of specific word meanings) (Beck, McKeown, and Kucan 2002; Nagy 1988). Within the confines of direct instruction, a new vocabulary

term is presented in the context of a sentence (contextual information), and the word's meaning is discussed with students (definitional information). Direct instruction incorporates research-based vocabulary-teaching strategies to activate students' prior knowledge and relate new word meanings to known words and concepts.

This instructional approach to vocabulary is appropriate for most subjects. However, Dunston was forced to rethink vocabulary instruction for mathematics when her preservice teachers had difficulty connecting information from her literacy education methods course to mathematics classrooms. In addition, first-hand experience taught her that constructing a sentence with sufficient contextual information was difficult to accomplish and led students to activate incorrect or unrelated prior knowledge. For example, when presented with the sentence,

A slice of pizza or apple pie is an example of an *acute* angle,

several students concluded that acute angles have at least one rounded or curved edge. A few others believed that *acute* is associated with the word *cute*; they thought the term meant that the angle is found in or on something pretty or attractive.

As a mathematics teacher educator, Tyminski has a different viewpoint on teaching vocabulary. He believes vocabulary terms should be introduced to students through active

engagement in mathematics where possible and that students should be allowed and encouraged to find ways to describe the phenomenon they are interacting with in their own words. The teacher's role then is to help students connect the formal mathematical vocabulary term with their current understanding of the idea or concept. When discussing a rectangular prism's appearance, for example, students often use words such as *corner* and *sides* to describe what they see. Further, students often use the word *side* to refer to both a face of the object (the six rectangular figures that comprise the shape) as well as the edge of the figure (the line segment formed by the intersection of two faces). Teachers should not discourage the use of these terms, but rather—

1. point out the confusion in using the same term to mean two different things;
2. introduce the mathematical vocabulary terms of *faces*, *edges*, and *vertices* (where two or more edges meet); and
3. encourage students to “speak mathematically” to ensure clear communication of intent.

We believe that combining the conceptual approach to mathematics vocabulary with research-based literacy strategies is a worthwhile path to explore and one that can improve students' mathematics learning. What follows is an examination of strategies for teaching vocabulary and their potential for improving students' understanding of mathematical concepts. After a brief discussion of the importance of learning mathematics vocabulary and the unique aspects of teaching it, we offer three instructional strategies that were designed for the English language arts classroom yet are especially suitable for vocabulary instruction in a middle school math class.

## WHY VOCABULARY IS IMPORTANT

Vocabulary knowledge is essential to student achievement because—

- vocabulary is strongly correlated to reading comprehension (Davis 1968; Fitzgerald and Graves 2005);
- vocabulary is a predictor of students' comprehension (Anderson and Freebody 1981; Nagy 1988) and content area learning (Espin and Foegen 1996); and
- lack of vocabulary knowledge can negatively affect learning content (Fisher and Frey 2008).

When confined primarily to the classroom, math vocabulary remains highly formalized. Thompson and Rubenstein (2000) write that although math vocabulary shares word meanings with English, it is controlled by and uniquely embedded in the field of mathematics. They also suggest that knowledge of math vocabulary is necessary for mathematics achievement but insufficient for attaining computational competence (Thompson and Rubenstein 2000).

Mathematics vocabulary instruction is particularly important in the middle grades because this is when “the serious development of the language of mathematics begins” and when mathematical learning focuses on numbers' multiplicative structures and relationships (Lappan 2000, p. 24). Developing mathematics vocabulary knowledge allows adolescents to expand their abstract reasoning ability and move beyond operations to problem solving. Math vocabulary is also inextricably bound to students' conceptual understanding of mathematics (Capraro, Capraro, and Rupley 2010; Capraro and Joffrion 2006; Kotsopoulos 2007). Various words frequently represent discrete constructs that are not related to other words, ideas, thoughts, feelings, or concepts.

For example, *integer* is only found in mathematics; it has a very specific meaning that does not shed light on other concepts or constructs.

Math vocabulary is also highly decontextualized. That is, math terms are not situated in everyday conversations or discussions because these words are rarely included as dialogue in the latest Hollywood production or generally found in novels, newspapers, or social media. Although some math terms are shared with everyday language, they have different meanings in mathematics. In other words, there is little likelihood that Patrick's students will simply pick up precise word meanings for math vocabulary as they work through problems.

## APPROACHES TO TEACHING MATH VOCABULARY

The language of mathematics presents challenges for English-only speakers and English language learners alike because words used in math have unique and specific meanings. For example, *table*, *origin*, and *leg* may already be present in a student's vocabulary but may not encompass the math concepts associated with the words. Terms such as *average* and *reflection* have precise mathematical definitions. Some words are uniquely related to mathematics (e.g., *integer*, *outlier*, and *algorithm*).

Even word combinations take on specific meaning in mathematics. *Value* by itself has one meaning; *absolute value* has a far different meaning. The word *inequality* has a common meaning inside and outside math class, but the symbols used to represent less than and greater than in a math sentences must be taught. Many students view the language of mathematics as being a foreign language (Kotsopoulos 2007), and some educators view mathematics as a second language (Jones, Hopper, and Franz 2008).

Both English language learners and English-only speakers need teacher

input to master the extremely narrow definitions of math terms to become conversant in mathematics classrooms. It is important to address vocabulary deliberately in math class. If not taught with its particular requirements in mind, then computation will move to the forefront, and vocabulary will lose its emphasis (Orton 2004).

The vocabulary teaching strategies described below are particularly beneficial to English language learners because they require students to think deeply, determine relationships, and connect new concepts and words to what they already know. In addition, these strategies visually convey meaning without using complex language or complicated sentence structure.

### THE FRAYER MODEL

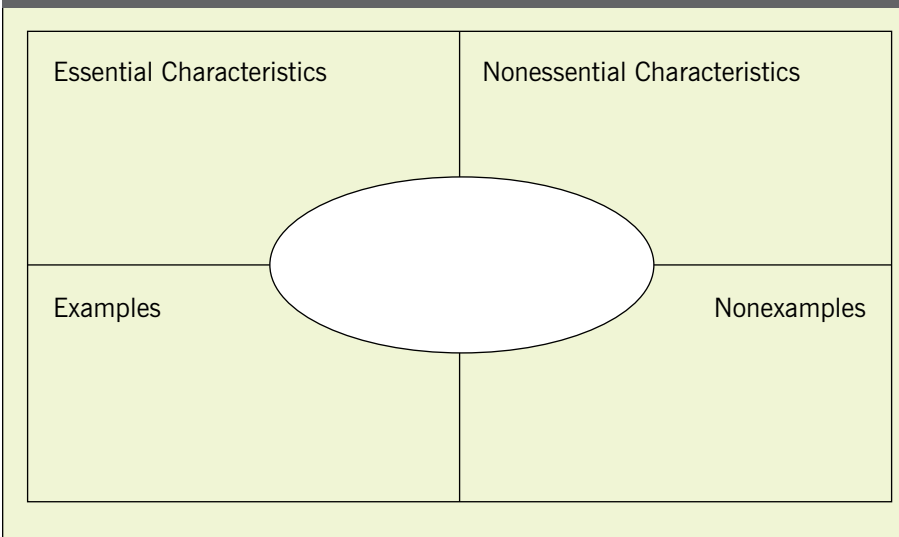
In developing a concept, knowing which properties that an object does *not* include is just as important as knowing which properties *are* included. For example, a guitar has strings, not keys. A guitar is strummed, not blown. The Frayer model (Frayer, Frederick, and Klausmeier 1969) is a graphic organizer that allows students to use inquiry to learn new concepts in mathematics and science. Students identify examples and nonexamples of a concept and differentiate between characteristics that define (or are associated with) the concept and characteristics that are interesting but not important. The following example will illustrate how the Frayer model can support the development of the terms *prime numbers* and *composite numbers*.

One approach to helping students conceptualize the idea of prime and composite numbers is through rectangular arrays. Students are asked to draw as many different rectangular arrays as they can whose area is equal to each set of numbers (e.g., the numbers 1–20). The numbers are then arranged according to the number of possible arrays (see **table 1**). The number of

**Table 1** The integers 1 through 20 are sorted by the number of possible rectangular arrays that can be formed.

Number of Possible Rectangular Arrays	1	2	3	4	5	6
1–20 number sorts	1	2 3 5 7 11 13 17 19	4 9 16	6 8 10 14 15	16	12 18 20

**Fig. 1** Using the Frayer model graphic organizer, students identify examples and nonexamples of a concept and differentiate between characteristics of a concept.



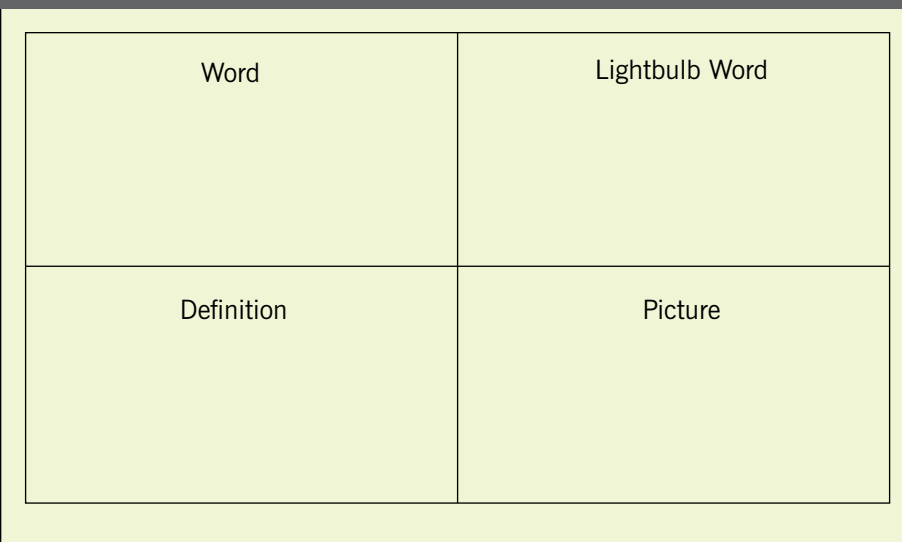
possible arrays, it turns out, is equal to the number of positive divisors for each number. We hope that students will notice after examining these data that some numbers (e.g., 2, 3, 11, and so on) have two distinct positive divisors and that other numbers (e.g., 4, 15, 20) have more than two positive divisors.

Using the Frayer model and the array table in **table 1**, we draw students' attention to numbers in the "2" column that have only two positive divisors. After introducing the term *prime numbers* to describe this group of numbers, we distribute copies of the Frayer model graphic organizer,

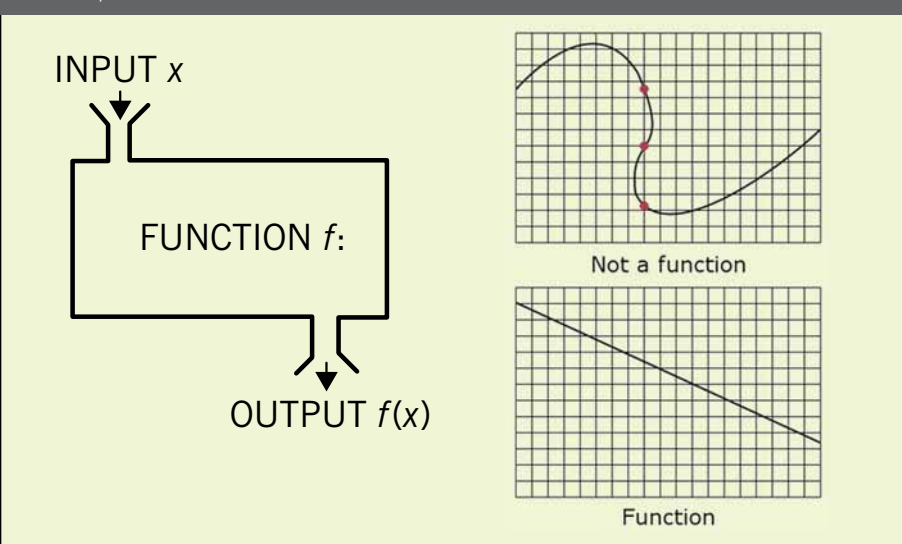
ask students to write the term in the oval (see **fig. 1**), and consider a series of examples and nonexamples. Students can either generate the list independently or the teacher can present the list of examples and nonexamples for students to consider.

Once the list is completed, students work in pairs or small groups to discuss essential and nonessential characteristics of the two lists. Nonessential characteristics may be interesting, but are not important. For example, the property that primes have two positive divisors greater than 1 is an essential characteristic; the fact that most primes

**Fig. 2** The four square graphic organizer allows students to associate many ideas with the term being explored.



**Fig. 3** Visual representations can help deepen students' understanding of a term; in this case, the term is *function*.



are odd is a nonessential characteristic. In our case, we ask students to list and discuss essential and nonessential characteristics of prime numbers. Next, we explain that the lists students generated are hypotheses that will be tested and revised as we consider example and nonexample pairs.

We present example–nonexample pairs and ask students to adjust their lists of characteristics. In general, new example–nonexample pairs should add specific defining characteristics that

will clarify students' understanding of the term. After these pairs are introduced, we discuss the essential and nonessential characteristics of the word or concept that students identified; explain and correct students' misunderstandings; and, when necessary, discuss essential characteristics that students missed. Students should record essential and nonessential characteristics on their Frayer model at this point and develop their own list of examples and nonexamples based on their under-

standing of defining characteristics.

The Frayer model teaching strategy requires students to engage in inquiry and hypothesis testing. This process takes time. Teachers should limit the number of words or concepts presented in each lesson to five or less. This allows sufficient time for testing the hypothesis and also ensuring that each student records accurate and complete information on the Frayer model so that it can be used as a reference or study aid.

#### FOUR SQUARE

Four square is a graphic organizer that is similar to, but not as complex as, the Frayer model. This graphic organizer is constructed using index cards divided into four quadrants (see **fig. 2**). The teacher provides the pronunciation and spelling of the term, and students write the term in the top-left quadrant. Next, the teacher explains the meaning of the term, and students write the definition in the bottom-left quadrant. Each student is encouraged to make a personal association with the term or choose a “lightbulb word” that will trigger the term’s definition. That word is written in the top-right quadrant. Lightbulb words and personal associations may differ from student to student. In the final quadrant, students either draw a picture or figure to remind them of the meaning or write an equation for the term.

We use the term *function* to illustrate the four square strategy. Teachers often begin students' exploration of functions through patterning and games like Guess My Rule that focus on the idea that a function assigns every input value to a unique output value. The birthday metaphor is another way to help students conceptualize the notion of a function. That is, everyone has one and only one birthday, but multiple people can have the same birthday.

Beyond conceptual explanations, a formal definition, and mathematical examples and nonexamples of

function, visuals like the function machine shown in **figure 3** help students develop a concept of function. The machine reminds students that input values ( $x$ -values) are substituted into the function equation and that simplifying the expression results in the output ( $y$ -value or  $f(x)$ ).

In algebra classes, the vertical line test is introduced as a visual cue that helps determine whether or not a graph represents a function. With this test, a student checks to see if a vertical line intersects a graph in more than one place. If it does, the graph does not represent a function because for this to happen, the  $x$ -values of the two points will be the same and yet result in different output values.

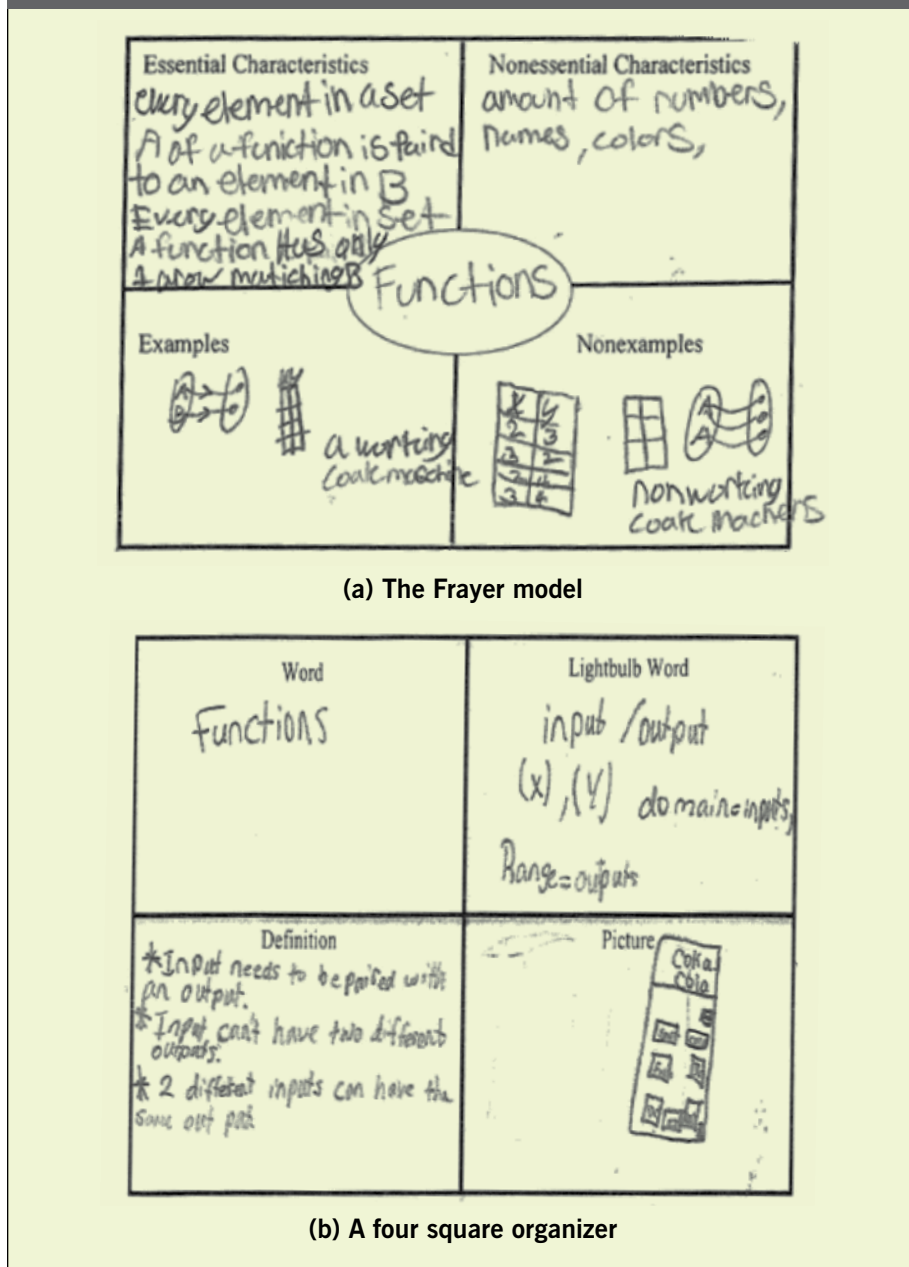
Four square allows students to mix and match representations and explanations to fit their own conception of what a function is and is not. A student may choose to match lightbulb words like *in-out*, *one and only one*, or *birthdays* with any equation (e.g.,  $f(x) = x + 3$ ), drawing, or graph to help them remember the concept of a function.

Tyminski taught an introductory lesson on functions with Mrs. Crane (a pseudonym) in her seventh-grade mathematics classroom. After playing Guess My Rule and comparing various representations of paired examples and nonexamples of functions (arrow diagrams, T charts, and ordered pairs), students were able to identify the essential characteristics of a function and generate a definition. Crane summarized the lesson by introducing the birthday metaphor to her students. Students completed either the Frayer model or four square graphic organizer to demonstrate their knowledge of functions (see **fig. 4**).

### FEATURE ANALYSIS

The feature analysis strategy (Johnson and Pearson 1984) is used to illustrate relationships between different terms or concepts. Some knowledge of the

**Fig. 4** The Frayer model (a) and four square organizer (b) illustrate how students view the term *function*.



terms and concepts and their meanings is necessary for students to use this strategy successfully. Features of the target words are listed across the top of a grid; vocabulary terms and concepts are found at the left (see **fig. 5**). Students consider one term at a time in relation to each feature. When a term has a feature, they check the box. An analysis of important features allows students to identify similarities and dif-







ferences and make predictions that are based on relationships between terms.

A feature analysis helps middle-grades students conceptualize and make sense of properties of and relationships between shapes and figures in geometry. Tyminski guided students as they used examples and nonexamples to generate definitions for *simple curves*, *closed curves*, *polygons*, *convex figures*, and *concave figures*. Following

**Fig. 5** The feature analysis strategy, which illustrates relationships between different words or concepts, is presented for various quadrilaterals.

		Features					
Terms		4 Sides	Opposite Sides Congruent	Opposite Sides Parallel	4 Right Angles	Adjacent Sides Congruent	Diagonals Are Congruent
	Quadrilateral						
	Square						
	Rectangle						
	Rhombus						
	Kite						
	Trapezoid						

**Fig. 6** This completed feature analysis used drawn figures rather than those identified by name.

Feature						
Simple	X	X		X	X	X
Closed	X		X	X	X	X
Polygon					X	X
Convex	X				X	
Concave				X		X

a group discussion, students used their definitions to complete a feature analysis of given figures (see **fig. 6**).

Using a feature analysis, students developed a connected view of the characteristics and their properties and made conjectures regarding relationships between various definitions.

### VOCABULARY STRATEGIES ARE EFFECTIVE

The positive effects of vocabulary instruction on students' learning are well documented, and the ability to speak the language of mathematics is

essential to understanding. We believe that these vocabulary strategies from the field of literacy are effective ways to help students assimilate the unique concepts and terms that they will encounter in mathematics. A successful use of the strategies presented here will require teachers to match instructional content with the appropriate teaching strategy while considering their students' prior mathematical knowledge.

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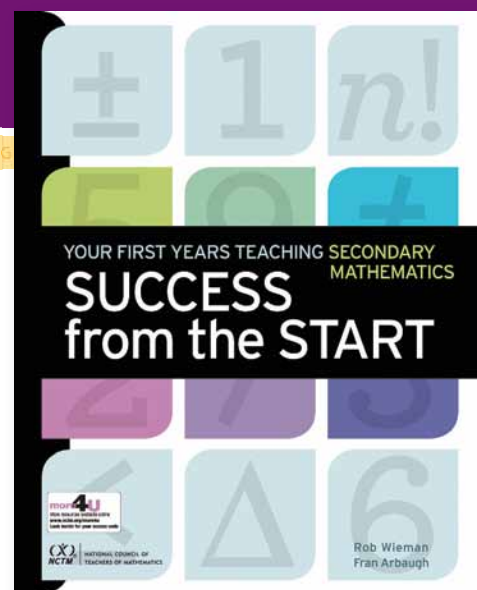
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