# Strategies that foster self-awareness, help regulate emotions, and encourage problem-solving perseverance can turn mathematical fight or flight into re-engagement.

"This problem triggered the run response in my brain."

Amy, seventh grade

Amy's reaction to a challenging story problem reflects the anxiety that many of us face when struggling with difficult mathematics problems. Recent research suggests that it is not simply experiencing anxiety that affects mathematics performance but also how we respond to and regulate that anxiety (Lyons and Beilock 2011). Most of us have faced mathematics problems that have triggered our "run response." The issue is not whether we want to run, but rather *how* we ultimately turn around and reengage with the problem.

This article examines both teachers' and students' emotional reactions to challenging mathematics problems and, more important, the strategies they use to cope with anxiety and to re-engage and grapple with these problems. These coping skills are embedded components of the first essential Standard for Mathematical Practice identified in the Common Core State Standards for Mathematics: "Make sense of problems and persevere in solving them" (CCSSI 2010, p. 6). Self-awareness and regulation are essential, and often ignored, components of mathematical problem solving. This article examines how these skills can be modeled, taught, and learned.

This work stems from a multiyear initiative designed to embed pertinent dimensions of social-emotional learning (SEL) into preservice teacher education. In selected classes, we are piloting specific strategies designed to develop both—

- 1. candidates' social-emotional skills for teaching; and
- 2. candidates' ability to foster students' social-emotional skills for learning.

Although many would view mathematics methods as an unlikely setting for this work, I suggest that teaching and learning mathematics is



## **TEACHER CANDIDATES**

I began exploring self-awareness and problem solving with the preservice teacher candidates in my mathematics methods course. Their emotional reaction and self-regulation when faced with difficult mathematics problems provided the initial insights into how to pursue this work with their students. I asked teacher candidates to describe their immediate emotional reaction after reading the following problem from a middle school textbook.

# Multi-Step Problem

The density of a substance is the ratio of its mass to its volume, written as a unit rate.

- a. Calculate A 500 cubic centimeter sample of sea water has a mass of 514 grams. Find its density.
- b. Calculate A 300 cubic centimeter sample of an iceberg has a mass of 267 grams. Find its density.
- c. Compare Which is denser, sea water or an iceberg? Explain why your answer is reasonable. (Larson et al. 2008, p. 263)

Some candidates expressed confidence:

- "Excited! Love math!"
- "Love them [word problems]; way better than just digits and symbols."

Many more expressed anxiety:

- "Oh \*#\*; this may take a while."
- "Nervous, I'm an English major. Ahhh!..."
- "Fear. Math is not a subject I feel confident in doing or teaching. This problem makes me feel frustrated and disappointed in my own math skills."

The following statements were made by the anxious, not the confident,

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teacher candidates. It was intriguing to hear the wealth of creative ways they used to talk themselves into re-engaging with the problem. They employed a variety of self-talk (Meichenbaum 1977) strategies, internal conversations with themselves, to shape their feelings and behaviors. These strategies helped them cope with their anxiety and regulate how they would re-engage in mathematical problem solving.

- "I take a deep breath and say, 'Break it down, it's OK, you can do this.' I start to draw a picture to help myself visualize...."
- "'OK, take it one step at a time. Who cares how long it takes to solve? When you're done, double check...'"
- "Reread the problem several times until [the] words and numbers start making sense. Draw/write out the problem. Work through it out loud. Make it visual so it makes sense. Ask questions."
- "Read the problem first, breathe, brainstorm, and solve what you know."

These prospective teachers recognized their stress and employed strategies to both reduce their anxiety

and make sense of the math. They alluded to classic mathematics problemsolving strategies identified by Pólya (1957) including—

- 1. understand the problem (i.e., "draw a picture to help myself visualize");
- 2. choose a strategy (i.e., "brainstorm, and solve what you know");
- 3. follow the strategy (i.e., "work through it out loud"); and
- 4. evaluate the strategy (i.e., "'when you're done, double check'").

To cope with their anxiety, they reminded themselves to slow down and breathe. To re-engage, they searched for and used known aspects of the problem. Their comments indicated a strategic integration of emotional self-awareness, coping, and problemsolving skills—all attributes that are well worth modeling and teaching to students.

# **SELF-AWARENESS IN** MIDDLE SCHOOL STUDENTS

The density problem was posed to an academically diverse group of sixthgrade through eighth-grade students in a rural school. I asked the students to describe their emotional reaction to the same problem. Although I knew that they had been exposed to the necessary math, their reactions mirrored those of my less confident teachers:

- "It triggered the run response in my brain." (Amy)
- "This doesn't make any sense to me. I'm confused with the first sentence. I just want to forget about it." (Mike)
- "It is really hard, and I would feel terrified of failing it. I haven't been taught this." (Gloria)
- "I can't do it. I do not know what to do." (Cecilia)
- "It looks hard and complicated." (Juan)

- "I begin to freak out as I read and reread the problem." (Cory)
- "What the heck are they talking about?" (Bill)

Some felt anger, and others appeared hopeless. Most felt some degree of fear. I asked them what they would say to themselves to cope with their feelings and talk themselves through the problem. Most looked baffled by the question and simply said they do not talk themselves through problems. They stop working or ask for help. Only one student, Cory, said he would reread the problem and look for parts he knew.

I showed them the kinds of strategies my teacher candidates used, but first I let them read some of my prospective teachers' reactions to the problem. They were amazed and highly entertained that teachers' emotional reactions to difficult problems could mirror their own.

We talked about recognizing how a problem might scare them and how to calm down and take their time. These self-talk strategies acknowledged and addressed students' emotional and physical reactions to the problem. We then practiced problem-solving selftalk for re-engaging and addressing the cognitive demands of the problem.

We studied the picture of an iceberg that appeared next to the problem in the textbook and decided to try Cory's strategy and "reread the problem." I read it aloud to allow struggling readers to think about the problem. Although they were "freaked out" at the academic language, students began to pick out words they knew ("solve what you know"). Mike knew about volume and mass and gave a credible description of density as the size of something in relation to its weight. We discussed the density of their math book compared with the density of a stuffed animal that is in the school's reading center. Gloria

Students felt that if successful future teachers were anxious when faced with a challenging math problem, then surely it was OK for them to "freak out."

commented that ice floats. The group pondered why and how that might be related to the density of the iceberg. They proposed that since icebergs float, they should be less dense than seawater.

I commented that students had essentially answered the problem by combining what they knew with what they figured out without ever doing any math. They exchanged the smug looks of students who had beat the system.

Nonetheless, we decided to engage in the math by trying the step-by-step approach next and built the ratio for density, density = mass/volume, from the sentence of the problem. I helped students link mass to grams and volume to cubic centimeters, and they built the ratio for the density of seawater, d = 514 g/500 cc. They eagerly set up the iceberg ratio, d = 267 g/300 cc, on their own. Amy noted that the seawater ratio was more than 1, whereas the iceberg ratio was less than 1, so seawater must be more dense than the iceberg. They had essentially solved the last part of the problem by acknowledging and coping with their emotional reaction, re-engaging, and taking time to make sense of the math. Mike, who initially wanted to "just forget it,"

insisted on staying to calculate the unit rates and prove that they were correct.

I asked the students to reflect on their initial reaction to the problem and what they had learned.

- "If I paid attention to what I knew instead of freaking out, I would have actually gotten the problem." (Amy)
- "It is quite simple when you calm down." (Gloria)
- "I could have done this really quickly if I wasn't freaking out." (Mike)

Students reflected that the math was actually easy. One student commented that "it was just all the words mixed together" that made the problem difficult. They were glad they had not had to tackle it alone and commented that they needed the teacher's help to understand the problem and help them find the parts they knew. However, when I asked them if they could imagine talking themselves through a problem like this on their own at some point, they agreed that with practice, they probably could use the coping and problem-solving strategies we had tried that day.

### **LESSONS LEARNED**

This case provides insights into how self-awareness and problem solving may be taught. First, it describes a lesson that could be aligned with teachers' content objectives at any point in a mathematics curriculum. The lesson could be introduced or revisited whenever students face a potentially intimidating problemsolving task. Teachers should choose the task carefully, finding a problem that they suspect students will find intimidating but that will require skills most students possess and that will offer multiple entry points and solution paths. Within this context, the lesson described in this case is

essentially a discursive frame including three steps: recognizing and acknowledging emotional reactions, developing self-talk and coping strategies, and providing cognitive scaffolding during the problem-solving process.

# **ACKNOWLEDGING EMOTIONAL REACTIONS**

It is essential to begin the lesson by asking students to describe their emotional reaction to the problem. Figure 1 provides some of the prompts that teachers could use to help students recognize their emotional reaction and its impact on how they approach (or avoid) the problem.

In my many years of teaching mathematics, I had never explicitly asked students how a problem made them feel. Prompting both groups teachers and students—to examine their emotional reaction to a problem provided space to discuss what we usually keep hidden. For the many students who reacted with anxiety, the discussion created a sense of safety in numbers as they heard similar sentiments from so many of their peers.

It was important to emphasize that feeling an emotional reaction to the problem was not wrong. Recognizing one's feelings and learning to cope with them was the objective. Describing their emotions paved the way for students to try the coping strategies and self-talk described in the following section.

Fig. 1 These prompts promote students' self-reflection.

# **Recognizing Emotions** and Their Impact

- How did this problem make you feel?
- · What did you say to yourself when you first read this problem?
- How do our emotions and beliefs influence what we choose to do?

Self-awareness and problem solving are essential not only for mathematics but also for life.

# **DEVELOPING SELF-TALK AND RE-ENGAGE STRATEGIES**

With both teachers and students, we brainstormed self-talk strategies after discussing their emotional reaction to the problem, both to relieve stress as well as to re-engage with the problem. Prospective teachers in my class had a wealth of such strategies, implying that they had considerable experience talking themselves through both their anxiety and the mathematics.

Students' repertoire of self-talk was far less developed, a finding that speaks to the need to develop students' coping strategies in mathematics. When faced with a difficult mathematics problem, many students do not try to figure it out. They simply stop and wait for the teacher to tell them what to do. Figure 2 provides examples of the kinds of self-talk that students could use to calm themselves and re-engage with difficult problems.

It is important to have students brainstorm ideas first, chart them, and try them, giving credit to the

Fig. 2 These self-talk strategies provide re-engagement entry points.

# **Self-Talk and Coping Strategies**

- Take a deep breath and relax.
- Take your time to re-engage.
- Reread the problem and find what you know.
- Take it one step at a time.

students who generated the ideas. Ask students to think about a time when they experienced a problem outside of the context of math. How did they overcome it? What kinds of things did they say to themselves in the process? These questions provide ways to start the conversation. If students are unable to suggest any coping strategies, it is important for the teacher to be ready with suggestions, thereby modeling self-talk both for reducing anxiety (take a deep breath, relax, take your time) and for re-engaging with the problem (reread it, find what you know, work step by step).

Alternatively, the teacher could use the preservice candidates' emotional reactions and self-talk described in this article as a discussion starter for this segment of the lesson. In the lesson described above, reflecting with the students on prospective teachers' emotional reactions to the problem and the self-talk that candidates generated moved the discussion forward. I believe that students felt that if successful college graduates—future teachers-were anxious when faced with a challenging math problem, then surely it was OK for them to "freak out." As my middle-grades students shared the same anxiety as my teacher candidates, they were willing to try some of the same coping strategies.

# PROVIDING COGNITIVE **SCAFFOLDING**

Supportive scaffolding took place throughout the lesson. In the first two steps of the lesson, the scaffolding that helped students recognize and cope with their anxiety cleared the way so that students could cognitively engage with the problem. In the third and final step, cognitive scaffolding helped them make sense of the problem and find places in the problem where their prior knowledge or experience could help them. I read the problem aloud.

Fig. 3 These cognitive-scaffolding strategies will promote moving toward a solution.

# **Cognitive Scaffolding**

- Assist struggling readers by reading the problem aloud.
- Help students use context clues in the text and pictures to make sense of the problem.
- · Provide positive specific feedback validating both students' knowledge and their use of selftalk or coping strategies.
- Honor incremental advances in solving the problem and multiple solution paths.
- · Do not tell students how to do the problem; validate their effective strategies.

We looked for words or scientific ideas (e.g., density) that at least some of the students knew. I gave positive specific feedback validating students' knowledge. Only once did I provide clarification: I stepped in to help them link volume and mass to their measurements in cubic centimeters and grams.

We discussed the picture of the iceberg and searched for clues in the text. Students shared what they knew and pieced their understanding together like the parts of a puzzle. I did not, however, tell the students how to do the problem. I helped them become aware of their feelings (step 1), cope with those feelings and re-engage (step 2), and persist in solving the problem (step 3) by making sense of it and piecing together what they already knew. Figure 3 identifies specific cognitive-scaffolding strategies that can be used to help students solve the problem themselves.

# THE VALUE OF EMOTIONAL **AWARENESS**

This case provides an instance of the intersection of mathematics and social-emotional learning skills. It demonstrates how self-awareness and problem solving interact when doing mathematics. It is a case that has changed my practice. I do not believe that I had ever before asked either adults or students how a mathematics problem made them feel. When I did, the range and depth of their emotions surprised me. As my students discussed their emotional reactions, they learned that they were not alone in their feelings.

Helping students develop the skills to recognize and regulate their emotional reaction set the stage for re-engaging with the problem and making sense of it. The discussion gave students the coping skills necessary to persist. Sense-making and persistence are foundational mathematical practices. Emotional awareness and regulation helped students engage in these practices. This lesson was time well spent. Self-awareness and problem solving are essential not only for mathematics but also for life.

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