Robert Mann

Muffin Mania

Problem

Goldilocks's grandmother hands a basket of freshly made peanut-butter-and-chocolate muffins to Goldilocks and says, "These are for you, darling, but they just came out of the oven and need to cool before you can eat them. Please set them outside on the porch. You can enjoy them later."

Goldilocks thanks her grandmother and carefully places the basket on the front porch of their forest home. She then goes inside to take a nap while the muffins cool.

Three bears stroll by the cottage and smell the wonderful aroma of the sweet muffins. They follow the smell right to Goldilocks's front porch and smile when they see both the nameplate on the door and the basket of marvelous muffins. Papa Bear approaches the basket first and eats exactly 1/4 of the muffins. "Mmm," groans Papa Bear, "these are yummy." Next, Mama Bear eats exactly 1/3 of the remaining muffins. "You are right, Papa," she declares. "These are yummy." Finally, Baby Bear goes to the basket and eats exactly 1/2 of the muffins left by Mama Bear. He licks his lips and says, "Mmm, much better than porridge." The three bears pat their full bellies, smile contentedly at one another, and continue their walk through the forest.

When Goldilocks awakens from her nap, she immediately runs to the porch to grab the basket of muffins. She is startled to discover that only 3 muffins remain in the basket. "What happened to all the muffins?" she exclaims. "I know there were more than 3 in this basket when I put it here. I wonder how many muffins were in the basket to begin with."

Use the information from this story to help Goldilocks determine how many muffins were in the basket. If Papa Bear took 1/4 of the original muffins, Mama Bear took 1/3 of what he left, Baby Bear took 1/2 of what remained, and only 3 muffins are left in the basket, how many muffins were in the basket when Goldilocks first put them on the porch? Once you have a solution, explain to Goldilocks how you came up with your answer.

Extensions

- How many muffins did each bear eat? How do you know? Are these results surprising to you?
- What if the bears ate 1/4, 1/3, and then 1/2 of the muffins and left 4 muffins for Goldilocks? How many muffins would have been in the basket originally and how many would each bear have eaten? What if only 1 muffin remained?
- If you know the number of muffins left for Goldilocks at the end of the story—3, 4, 5, 1, and so on—can you explain how to find the total number of muffins that the grandmother made and the number that each bear ate?
- Suppose the bears eat the muffins in reverse order. First, Baby Bear takes 1/2 of the original muffins; next, Mama Bear takes 1/3 of what is left; finally, Papa Bear takes 1/4 of the remaining muffins, again leaving just 3 in the basket for Goldilocks. How many muffins did each bear eat and how many muffins were in the basket originally?

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The goal of the "Problem Solvers" department is to foster improved communication among teachers by posing one problem each month for K–6 teachers to try with their students. Every teacher can become an author: pose the problem, reflect on your students' work, analyze the classroom dialogue, and submit the resulting insights to this department. Every teacher can help us all better understand children's capabilities and thinking about mathematics with their contributions to the journal. Remember that even student misconceptions provide valuable information.

Classroom Setup

Allow your students to work on this problem in small groups. Read the problem aloud to your students and spend time explaining the situation. Having students act out the story and the removal of the muffins may be helpful. You may need to review fractions such as 1/4, 1/3, and 1/2 with your students. For students with limited experience with fractions, you may want to change the fraction of muffins removed each time to 1/2. Papa Bear would take 1/2 of the original number of muffins, Mama Bear would take 1/2 of what is left, and Baby Bear would take 1/2 of the remaining muffins. This modification allows younger students to concentrate on only one fraction and makes the calculations at each stage a little easier to complete.

You probably will need to reiterate that each bear takes his or her respective amount from what is left in the basket at his or her turn. If your students struggle with the problem or have difficulty working with the fractions, you might suggest that they start at the end of the story and work backward. You can prompt this thinking by asking, "If Goldilocks is left with 3 muffins, how many did Baby Bear take when he took one-half?" Do not, however, propose this solution strategy immediately. Instead, encourage your students to devise their own strategies and to use words, manipulatives, pictures, tables, or other methods to investigate and explore the Muffin Mania problem.

Once students have solved the original problem, direct them to the extensions and ask them what happens when the number of remaining muffins is changed to 4 or 1 or any other number. They may be surprised by how quickly they now can determine a solution for any value of remaining muffins. In addition, have the students explore what happens when the order of the fractions is reversed as indicated in the last extension. If 3 muffins remain at the end, did each bear still eat the same number of muffins that he or she did originally? Will the total number of muffins made remain the same? Encourage your students to explore these extensions as well as any other related questions or scenarios that you or they propose. ☛

Where's the Math?

The Three Bears achieve "sweet revenge" by consuming different fractional amounts of Goldilocks's muffins. Students may be surprised to discover that each bear eats the same number of muffins, even though they each take a different fraction of the muffins. This discovery may help students better understand the nature of fractions and may help them realize that to correctly interpret a fraction, they must first identify the whole. Consequently, students working on this problem should gain a better understanding of fractions, division, multiplication, and number sense.

In addition, students who investigate the extensions to the Muffin Mania problem should begin to make generalizations and, in the process, begin to recognize the power and benefits of such generalizations. Students who devise a method to solve any situation given the remaining number of muffins should be impressed by the magnitude of their solution and methodology. They also may be impressed to discover that such generalization techniques are an important element in algebra, geometry, and other areas of mathematics.

As the students work on solving the Muffin Mania problem and its extensions, they also will be engaging in problem solving and critical thinking. As they explain and extend their solutions, students will use multiple representations to communicate mathematics and will make connections among the various methods, representations, and insights. Finally, as children attempt to justify their answers and generalizations to their classmates, they will be actively engaged in proof and logical argument. We hope that the Muffin Mania problem instills in the students a hunger for mathematical reasoning and problem solving that is matched only by their hunger for delicious peanut-butter-and-chocolate muffins!

As a class, discuss the proposed solutions and the methods that students used to determine these solutions. Have students justify their answers and explain their thinking. Collect student work, make notes about interactions that took place, and document the variety of student approaches that you observed in your classroom. We also encourage you to take photographs of your students as they work on the problem and to extend the problem into other areas such as literature, art, and personal experience. (Receiving student artwork of the pilfering bears or student tales of similar camping experiences would be exciting!) Feel free to adapt the problem to fit the level and experience of your students and to present it in a way that you believe is most engaging and understandable to them. As you reflect on your experience with this problem, keep in mind the following questions:

- What difficulties did students have in understanding the problem?
- How did students approach this task?
- What strategies did students try?
- Were any student responses or interpretations surprising to you?
- What questions or justifications arose from student explanations?
- Were the students surprised by any of the results or solutions?
- What sorts of patterns or generalizations resulted from the investigation of this problem?

We are interested in how your students responded to this problem or your adaptation of the problem and how they explained or justified their reasoning. Please send us your thoughts and reflections. Include information about how you posed the problem, samples of students' work, and any photographs that you would like to share. Send your results with your name, grade level, and school by **April 1, 2004,** to Bob Mann, Mathematics Department, Western Illinois University, Macomb, IL 61455. Selected submissions will be published in a subsequent issue of *Teaching Children Mathematics* and acknowledged by name, grade level, and school unless otherwise indicated. ▲

(Solutions to a previous problem begin on the next page.)

Kim Hartweg

Responses to the Picasso Masterpiece Problem

The problem appearing in the February 2003 "Problem Solvers" section was stated as follows:

> Picasso, a famous artist of the 1900s, sometimes used geometric design in his art to express his moods or dreams. Pretend you are a modern-day Picasso creating a computer-generated masterpiece designed to help young mathematics students better understand various geometric terms. Use a paint or draw computer program to create your masterpiece and include—
>
> - a polygon with no lines of symmetry; and
> - two congruent figures with at least two lines of symmetry.
>
> Identify the geometric figures by labeling the drawings. Explain how you decided what to draw. If you do not have access to a computer, you can create your masterpiece with another medium such as paper and pencil or crayons.

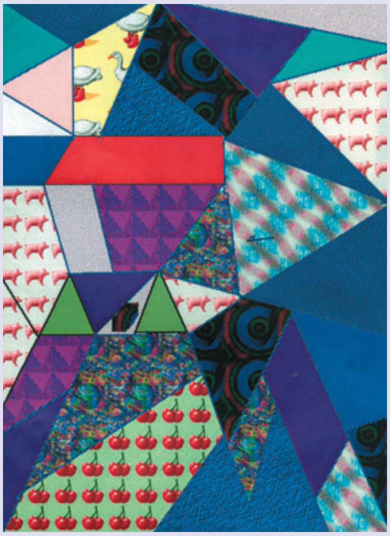
This problem addresses NCTM's Geometry Standard, which emphasizes the use of visualization, spatial reasoning, and geometric modeling to solve problems. By allowing students to explore geometric spatial reasoning problems such as this, teachers can help all students develop an intuition about relationships among various shapes. Tasks that students will complete while solving this problem include identifying, describing, classifying, and comparing relationships using points, lines, and plane and solid figures while also developing an understanding of congruent, symmetrical, and non-symmetrical figures.

Kate Burton, a fifth-grade science teacher at an independent school in Florida, created a problem-solving club for fifth and sixth graders. The club has an open admission policy in which students can decide each week whether the problem presented interests them. Students who choose to participate each week use their free time to complete the problem and discuss their findings at lunch on Mondays. Burton described her students' progress:

> The problem-solver's club worked on the problem as it was written, with no modifications. I suggested using the technology programs that were on our school computers and that I knew the students had experience with. These programs included AppleWorks draw and paint, Microsoft Office draw and paint, and HyperStudio. Each student worked individually on his or her own masterpiece, but, as always, I encouraged the students to talk to one another about their ideas and give help if someone was stuck. After some initial struggles during the week with pictures that weren't being saved where we could find them, the students found a lot of success and enjoyed seeing their peers' creations. Many students started their masterpieces with HyperStudio only to abandon the program when they discovered the more sophisticated tools in Microsoft Office and AppleWorks.

Figure 1

Tori uses an equilateral triangle and a trapezoid to meet the problem's requirements.

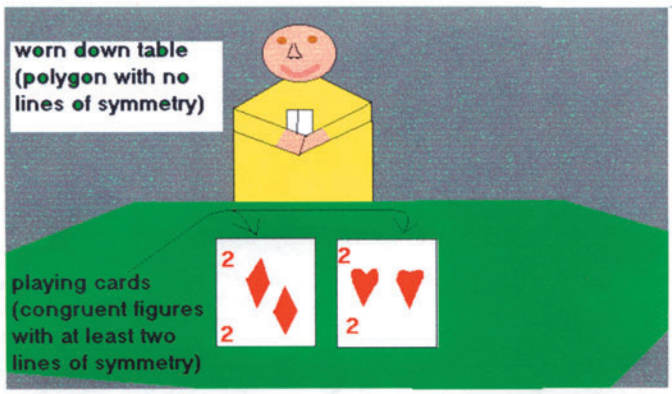


I chose these because I looked up what congruent figures were. They are the exact same figures. Then I looked up what a line of symetry is. It said a figure that you can cut in half and they will both be the same. A triangle was the perfect fit! A triangle is congruent and you can cut it in half both up and diagnally. "Perfect" I said! The Polygon with $\boxed{\text{no}}$ line of symetry.

Well I thought about all the shapes until I thought abot the rectangle! It was perfect exept, I had lines of symetry. But what if I could make one of the small lines diagnol. Yes! It worked! So there you go! That is how I figured it out!

Figure 2

Nick gets his inspiration from his brother.



I thought of drawing a person playing cards because my brother was playing with cards while I was trying to think of a picture for this assignment.

• made figures congruent using size measurments
• my brother was playing cards so I dicided to draw a person playing cards

I was truly surprised to see the finished products. I expected that most of the kids' masterpieces would be "crazy quilt" types with shapes synthesized to fill the two-dimensional space, much like Tori's picture [see **fig. 1**]. Instead, most were pictures that looked like something. This group did a good job of working independently without worrying that their picture didn't look as "good" as that of the kid sitting next to them.

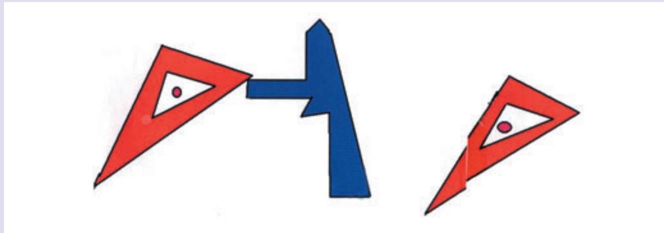
I was impressed with Tori's thorough explanation of how she came to decide what figures to include to meet the problem's requirements [see **fig. 1**]. Her first challenge was to identify the vocabulary in the problem, and with that accomplished, she did a good job of rationalizing what to do next. When looking for her polygon with no lines of symmetry, she decided the rectangle would be a good shape to use with some modification. Notice how Tori describes the making of her trapezoid by making one of the "small lines diagonal."

Many of my students got their inspiration for what to draw from things they saw around them. Nick's inspiration for the card player was seeing his little brother playing cards [see **fig. 2**]. Brittany saw a woman wearing a necklace that had no lines of symmetry, which gave her a good start [see **fig. 3**]. Others were like Becky, who said, "I just started doodling and it all came to me" [see **fig. 4**]. Rachel started with what she thought was the hardest part—the figure with no lines of symmetry—and in that she saw a mountain that she decided to build on [see **fig. 5**].

The aspect that I found most interesting was the technology techniques that the students used to make their congruent shape with two lines of symmetry. Brittany and Rachel [see **figs. 3** and **5**] used the copy feature on AppleWorks and Microsoft Office. Becky [see **fig. 4**] tried to redraw the figures by hand. Dane [see **fig. 6**] used a good strategy by redrawing his congruent rectangle on top of the other one so that he knew it would match. When he had the second figure, he then dragged it to where he wanted it in the overall piece. I thought the most ingenious technique was the one that showed the greatest knowledge of how his draw program worked. Nick used the size measurements shown in the dialogue box when he was drawing the rectangles

Figure 3

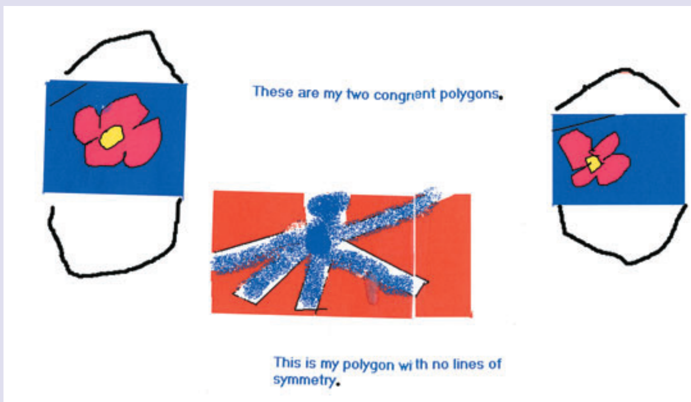
Brittany is inspired by a necklace.



I figured out what to draw because I was in the car and saw a lady with a cross neclass and the neckless was really wierd it was no lines of semetry I got them congruet because of trial and air because I Just started drawing.

Figure 4

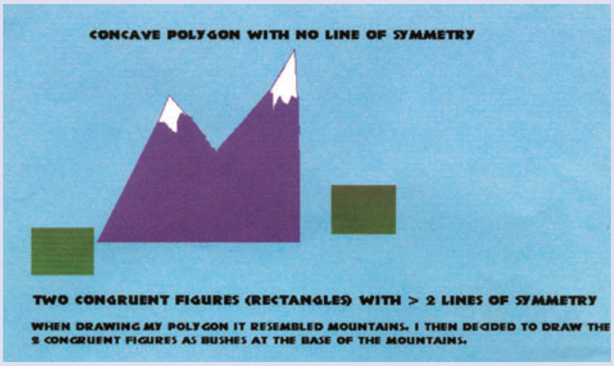
Becky experiments with the program to find a solution.



To get my picture I just started doodling and it all came to me.
I also looked up the word "congruent" in my Reference book.

Figure 5

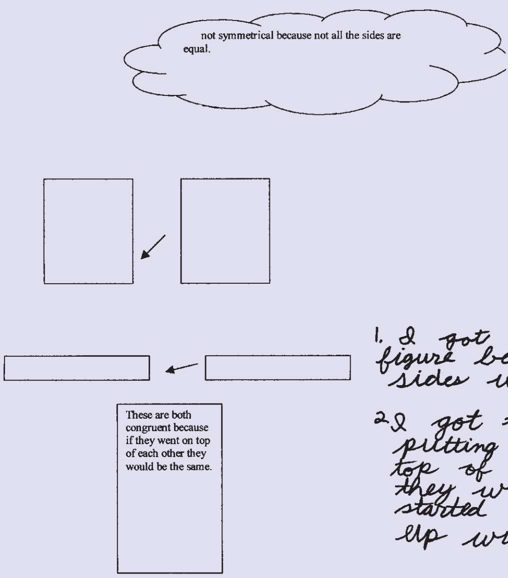
Rachel uses the program's copy feature to create two congruent figures.



I made a figure congruent by using the copy button on the computer. I drew different shapes on a piece of paper and my 3rd shape looked like mountains.

Figure 6

Dane creates congruent pairs of rectangles by forming the second rectangle on top of the first one.



1. I got a nonsymmetrical figure because not all the sides were equal.

2. I got them congruent by putting the shape on top of each other so they were equal. I just started drawing and came up with it.

for the playing cards [see **fig. 2**]. He realized that he could re-create the same-size rectangle by remembering the sizes.

I was incredibly pleased with how well this project worked. I enjoyed the opportunity to have fifth and sixth graders work together and help one another figure out the best approach for finding a solution. I really think you get the best out of all the individuals involved if you have mixed age groups. The students encouraged one another and were very excited during the Monday lunch. They couldn't wait to see what their friends and classmates had created. All the students demonstrated a lot of pride as they explained how they were inspired and their strategies for overcoming the "problem" parts of this problem.

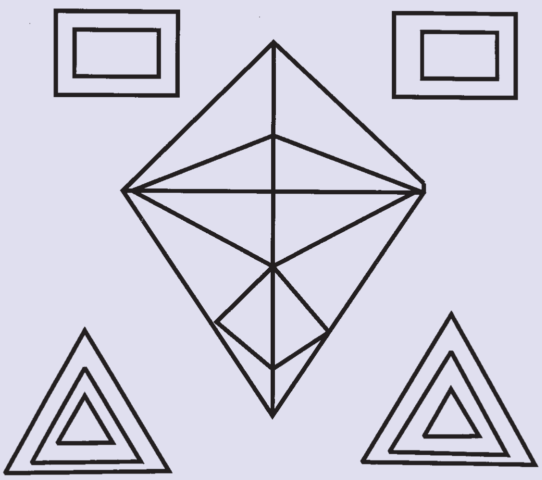
Sara Jenkins from Lattie Coor School in Avondale, Arizona, is a technology teacher who works with more than six hundred fifth- and sixth-grade students integrating subject matter throughout various explorations. Jenkins had discussed geometric shapes earlier in the year, and the current feature exhibit at the Phoenix Art Museum was "El Greco to Picasso." The art teacher showed students posters of Picasso's work and created a bulletin board. The library also had a book highlighting Picasso that was part of the school's Accelerated Reader program. Needless to say, Jenkins thought the Picasso Masterpiece problem was "kismet" when it appeared in the February issue of *Teaching Children Mathematics*:

During the first period, I read the library book aloud, highlighted some of Picasso's work and Cubism, and then had students take an Accelerated Reader quiz for comprehension. The next period, I showed them the CD "Artrageous," which features one of Picasso's paintings and allowed us to explore the work close-up. Next, I showed students how to use the draw program in AppleWorks and gave them their assignment.

I tried to be as brief as possible: Students were to draw a masterpiece using polygons and containing at least two congruent polygons with two lines of symmetry. It could be abstract or it could be something "real." It was to fit on one piece of 8 1/2 × 11-inch paper. Because we do not have a color printer, it had to be printed in black and white. They could,

Figure 7

Cecy uses many geometric terms to describe her masterpiece.

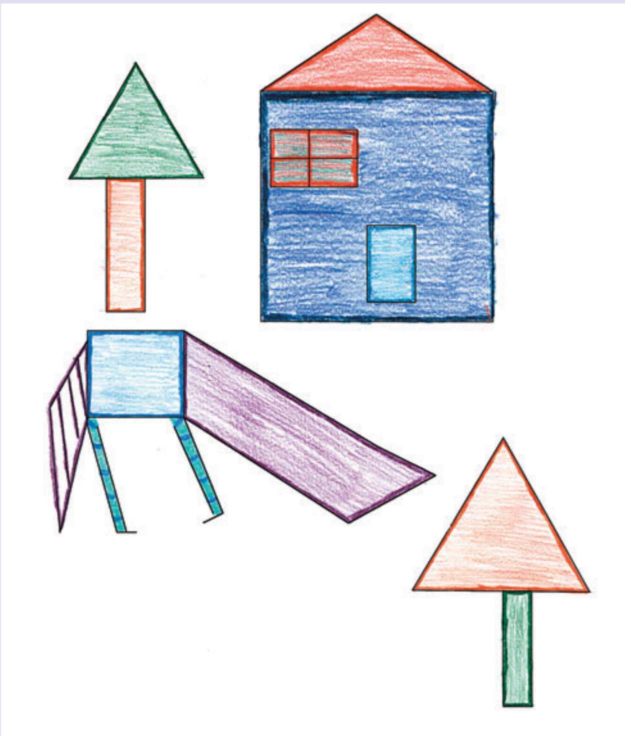


"It is a rhombus with triangles on the bottom and rectangles on the top. Inside of the rhombus are triangles and no parallel lines. There are 47 lines and the two triangles are congruent. The two rectangles are congruent too. I have 3 quadrilateral shapes.

"When I was drawing this I was thinking that it was a diamond cut out in a beautiful way. It was the famous ring in the whole wide world. With the triangles I was thinking I was in Egypt, that I actually lived there with my mom, sisters, and brothers during old times. We were the ones that owned the beautiful diamond of Egypt. Everybody wanted it because it was the most famous diamond in Egypt. Well, the rectangle I just thought it was a cool mirror."

Figure 8

Perla places her congruent and symmetrical figures in a park setting with a clubhouse.

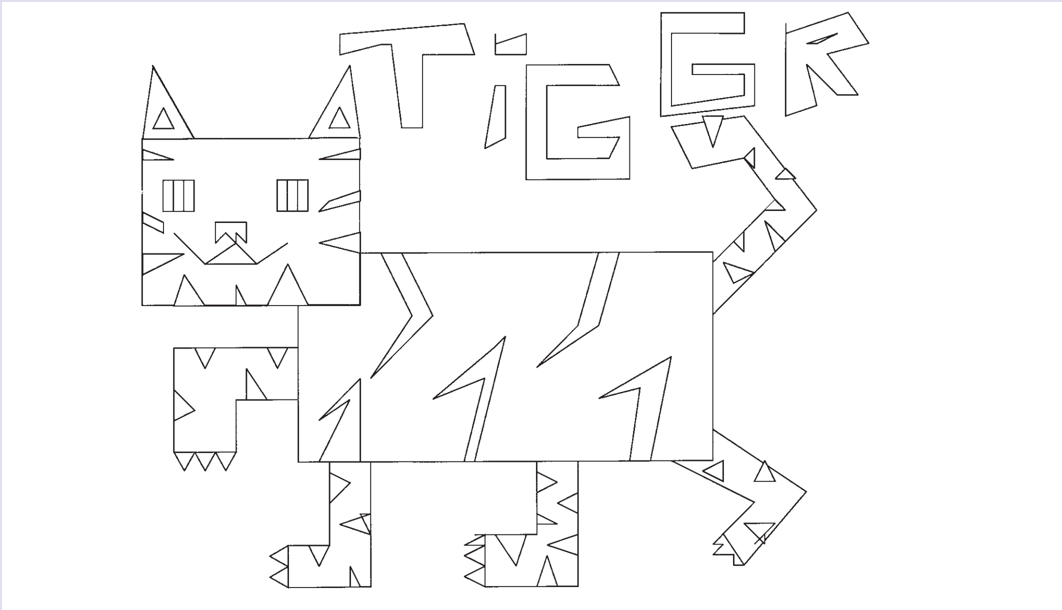


"I started making my picture by doing a square, then putting a triangle on top of it, which looked like a house. Then I put a door and a window to the house. After that I thought of putting something in front so I put two trees. I also made a slide in between the trees. The trees have symmetry, the house and slide are not. I used only straight lines, triangles, and squares on my drawing. I think it kind of looks like a clubhouse.

"Once upon a time there was a park that people didn't go to anymore. There was a little boy named Alex, who wanted people to go back to the park because it looked lonely. With the help of his friends he built a clubhouse and a slide. He thought that would make the park look more interesting. As soon as people saw the clubhouse and slide they went to the park and played. Alex was happy because he saw people having fun. Also the park wasn't lonely anymore."

Figure 9

Ellie's elaborate tiger is composed of many different polygons.



"Long ago when lions were kings of the jungle, and tigers were only lonely peasants with mundane lives, or so they thought, mysterious happenings were interrupting the tigers' daily routines. They were getting sick of it! One day while Koobanka, the youngest of all workers, was going on with his daily routine, a storm came upon him. He wanted to stop everything that was happening, so one day he went to the lions and demanded to know what was happening. The lions wouldn't tell him anything he wanted to know. One day when all the lions were sleeping (which they did often) he discovered that the lions were all robots! They were made by an evil scientist planning to take over the world. What was he going to do? He decided to find out where the scientist lair was. He found out that it was in an apple tree in the center of the forest. He found the door. Now a clever plan came to him. Since all those terrible things were happening to the tigers, he decided to do his daily routine like usual. OH NO! A storm came. Just like the tiger expected, he opened the door to the tree and everything got ruined by the water! It was all over and the tigers were free!

"The tiger's ears are congruent. The head is a square and the body is a rectangle. All the paws are hexagons and the tail is an octagon. The eyes are also congruent. They are made of two squares overlapping each other. The stripes of the tiger are polygons. The stripes on the body are hexagons. The stripes on the tail, paws, and head are triangles."

however, use various patterns to shade their figures. After students printed a copy, they could save a color original in their folder on the computer. They could choose whether or not to color their printed copy.

During the fourth period, Jenkins told students to use a word-processing program to write about the masterpiece. She instructed students to write one paragraph discussing the mathematics and one paragraph either describing how they created their picture or creating a story to go along with their masterpiece. Jenkins described the problems that students encountered and the results:

Students encountered few difficulties with this exploration. I used mathematics vocabulary such as *congruent* and *symmetry* many times. Most of my students are second-language learners, so we practiced finding symmetry by folding paper. We also went over the fact that the picture had to include a polygon; no curved lines were allowed. Some students seemed unable to remember that a circle, an oval, or even a square-like shape with curved corners is not a polygon.

The use of computer drawing programs helped students gain understanding. Jenkins reported that once students became comfortable with using the computer program's draw features, "they really liked using the polygon tool to click and draw all kinds of shapes." See **figures 7, 8,** and **9** for examples provided by Jenkins's students. Many of her students' masterpieces included creative and interesting stories.

Teachers who presented this problem to their students reported that the students enjoyed the activity and were excited about the possibility of having their work published in a journal. Jenkins also discussed how "difficult it is for the classroom teacher to have the technology and time to do geometry." This problem gave her the opportunity to integrate the use of technology into the mathematics curriculum.

We wish to thank the following teachers and students for their responses:

Kate Burton's fifth and sixth graders, Florida
Sara Jenkins's fifth and sixth graders, Lattie Coor School, Avondale, Arizona ▲