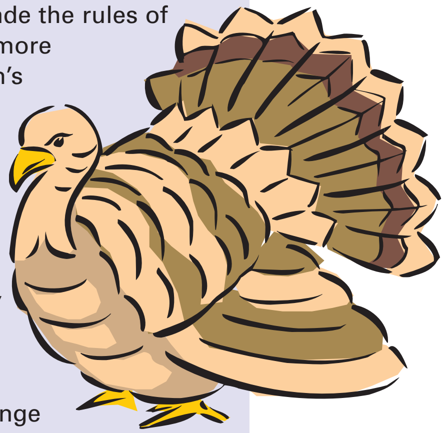
Talking Turkey

Problem

Sam loves to help around the kitchen. With Thanksgiving approaching, there is plenty to do! Sam offers to help determine how big a turkey the family should buy for the dinner. He finds some "rules of thumb" for buying turkeys. The suggested weight range is from one pound to one and one-half pounds per adult, and three-quarters of a pound per child, if the family wants leftovers; or three-quarters of a pound to one pound per adult, and one half-pound per child, if the family does not want leftovers. The people who made the rules of thumb do not know Sam's family. The two teenagers in Sam's family eat more than most of the adults, and Uncle Roy eats more than anybody else. Sam's sister Judy does not really like turkey, so she will fill up on dinner rolls and just eat a little to be polite. It is hard to predict what the three young children (Uncle Roy's kids) will eat—it depends on their mood. Sam loves turkey, but he guesses that the standard amount per child will be about right. Sam also knows that Uncle Roy will not take leftovers with him after dinner. Sam's family likes leftovers, but not too many.

How big a turkey should Sam recommend to feed the ten people (Sam, Mom, Dad, two teenage siblings, Judy, Uncle Roy, and Uncle Roy's three kids) who will be eating Thanksgiving dinner? Explain your reasoning.



Variations: This problem can be modified for younger students by changing the number of people in the family. Teachers can take out the range of choices for the amount of turkey per person, and make the people involved "standard eaters" rather than interesting individuals. The problem can be extended for older students to plan more of the meal. The author did research on the Internet to find the "rules of thumb" for amounts of turkey. Students with different traditions could share different menu items they have for Thanksgiving, and the class could do research to help plan for their meals. This problem is a great example of using mathematics in a real-life situation.

The goal of the "Problem Solvers" department is to foster improved communication among teachers by posing one problem each month for K–6 teachers to try with their students. Every teacher can become an author: pose the problem, reflect on your students' work, analyze the classroom dialogue, and submit the resulting insights to this department. Every teacher can help us all better understand children's capabilities and thinking about mathematics with their contributions to the journal. Remember that even student misconceptions provide valuable information.

Classroom Setup

Allow your students to work in small groups. Spend time discussing this problem with your students, but avoid giving too much guidance. The problem should generate lively discussion as students try to decide on the best size of turkey.

Encourage your students to use words, manipulatives, pictures, tables, or other methods to experiment, organize, and explain their thinking about the problem. Collect student work, make notes about interactions that took place, and document the variety of student approaches that you observed in your classroom. Feel free to adapt the problem to fit the level and experience of your students. As

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Where's the Math?

Many students probably have asked their parents at this time of year, "How do you know how big a turkey to get?" They will find out through this problem that selecting a turkey is not a cut-and-dried process. Estimation skills are necessary, yet there is a certain urgency to be relatively close in the estimation. Overshooting on either the high end or the low end by significant amounts will cause problems with the meal. "Rules of thumb" exist, but they are just guidelines; they must be modified for specific situations. Mathematical reasoning abounds in this problem as the students communicate about why they believe they should buy one size turkey rather than another. The problem also provides many opportunities to practice arithmetic skills, including operations with fractions, while solving a very practical problem.

you reflect on your experience with this problem, keep in mind the following questions:

- What difficulties did students have in understanding the problem?
- How did students approach this task?
- What strategies did students try?
- Were any student responses or interpretations surprising to you?
- What questions or justifications arose from students' explanations of their plans?

(Solutions to a previous problem begin on the next page.)

Share Your Student Work We are interested in how your students responded to the problem and how they explained or justified their reasoning. Please send us your thoughts and reflections. Include information about how you posed the problem and samples of students' work, or even photographs showing your problem solvers in action. Send your results with your name, grade level, and school by **January 1, 2006,** to Barbara Britton, Mathematics Department, Eastern Michigan University, Ypsilanti, MI 48197. Selected submissions will be published in a subsequent issue of *Teaching Children Mathematics* and acknowledged by name, grade level, and school unless otherwise indicated. ▲

Solutions to the Winning Strategy Problem

The Winning Strategy problem that appeared in the November 2004 issue of *Teaching Children Mathematics* read as follows:

Anthony has a winning strategy. He always wins when he plays this game! Here's how to play the game with Anthony:

> Start with 14 counters or game chips.
> Take turns picking up one or two counters at a time.
> Whoever takes the last counter wins.
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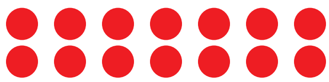
> 
>
> Can you figure out Anthony's strategy for winning?

Playing this game was fun and it was easy to learn; however, finding a winning strategy turned out to be quite a challenge. Students in Kristi Krieg's class at Edna Louise Spear Elementary School in New York played the game in pairs. When most of the students felt they had a strategy, they switched partners so they could test their "winning strategy" on someone else. Finally, they got back together with their original partner to discuss their strategies and solicit feedback. Kristi Krieg said that after the class came together to discuss strategies that the children felt were effective, the students wrote a paragraph to explain their thinking.

Alexandra explained her "aha" moment for determining the winning strategy for this game:

> The strategy that I used was not that simple. What you do is plan ahead. If there were four counters left you would take one because the person who ends up with three counters loses. No matter if they take two or one, they still will lose.

Patrick also figured out that you really want to avoid putting yourself into a situation where there are three counters remaining and it is your turn. Patrick explained that in some circumstances, you may need to count on luck:

> [If there are three counters left] then you would have thought you lost, but here's what to do. Pick one so there are two left. Then your partner would be so surprised he might stop thinking clearly and pick one.

Gabe, another classmate, added the following insight into determining a winning strategy:

> If they pick one, you pick two, or if they pick two, you pick one. Then you figure out a way (depending on what they pick) to make it their turn when there are three counters left.

It was their friend Connor, however, who explained that it is essential to go first. He summed up a surefire way to win at this game:

> If you go first, you can take two; then if he or she takes one, you take two, and if they take one and then you take two and you win.

That really is the key; you want three counters remaining when your opponent has his or her last turn. This guarantees a win because whether the other player takes one or two counters, you are able to pick up the remaining and last counters. When you play first and remove two counters, twelve counters (a multiple of 3) remain. A strategy is to ensure that the number of counters on the board is always a multiple of 3. Hence, if your opponent removes one counter, you should remove two. If

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your opponent removes two counters, you should remove one. In other words, do just the opposite of your opponent. Continue until only three counters remain and it is your opponent's turn. No matter what the other player does, you are guaranteed a win.

Kristi Krieg wrote that many of the students realized that different strategies worked depending on whether they went first or second, and whoever had three counters left on his or her turn would lose. The hardest part of this problem for her students was explaining their strategies in writing.

Mary Kay Varley from Fort Worth Country Day School introduced this problem to her students by playing the game on the overhead. The students tried to beat the teacher. After they failed to beat her, Varley explained that a winning strategy is a method of winning each and every time; winning does not have to be the result of luck or someone else's mistake. She told the class that she had figured out a winning strategy. She also explained that some games give an advantage to the player who goes first, and in other games, the second player may have an advantage. She told the students to think about this as they tried to figure out a strategy for winning.

Varley observed her students as they worked to find a winning strategy. Several of the students noticed that once there were three counters left, one person was guaranteed a win. Occasionally, the class would pause to share new insights. This helped motivate other students to persevere on the problem. Other students watched for clues as Varley played, trying to figure out her secret. They looked for patterns in the way she was removing the counters. Students were encouraged to explain in their own words a winning strategy for the game. For homework, students were to play Anthony's game at home and see if their parents could figure out their strategy.

The Winning Strategy problem is just one version of a family of similar NIM games. A variation, sometimes referred to as classic NIM, was also posed in the November 2004 issue of *Teaching Children Mathematics.* To play classic NIM:

> Arrange the counters in three rows with three in one row, four in a second row, and five in a third row.

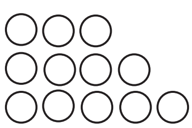


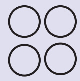
Figure 1

One, one, one trapping move

Scenario: If my opponent takes one, then I take one from another row and my opponent takes the final one. Opponent loses.

Figure 2

Two, two trapping move

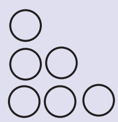


Scenario A: If my opponent takes one from either row, then I remove the entire other row of two. My opponent takes the remaining piece and loses.

Scenario B: If my opponent takes an entire row of two, then I remove one from the remaining row. My opponent takes the remaining piece and loses.

Figure 3

Andrew's one, two, three trapping move



Scenario A: If you take the entire row of one, I take one from the row of three, making the two, two trap.

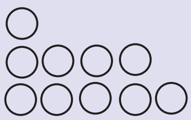
Scenario B: If you take the entire row of three, I take the entire row of two, leaving you with the last counter.

Scenario C: If you take the entire row of two, I take the entire row of three, leaving you with the last counter.

Scenario D: If you take one from the row of two, I take two from the row of three, making the one, one, one trap.

Figure 4

One, four, five trapping move



Scenario A: If opponent takes one from the top row, I should take one from the row of five to get to the four, four trap.

Scenario B: If opponent takes one from the row of four, I should take three from the row of five to get to the one, two, three trap.

Scenario C: If opponent takes two from the row of four, I should take two from the row of five to get to the one, two, three trap.

Scenario D: If opponent takes three from the row of four, I should take four from the row of five to get to the one, one, one trap.

Scenario E: If opponent takes the entire row of four, I should take the entire row of five.

Scenario F: If opponent takes one from the row of five, I should take the row of one to get to the four, four trap.

Scenario G: If opponent takes two from the row of five, I should take two from the row of four to get to the one, two, three trap.

Scenario H: If opponent takes three from the row of five, I should take one from the row of four to get to the one, two, three trap.

Scenario I: If opponent takes four from the row of five, I should take three from the row of four to get to the one, one, one trap

Scenario J: If opponent takes all the row of five, I should take all the row of four.

Take turns taking as many counters as you want, but from only one row. (You can take an entire row if you want.) Whoever takes the last counter loses.

Frank Hatcher from Upper Arlington, Ohio, wrote that variations of NIM have been a personal favorite in his more than thirty years of teaching. His students love the game as well. He starts by playing a few games with students. Next, the students play a number of games with their peers. Homework for the week is to play NIM with their families and to try to come up with a trapping move. A trapping move is a move that leaves the opponent in a position that is impossible to win. **Figures 1** and **2** show examples of trapping moves. The strategy is to build on previously determined trapping moves. Students look for new trapping moves by causing their opponent to fall into a known trap. Hatcher explained to the class that one problem-solving strategy is to "look at a simpler problem." As students try to find a winning strategy for classic NIM, they break the problem down into a simpler problem by looking for trapping moves. After a week, students share their trapping moves with the class. The defending student is challenged to clearly show that he or she has found a genuine trapping move. These trapping positions, along with the names of the students who devised them, become a display on the classroom wall. In **figure 3,** Andrew explains the one, two, three trap.

After the students created the trapping moves, Hatcher's class determined that to win the classic NIM game, a winning strategy is to go first and to remove two counters from the top row of three, so that your opponent falls into a one, four, five trap (see **fig. 4**). Hatcher wrote that his students become experts at playing NIM using the trapping moves and love to challenge parents and visitors to a game.

A special thanks to those teachers and students who made contributions:

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Kristi Krieg, Edna Louise Spear Elementary School, Port Jefferson, New York
Mary Kay Varley, Fort Worth Country Day School, Fort Worth, Texas ▲

Correction

In the August 2005 issue of *TCM*, page 8, the scoring rubric for the Contig game illustrated in figure 2 was incorrect. The corrected information is below. We apologize for this error.—*Ed.*

In ***figure 2,*** *for example, 3, 6, and 19 are worth 2 points; 4, 5, and 22 are worth 3 points; and 20 is worth 4 points.*