Glenn Gebhard

A Mathematical Cornucopia of Pumpkins

The "Investigations" department features children's hands-on and minds-on explorations in mathematics and presents teachers with open-ended investigations to enhance mathematics instruction. These tasks invoke problem solving and reasoning, require communication skills, and connect various mathematical concepts and principles. The ideas presented here have been tested in classroom settings.

A mathematics investigation—

- has multidimensional content;
- is open ended, with several acceptable solutions;
- is an exploration requiring a full period or longer to complete;
- is centered on a theme or event; and
- is often embedded in a focus or driving question.

In addition, a mathematics investigation involves processes that include—

- researching outside sources;
- collecting data;
- collaborating with peers; and
- using multiple strategies to reach conclusions.

Although this department presents a scripted sequence and set of directions for an investigation in this particular classroom, *Principles and Standards for School Mathematics* (NCTM 2000) encourages teachers and students to explore multiple approaches and representations when engaging in mathematical activities. This investigation centers on measurements of pumpkins or other seasonal fruits, the appropriate use of measurement tools, and the relationship among the measured attributes.

NCTM's Algebra Standard asks students to represent, analyze, and generalize a variety of patterns with tables and graphs. It also expects them to model and solve contextualized problems using various representations, such as graphs, tables, and equations. The Measurement Standard goals addressed in this investigation include understanding measurable attributes of objects and the units, systems, and processes of measurement; understanding relationships among units; and converting from one unit to another. Most important, this investigation offers students practice in selecting and applying techniques and tools to accurately find length, mass, and volume to appropriate levels of precision.

NCTM's Data Analysis and Probability Standard asks students to formulate questions that can be addressed with data and to collect, organize, and display relevant data to answer them. Graphing—in particular, making scatter plots of the data—is also an important part of this investigation. Students make conjectures about relationships between two attributes on the basis of the scatter plots, use those conjectures to formulate new questions, and then plan new explorations to answer them.

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Edited by **Cornelis de Groot**, *degrootc@newpaltz.edu, State University of New York, New Paltz, NY 12561. This section is designed for teachers who wish to give students in grades K-6 new insights into familiar topics. Classroom teachers may reproduce this material for use with their own students without requesting permission from the National Council of Teachers of Mathematics. Readers are encouraged to submit manuscripts appropriate for this section by accessing* **tcm.msubmit.net**.

The Investigation

Learning goals

This four-day investigation is an opportunity for students to measure attributes of a pumpkin. They then analyze these data to see if they can identify relationships or trends. Connecting measuring activities with data analysis allows students to address the natural connections between mathematics and scientific activities. In these activities, students practice the use of mathematics process

Figure 1

Students using the "book" method to measure their pumpkin's height

skills, such as mathematical representation, as well as their science process skills, such as inferring and predicting. Measurement and data are the ripest content strands to make such connections. This content is the core of modern curriculum structures. At the heart of this investigation is the opportunity for students to gain confidence in asking their own questions and using mathematical tools and analysis to answer these questions. This investigation incorporates the use of spreadsheets for the purpose of collecting and representing data; a copy of this file may be obtained from the author via e-mail at ggebhard@hornell.wnyric.org. You can organize the entire investigation so that the students work with data and graphs on paper, although you may need to allot some additional time to accomplish this.

The activities in this investigation were used in grades 3, 4, and 6 mathematics and science classes in the Hornell City School District, Steuben County, New York, in the Finger Lakes region. The sixth graders were able to pose slightly more sophisticated questions, such as "How does a pumpkin's circumference relate to its mass?" and "Given a pumpkin's circumference, can you predict its mass, and vice versa?" The fourth graders did not tackle the issue of volume. They measured equatorial circumference, not polar circumference. Consequently, they did not concern themselves with such volume-related questions as "How round is this pumpkin?"

Objectives of the investigation

The students will—

- use appropriate tools to measure the mass of a pumpkin and the mass of its seeds as well as its volume, equatorial circumference, polar circumference, diameter, and height;
- use appropriate counting methods to determine the number of seeds in a pumpkin;
- pose and investigate questions about the data and the relationships between some of the measured attributes;
- use Excel spreadsheets (or other spreadsheet software) to draw scatter plots of their data to analyze relationships among variables; and
- discover the smallest box or cube into which their pumpkin will fit.

Materials

All materials are for groups of 3 or 4 students, unless indicated otherwise.

Lesson 1

- one pumpkin or other seasonal fruit that has some variability in size (e.g., watermelon, orange, grapefruit, etc.)
- a metric measuring tape and centimeter ruler
- one data table per student (**table 1**)
- a pan balance, gram mass set, and extra kilogram masses as needed (see **table 2** for a range of masses needed)

Table 1

Template for Sixth-Grade Data Table for Lesson 1

Note: For use with other grades, eliminate the polar circumference column.

Pumpkin Number	Mass (g)	Volume (cc)	Equatorial Circumference (cm)	Polar Circumference (cm)	Diameter (cm)	Height (cm)	Seed Count	Seed Mass (g)
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
mean								
median								

Table 2

Completed Sixth-Grade Data Table for Lesson 1

Pumpkin Number	Mass (g)	Volume (cc)	Equatorial Circumference (cm)	Polar Circumference (cm)	Diameter (cm)	Height (cm)	Seed Count	Seed Mass (g)
1	5100	7420	72	83	18	29	738	86.5
2	2505	2950	48	62	23	23	735	62.8
3	4200	6700	70	73	23	23	419	93.5
4	2000	1680	49	49	16	14	437	76.7
5	6500	7500	73	81	23	26	739	96.0
6	5500	6450	67	70	28	37	914	63.0
7	5900	6800	76	88	24	27	523	97.7
8	1445	1400	48	47	16	14	469	59.6
9	2550	3400	59	60	19	17	589	109.1
10	6890	10200	77	85	25	31	676	173.9
11	4270	7500	77	79	24	24	344	74.5
12	3035	4500	64	64	20	22	497	109.0
13	1569	2770	60	54	18	13	756	80.0
14	2181	2830	62	55	20	26	668	101.0
15	7600	10800	77	93	26	34	920	191.5
16	11000	15540	94	101	29	34	990	139.0
mean	4515	6153	67	72	22	25	651	101
median	4235	6575	68.5	71.5	23	25	672	95

For the students—
- a metric or customary spring scale (or bathroom scale)
- Earth globe

For the teacher—
- chart paper and markers

Lesson 2

For the students—
- a large supply of interlocking centimeter cubes
- one 5-gallon bucket with a 1/2-inch hole drilled 2 inches from the top rim
- a long plastic tray wide enough to allow the bucket to fit (i.e., deep enough to hold several liters of water)
- a 1-liter graduated cylinder
- a plastic 1-liter box (a cubic decimeter)
- a cubic meter made either from 12 triangular meter sticks and plastic corner connectors or from 1-meter pieces of lath fastened together (optional)

Lesson 3

For each group of students—
- newspapers and paper towels

For the teacher—
- a knife or pumpkin carver set

Lesson 4

For the students—
- Excel spreadsheets

Previous knowledge

Students should know how to use a ruler and centimeter tape to measure their pumpkin's length, diameter, height, and circumference to the nearest centimeter. They also must be able to use a balance to measure mass to the nearest gram or nearest 50 grams, depending on the measuring device used, and read a graduated cylinder to the nearest milliliter.

Figure 2

Students using the "book" method to measure their pumpkin's diameter

Most of the teachers in our school district use a variety of AIMS (Activities Integrating Mathematics and Science) materials. Previous knowledge and skills have been developed, in part, through the use of these activities. For all these activities, we begin with real-world models, something that can be manipulated. Then, by counting or measuring, we get both usable data and a new level of abstraction. When we graph the data, we see their pictorial form. Next, we make generalizations and seek formulas. In a perfect lesson, students would then pose a new question to lead back to the real world. They can then tackle the new problem with the skills practiced earlier.

The students in our district have had collective experience in this approach to science and mathematics. Following this model of instruction, they have answered such questions as "How much of a banana is edible?" and "How much sugar is in this gum?"

Lesson 1

Posing interesting questions and making measurements

This initial lesson focuses on the question "What measurements can we collect regarding these pumpkins?" The initial brainstorming session revealed that we can measure the pumpkins' mass, weight, volume, circumference, diameter, and height. Because one anticipated interesting question may be "How round is your pumpkin?" the students should measure both equatorial and polar circumferences. The closer these measurements are to each other, the more spherical is the pumpkin. Use a globe to demonstrate.

Finding the mass of the seeds was an idea that I had to coax from the students by asking, "Does a heavier pumpkin have heavier seeds?" During the brainstorming session, have the students record on chart paper all their ideas about measuring pumpkins, even ideas that will not be part of this investigation. Some of these ideas included measuring stem length, temperature, wall thickness, and the time it takes for a pumpkin to fall to the ground and smash when dropped from a window or the time it takes for a pumpkin to decompose. As with any brainstorming activity, the quantity of ideas is prized, so try to maintain an infectious frenzy of responses for as long as possible. The chart paper with these ideas stays up in the classroom for the entire investigation (the list will be used again in lesson 4).

When the students have exhausted all ideas for measuring pumpkins, a new question that you can pose—"What can we count?"—leads naturally to the idea of counting seeds. Accept all ideas relating to counting and record them all on the chart paper. When you return to this list in lesson 4, you can choose the questions to be investigated or have your students help choose these. In this investigation, I moved the students in the direction of the "interesting questions" discussed here.

At this point, briefly discuss mass and weight with the students. For the purposes of this specific investigation, the differences between the two are not important, although students must realize that mass and weight are not the same. Mass is the amount of matter in an object and is measured in grams using a balance. Weight is the amount of gravitational pull on an object and is measured in gram-force units by using a spring scale. Weight decreases as altitude increases, disappears entirely in free fall (or in orbit), and is less on the moon than on Earth. In contrast, mass does not change with any of these conditions.

In this investigation students should, when possible, measure length to the nearest centimeter and measure mass to the nearest gram. For smaller pumpkins use a pan balance to measure mass, and for larger pumpkins use a spring scale or a bathroom scale. Note that spring and bathroom scales technically measure weight, but we assume that gravity will not change during these measurements, so we can set the weight to its mass equivalent. The floor-type spring scale that the learners in this investigation used had its smallest markings at 1/2 kilogram

(500 grams). Because one can generally estimate a scale to 1/10 the size of the smallest graduation, our accuracy was to the nearest 50 grams.

To measure the pumpkin's height, have the students set the pumpkin on a flat surface, such as a desk or table. Next to the pumpkin they should position a ruler on and perpendicular to this flat surface. Placing a book on top of the pumpkin may help the students read the ruler measurement more accurately (see **fig. 1**). When measuring diameter, have the students place their pumpkin on a ruler between two notebooks or textbooks held perpendicular to the desk or table (see **fig. 2**). Placing the end of one book on the zero mark eliminates the need for subtraction. Remove the book at the zero mark, then the pumpkin. The remaining book's position on the ruler will indicate what the diameter is. Of course, if a group of students has another valid way to measure the pumpkin, let them defend it first and then demonstrate it!

Have the students, working in pairs, record their data and share them with other groups. One group member can record the data on a class chart and type it into the prepared spreadsheet (see **table 2**). Entries are made directly into each cell on the data sheet of the Excel file. Make sure that students enter only numbers and not the units, which are indicated in the heading of each column (spreadsheet programs will not recognize the value of a number if units are included). Also, make sure that the students enter data only on the data sheet of the Excel workbook. The data sheet is linked to all the other sheets; thus, changes on this sheet will automatically update the information on all other graphs drawn by the program.

Go back to the suggested measurement ideas that the class listed earlier on the chart paper and have the students generate a question for each one. For example, for the suggestion about measuring a pumpkin's volume, the question could be "How much space does this pumpkin take up?" These will become our "interesting questions." Interesting questions include (1) something we do not know and (2) something we can find out using our data.

Students were asked to pose five interesting questions about pumpkins. They were allowed to work alone or together. We compiled the questions at the beginning of lesson 2.

During this time, various themes emerged in the students' questions. Many focused on life science questions about the pumpkins themselves and their various structures (e.g., stem, seeds, exterior skin, etc.). Another group discussed physical science questions such as "How much time does it take for a pumpkin to smash on the ground?" and extended that line of questioning. This led, naturally, to my question "Do larger pumpkins fall faster than small ones?" Such is the richness of an inquiry-based lesson.

Lesson 2

Measuring volume and cubing pumpkins

This lesson consists of two different investigations that can be set up as stations for the students to rotate to in small groups or pairs. For the volume station, prepare the bucket and tray before the lesson. The hole near the top of the bucket allows the water to fill the bucket without spilling over the rim, an important consideration when submerging each pumpkin. Each pumpkin will need to fit inside the bucket, so select small pumpkins for measuring. (Finding the volume by displacement of very large pumpkins is a challenge in a typical classroom.) The displaced water will pour out through the hole into the tray. The students then empty the water in the tray into the graduated cylinder for measurement (**fig. 3**).

Begin this lesson by reading "Archimedes, The Greek Streaker" from *Historical Connections in Mathematics* (Reimer and Reimer 2002), an account of how Archimedes solved the question of whether King Heron's crown was made of pure gold or an inferior material. According to legend, Archimedes, while sitting in a bath, noticed that he displaced some water; he further realized that

Figure 3

Students submerging their pumpkin and collecting the displaced water

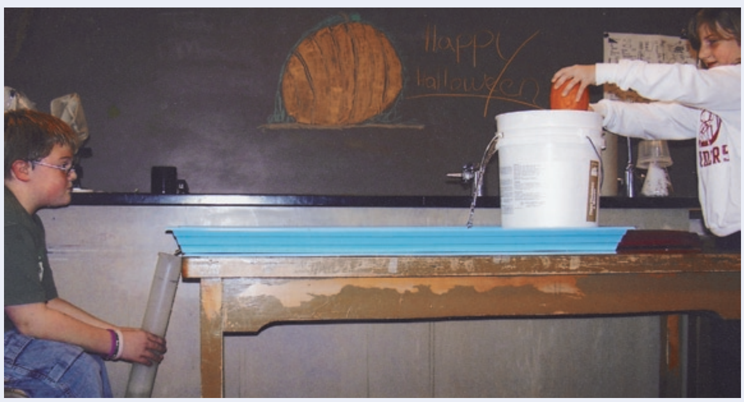


Figure 4

Students with their completed cubic frame

Figure 5

Students measuring the circumference of their pumpkin

the displaced volume was equivalent to the volume of his body. As a result of this discovery, he submersed an object of pure gold in water and then submersed the crown so that he could compare the volume of the displaced water and determine whether the crown was authentic.

Tell the students that they will apply Archimedes' laws of density and buoyancy by collecting the water their pumpkin displaces and then measuring it by using the graduated cylinder. Demonstrate that one liter of water in the graduated cylinder will precisely fill the liter box. Make sure the liter box is free of soap, which will cause the surface tension of the water to break and possibly allow a little to leak from the top of the box. For students who do not have a well-developed sense of conservation of volume, this experiment will be an eye-opener.

Make the connection that the liter box will hold 1000 cubic centimeters (10 cm × 10 cm × 10 cm) and that a cubic meter will hold 1000 liter boxes (again, 10 dl × 10 dl × 10 dl). The cubic meter will also hold one million cubic centimeters (100 cm × 100 cm × 100 cm). This minilesson in scale and dimension provides a new way to visualize the concept of one million.

Taking turns, the students in each group fill the bucket until the water just overflows the hole, empty the tray, and then slowly submerge the pumpkin. Most pumpkins float, so the students must submerge them gently, making sure that the stem and their fingers are not under water. Water will pour from the hole into the tray. When no more water flows out, the students remove the pumpkin, empty the water in the tray into the graduated cylinder and measure, to the nearest milliliter, the displaced water that has accumulated. One milliliter of liquid volume is equivalent to one cubic centimeter of solid volume. Thus this method allows us to determine the volume of the pumpkins. Have the students record their data and share them with other groups by using the class chart and spreadsheet.

Students who are not working at the volume station should be constructing the smallest cube frame that their pumpkin will fit into. Using interlocking centimeter cubes, they should construct a square or cube-shaped framework, beginning around the base of the pumpkin and then, being careful to keep the edge length constant, building the sides and top of the square (**fig. 4**). A real-world context for this problem would be shipping a pumpkin in a cubic box and finding the smallest box possible to minimize shipping cost.

Discussion of the "interesting questions" is an important ongoing part of the investigation. Most questions that students generate are simple measurement questions. After all have been recorded on the chart, ask a new question (if the students do not pose such questions themselves) that moves into the realm of measuring between variables. Use this general formulation: "If I know a pumpkin's ___, can I predict ____?" For example, you could ask, "If I know a pumpkin's circumference, can I predict its diameter?" The students can ask and record this type of questioning in their small groups and then, as a class, pull the questions together into a chart. Alternatively, you could have the students generate these questions for homework.

As the students worked on this task, one rather quiet, thoughtful girl came up with her data sheet and asked, "If we have all these columns filled in, can't we ask questions about any two of them?" The answer, of course, was, "Yes, I suppose we can. How many questions would that be?" She went away to ponder the solution, not realizing that she was pondering combinations.

Lesson 3

Counting seeds and looking for relationships

Take some time to discuss what a "typical" pumpkin is like. Making sure that students think about a "typical" pumpkin sets up the transition to examining the relationships among the pumpkin's various measured attributes. After the students have measured each pumpkin's mass, have them record the measurement on the pumpkin itself with an indelible marker. Then have them arrange the pumpkins in a line from least mass to greatest and also identify, according to mass, the median pumpkin. Repeat this procedure to illustrate other attributes. As students are taking other measures, they can record these on the pumpkins as well. At this time, they might also find it useful to calculate the mean and the median as well as the mode of the data set or construct an empirical sampling distribution and then analyze these data.

Before cutting open any pumpkins, check any student measurements that appear to be too far off. For example, make sure that the circumference is roughly three times the diameter or height. Although no pumpkin is perfectly round in all three dimensions, finding the ratio of circumference to diameter is a good way to recheck measurements (see **fig. 5**). A worksheet with the spreadsheet file Checker (see **table 3**) calculates the accuracy of students' measurements. It allows for the circumference-diameter ratio to vary ±25 percent from π; otherwise, it flags this ratio in red. Green indicates an acceptable measurement; recheck red ones before dissection during lesson 3.

Review the questions being investigated today: "How many seeds are in this pumpkin?" and "Do larger pumpkins have more seeds than smaller pumpkins?" The student groups should discuss and agree on a strategy for counting seeds. A class discussion about the various methods is useful after each group has decided on a strategy. Most of the groups participating in this investigation chose to collect the seeds in piles of 10, 20, 25, or 50.

Slice off the top of each pumpkin, and let the fun begin! (Have plenty of newspapers and paper towels on hand for this lesson!) Earlier, when I asked the students for a prediction, almost everyone thought that the smallest pumpkins would have the least number of seeds. As we cut open one of the smallest, everyone was amazed at the concentration of seeds and the lack of airspace; the larger, taller pumpkins contained mostly air. As the counting progressed, it became clear that larger pumpkins do not tend to have more seeds than smaller pumpkins.

Table 3

Checker Spreadsheet (Excel)

Pumpkin Number	Equatorial Circumference (cm)	Diameter (cm)	Ratio	Polar circumference (cm)	Height (cm)	Ratio
1	72	18	4.00	83	29	2.86
2	48	23	2.09	62	23	2.70
3	70	23	3.04	73	23	3.17
4	49	16	3.06	49	14	3.50
5	73	23	3.17	81	26	3.12
6	67	28	2.39	70	37	1.89
7	76	24	3.17	88	27	3.26
8	48	16	3.00	47	14	3.36
9	59	19	3.11	60	17	3.53
10	77	25	3.08	85	31	2.74
11	77	24	3.21	79	24	3.29
12	64	20	3.20	64	22	2.91
13	60	18	3.33	54	13	4.15
14	62	20	3.10	55	26	2.12
15	77	26	2.96	93	34	2.74
16	94	29	3.24	101	34	2.97

The ratios will be automaticallly filled in as the measurement data are entered into this spreadsheet.
Note: When the number in the ratio columns turns red, it is advisable to remeasure the associated circumference and height. The color red means that the ratio is off by more than 25 percent.

The students who predicted that the larger pumpkins would have the most seeds retorted that at least their seeds were bigger than the others. This led to another interesting question: "Do larger pumpkins have larger seeds?"

When the students have finished counting the seeds and recording the results on the class data table and in the Excel spreadsheet (see **table 1**), have them spread the seeds out evenly on paper towels to dry overnight. (To avoid confusion, label the towels with group names or numbers.) We are interested not in wet seed mass but in dry seed mass—the difference can be significant. The next day have the students find the mass of the dry seeds.

Lesson 4

Exploring interesting questions with scatter plots

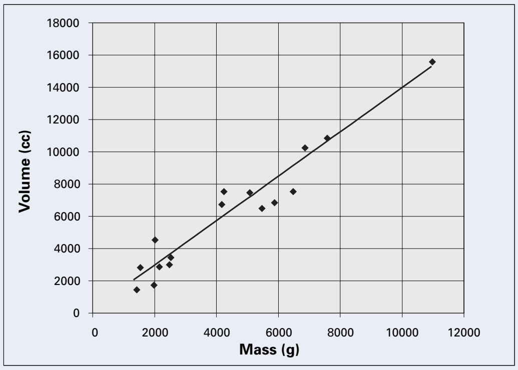
Investigations such as this one need to have a general direction but must be flexible enough to

Figure 6

Mass-volume spreadsheet and scatter plot

Mass (g)	Volume (cc)
5100	7420
2505	2950
4200	6700
2000	1680
6500	7500
5500	6450
5900	6800
1445	1400
2550	3400
6890	10200
4270	7500
3035	4500
1569	2770
2181	2830
7600	10800
11000	15540

Do more massive pumpkins have greater volume?



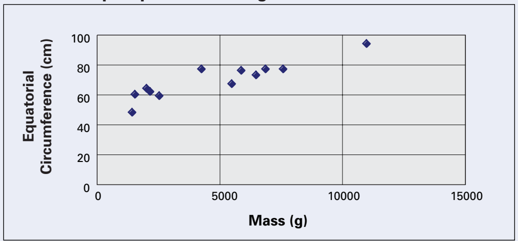
The data in this scatter plot will be automatically filled in as the mass and volume data are entered into the spreadsheet.

Figure 7

Mass–equatorial circumference spreadsheet and scatter plot

Mass (g)	Equatorial Circumfer-ence (cm)
5100	72
2505	48
4200	70
2000	49
6500	73
5500	67
5900	76
1445	48
2550	59
6890	77
4270	77
3035	64
1569	60
2181	62
7600	77
11000	94

Do heavier pumpkins have larger circumferences?



The data in this scatter plot will be automatically filled in as the mass and equatorial circumference data are entered into the spreadsheet.

follow students' inquiries and thus retain authenticity. Students will certainly generate more questions than could ever be researched in the mathematics or science classroom. With careful guidance, however, their questions can all be examined for merit and listed on chart paper. This process gives all participants ownership in the lessons, an important goal in maintaining motivation in both mathematical analysis and scientific inquiry.

Seeing trends in a data table can sometimes be difficult, but graphs such as scatter plots are wonderful and appropriate tools to quickly spot trends (or the lack thereof). This lesson takes time to prepare because it is based on graphs and scatter plots generated by spreadsheets. Decide ahead of time which relationships to discuss with the class, taking into account the students' grade level and interest. You may need to modify the accompanying spreadsheets to fit your data. All tables and charts are linked to the data sheet, so make all changes to your class data there. A list of the linked spreadsheets and their functions follows:

- *Mass–volume spreadsheet* (**fig. 6**)—graphs the relationship between the pumpkin's mass and its volume; answers the questions "Do more massive pumpkins have greater volume?" and "Can we predict a pumpkin's volume given its mass?"
- *Mass–equatorial circumference spreadsheet* (**fig. 7**)—graphs the relationship between mass and equatorial circumference; answers the questions "Do heavier pumpkins have larger circumference?" and "Can we predict a pumpkin's circumference given its mass?"
- *Mass–seed count spreadsheet* (**fig. 8**)—graphs the relationship between pumpkin mass and seed count; answers the question "Do heavier pumpkins have more seeds?"
- *Mass–seed mass spreadsheet* (**fig. 9**)—graphs the relationship between the pumpkin's mass and the collected seeds' mass; answers the question "Do heavier pumpkins have a heavier collection of seeds?"
- *Roundness spreadsheet* (**fig. 10**)—the conditional formatting of this spreadsheet graphs the relationship between polar circumference and equatorial circumference. Using "if-then" statements, it identifies tall, skinny pumpkins (those with a difference of less than 5 cm), short fat pumpkins (those with a difference greater than 5 cm), and round pumpkins (those with a difference of about 5 cm) and colors the data red, yellow, and green, respectively.

When the students were discussing whether heavier pumpkins had greater volume, their unanimous intuitive and unsurprising prediction was yes. When they saw the graph of the two variables (**fig. 7**), however, several "ahas"—indicating leaps of understanding—resounded from the group. Clearly, the visual learners in the class could grasp the data when transformed into pictorial form. This example was a good first look at scatter plots. The ordered pairs lined up nicely on the graph, and although there were some data points above and below the trend line, it was easy for the students to see that these two variables were related. One student remarked that mass and volume must be related because the dots all lined up so well.

The mass-circumference scatter plot (**fig. 7**) showed a noticeable tendency to form a general trend but not as strongly as the volume-mass connection did. One student commented that he could see where a line could go but that it was more spread out than the previous graph. I encouraged this kind of qualitative reading of the graph, considering that these students are 11 and 12 years old.

When the students plotted the data, most saw clearly whether a strong correlation between variables existed. How the class reacted to the seed count–pumpkin mass data and chart (**fig. 8**) was equally important. These points are very spread out, so much so that a student noted, "It's hard to see a line here. I don't see how these two things are related!" Out of the mouths of babes—it turns out that the mass of a pumpkin is a poor predictor of the number of seeds it contains.

Beyond the Lesson

The pumpkin seeds can be dried and stored until spring. Some of the third graders planted seeds in May, watched them germinate, and then took them home to grow over the summer. Then, in the fall, when they were fourth graders, they brought back a pumpkin! Taking the seed through the entire life cycle satisfies a number of science standards (Rutherford and Ahlgren 1991). It is also a fun family project for the summer. To encourage good habits of mind while the plants grew, we supplied the students with observation and measurement sheets.

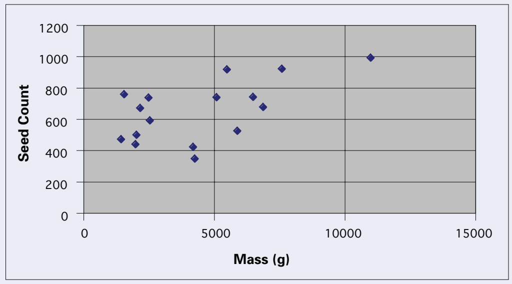
If you are working with students who have discovered π, the volume formula for a sphere—$V = (4/3)\pi r^3$—has meaning here. On the most spherical pumpkins, the cube edge length will be the same as the diameter of the pumpkin. Half this length, of course, is the radius of the pumpkin.

Figure 8

Mass–seed count spreadsheet and scatter plot

Mass (g)	Seed Count
5100	738
2505	735
4200	419
2000	437
6500	739
5500	914
5900	523
1445	469
2550	589
6890	676
4270	344
3035	497
1569	756
2181	668
7600	920
11000	990

Do heavier pumpkins have more seeds?



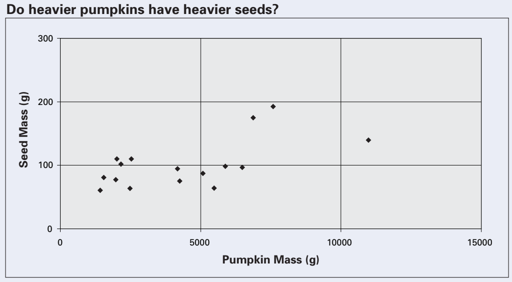
The data in this scatter plot will be automatically filled in as the mass and seed count data are entered into the spreadsheet.

Figure 9

Mass–seed mass spreadsheet and scatter plot

Mass (g)	Seed Mass (g)
5100	86.5
2505	62.8
4200	93.5
2000	76.7
6500	96.0
5500	63.0
5900	97.7
1445	59.6
2550	109.1
6890	173.9
4270	74.5
3035	109.0
1569	80.0
2181	101.0
7600	191.5
11000	139.0

Do heavier pumpkins have heavier seeds?



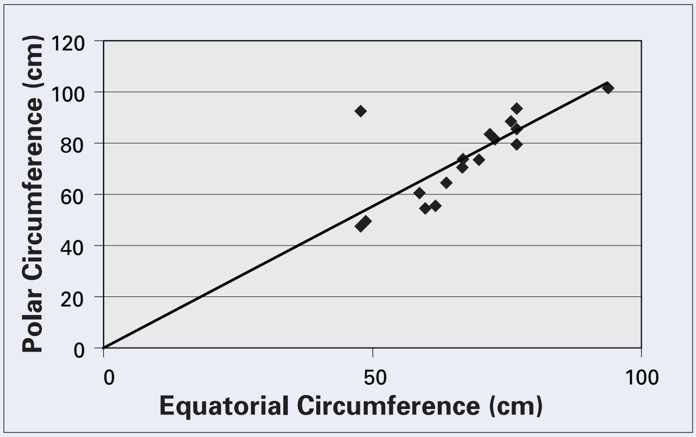
The data in this scatter plot will be automatically filled in as the mass and seed mass data are entered into the spreadsheet.

Figure 10

Roundness spreadsheet and scatter plot

Pumpkin Number	Equatorial Circum-ference (cm)	Polar Circum-ference (cm)	Equatorial Circumfer-ence–Polar Circum-ference (cm)
1	72	83	-11
2	48	92	-44
3	70	73	-3
4	49	49	0
5	73	81	-8
6	67	70	-3
7	76	88	-12
8	48	47	1
9	59	60	-1
10	77	85	-8
11	77	79	-2
12	64	64	0
13	60	54	6
14	62	55	7
15	77	93	-16
16	94	101	-7

How round is your pumpkin?



The data in this spreadsheet will be automatically filled in.

Legend
- A short, fat pumpkin (data point falls below the line of best fit)
- A round pumpkin (data point falls on the line of best fit)
- A tall, skinny pumpkin (data point falls above the line of best fit)

Compare the measured volume (by displacement) with the calculated volume (by formula).

Reflections

For investigations in mathematics and science to engage students, they must be fun. Our pumpkin mathematics project incorporated a spirit of wonder and inquiry and met our goal of authentic practice in measurement. Spurring children to ask interesting questions, to arouse their curiosity in a thoughtful way, and to assist them in analyzing data were all complimentary components of this unit.

The students were enthusiastic about using the authentic data they themselves had gathered about their group's pumpkin. They made huge leaps in the way that they thought about how a pumpkin could be measured and, on the basis of their data analysis, what could be predicted about another pumpkin. This investigation clearly showed that, although most students had predicted the opposite, larger pumpkins do not produce more seeds; indeed, some of our smallest pumpkins had the most seeds. As the class was discussing our interesting questions, one student declared, "I don't know about that, but I bet we can find out!" The spirit of inquiry evidenced by these elementary school students is common to all scientists and mathematicians.

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