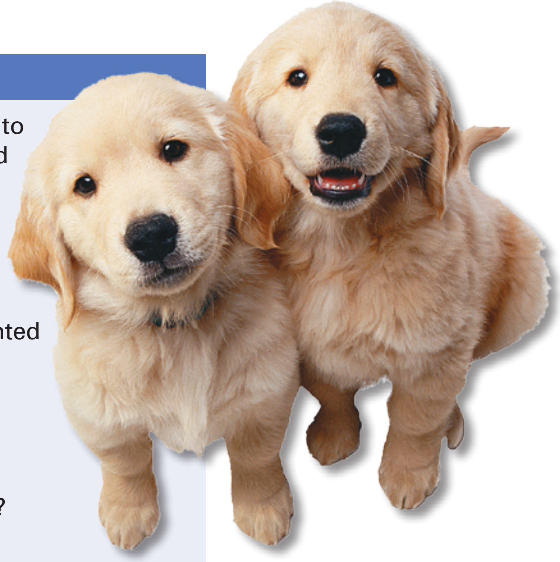
# Walk for the Paws



## Problem

The Orange Beach Animal Shelter holds a Walk for the Paws event every year to raise money for dog care and adoptions. People sign up to walk three miles and may bring their dogs with them. The cost is $15 per person and $5 per dog. Every walker gets a "Walk for the Paws" shirt (for people) or a collar (for dogs).

This year's event is a big success. At the finish line, Evan and Crystal are helping count participants. Evan is told to count each person and each dog. Crystal is told to count the number of legs that cross the finish line. When everyone has finished, Evan has counted a total of 1,000 people and dogs, and Crystal has counted 2,500 legs. (Assume that each person had two legs and each dog had four legs.)

On the basis of information above, answer the following questions:

1. How many people participated in the Walk for the Paws?
2. How many dogs participated in the Walk for the Paws?
3. How much money did the Orange Beach Animal Shelter raise from this event?
4. How could next year's Walk for the Paws raise $15,000?

**Variations**

Younger students may be given more manageable numbers to work with by using 100 people and dogs and 250 legs. Also suggest that they explore a similar but easier situation with pictures to get a sense of how the number of people and dogs is related to the number of legs. Omit question 4 for younger students.

The goal of the "Problem Solvers" department is to foster improved communication among teachers by posing one problem each month for teachers of grades K–6 to try with their students. Every teacher can become an author: Pose the problem to your students, reflect on your students' work, analyze the classroom dialogue, and submit your resulting insights to this department. Through contributions to the journal, every teacher can help us all better understand children's capabilities and thinking about mathematics. Remember that even students' misconceptions provide valuable information.

## Classroom Setup

Unfortunately, millions of dogs are lost or abandoned each year and end up in one of over 5,000 animal shelters in the United States. The shelters rely on volunteers and donations to help care for a large number of animals and try to arrange adoptions. Walk for the Paws and the names in this problem are fabricated, but many animal shelters sponsor events just like this to raise money, awareness, and volunteer support. An Internet search of "Walk for the Animals" will bring up thousands of results.

Generate student interest by first discussing pets (specifically, dogs) that the students own or know. Let students know that, according to the Humane Society of the United States, over 73 million households own dogs in the United States and about 40 percent of all households have at least one dog. Tally how many students in the class have a dog. Have older students figure out what percentage of the class owns dogs.

Discuss the problem statement with students but avoid giving too much guidance. Allow students to work with a partner or in small groups. Young children who understand the concept of multiplication but have not learned to do long multiplication problems may use a calculator.

Many students commonly use a guess-and-check method for solving this problem. Although this method can be time consuming, students can use a guess-and-check method with logic and mathematical reasoning. For instance, if students assume that all 1,000 participants in the problem were people, they get an answer of 2,000 legs. Likewise, if all 1,000 participants were dogs, 4,000 legs result. Given the information generated from these two strategic guesses, students will notice that a lot more people than dogs were at the event. They will be able to make more reasoned or educated guesses. Encourage students to keep track of their guesses

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and discuss with them ways they can organize their guesses, such as creating a table, so that they do not have duplicates and can use the information from each guess to inform their next guess.

An alternative method of tackling the Walk for the Paws problem is to use a diagram, an especially useful way for younger students who may need a visual understanding of the problem. An option for students solving the modified version of the problem is to start by drawing 100 circles to represent the event participants and then adding two legs to each. They can then slowly add two more legs to each circle until they get to 250 legs.

Experimenting is a key component of solving any question or challenge and especially of this problem, which lends itself to multiple solution approaches. Encourage students to think of more than one strategy. Ask them to use manipulatives, pictures, words, or other methods to clarify their explanations. Here are some suggested prompts to scaffold students' work and encourage them to document their journey to a solution:

- How many legs would there be if all 1,000 participants were people?
- How many legs would there be if all 1,000 participants were dogs?
- Describe the strategy you used to try to solve the problem.
- Explain how you decided on this strategy. What was your thinking?
- What are some other strategies that might work?

## Where's the Math?

This problem deals with logical reasoning and algebraic thinking as students must identify relationships among variables and recognize patterns. Although formal algebra is taught in the secondary grades, having opportunities to explore and develop algebraic thinking in the elementary grades helps students prepare to understand and apply algebra later on. Instead of looking at algebra through symbols and equations, this problem begins with a real-life example. Developing concepts of algebraic reasoning from within a meaningful context makes them more accessible to students.

Because this problem can be solved by a combination of methods, students can be led into great discussions about using different methods to solve the same problem. During the sharing stage, students get a chance to learn from one another and see different approaches that have worked or have been tried by others. Listening to peers encourages students to try new strategies and analyze the efficiency of their own.

This problem can also be modeled by using a symbolic algebraic approach with a system of equations in two variables, although it is an unlikely strategy to emerge among students in lower elementary grades.

Collect students' work, make notes about interactions and discussions that took place, and document the variety of student approaches you observed. As you reflect on your experience with the problem, keep in mind the following questions:

- What difficulties did students have in understanding the problem?
- What strategies did you see students using to solve the problem?
- Were you surprised by any students' responses or interpretations?
- What extensions to this problem did you or your students pose?
- What did your students learn from investigating this problem?

## Share Your Students' Work

We are interested in how your students responded to the problem and how they explained or justified their reasoning. Please send us your thoughts and reflections. Include information about how you posed the problem, samples of students' work, and photographs showing your problem solvers in action. Send your results with your name, grade level, and school by **July 1, 2008**, to Mark Ellis, College of Education EC-190, 800 North State College Boulevard, Fullerton, CA 92834. Selected submissions will be published in a subsequent issue of *Teaching Children Mathematics* and acknowledged by name, grade level, and school.

## Source

Humane Society of the United States. "Pets for Life." www.hsus.org/pets/.

*(Solutions to a previous problem begin on the next page.)*

## Additional Resources

Did you know NCTM has published a collection of some past "Problem Solvers" columns?

Sakshaug, Lynae E., Melfried Olson, and Judith Olson. *Children Are Mathematical Problem Solvers*. Reston, VA: NCTM, 2002.

Visit nctm.org/catalog for more on this and other NCTM resources, including professional development offerings, other publications, and online resources. ▲

# Solutions to the Characteristics of Shapes Problem

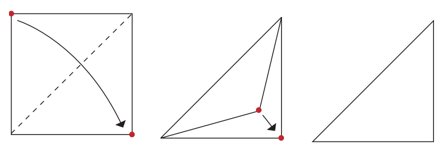
The Characteristics of Shapes problem helps students develop spatial visualization skills and better understand geometric terminology. The original problem, in the May 2007 “Problem Solvers” section, was stated as follows:

## Problem

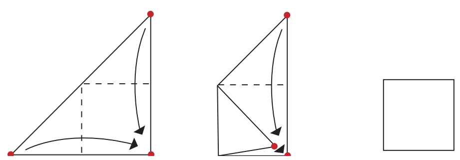
Jennifer cut a 4-inch-by-4-inch square from a piece of paper. She then folded the square in half along the diagonal to form a triangle (**step 1**). What are some characteristics of the resulting triangle? Jennifer then folded the vertices at the two 45-degree angles of the triangle down to meet the vertex at the right angle of the triangle (**step 2**). The result was a square.

Take a square piece of paper the same size as Jennifer’s and repeat the folds that she made. Unfold the paper. Now, try to see what shapes you can make by refolding the paper—but only along the crease lines (**step 3**). As you make a shape, fill in the chart (**table 1**) to show that shape’s attributes. How many of the shapes are similar? How many of the shapes are nonsimilar? What relationships do you see between the number of sides and the other attributes, or characteristics, of the shapes? Can you create a 5-sided figure? Why or why not?

**Step 1: Fold the square in half along the diagonal to form a triangle**



**Step 2: Fold down the triangle’s vertices at the two 45-degree angles to meet the vertex at the triangle’s right angle, thus forming a smaller square**



**Step 3: The unfolded square and its resulting creases**

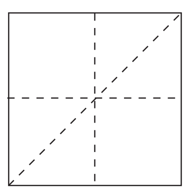


**Table 1**
Chart for Recording Characteristics of Shapes

| Sketch of the Shape | Name(s) for the Shape | Number of Sides | Other Attributes |
|---|---|---|---|
| | | | |
| | | | |

**Challenge**
Add columns to the table to record other specific characteristics, such as the following:

- area of the shape
- perimeter of the shape
- types of angles contained in the shape
- number of pairs of parallel sides

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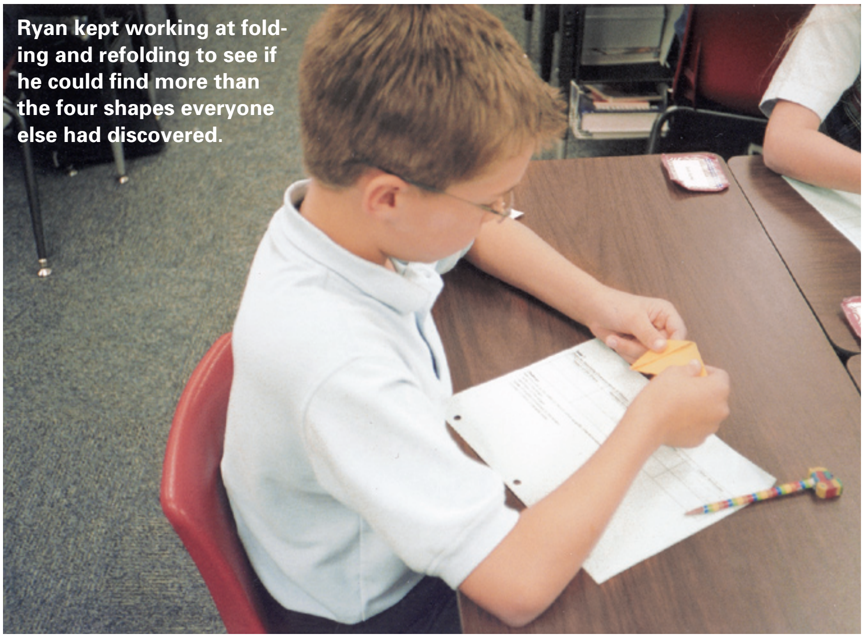
*Edited by* ***Joyce Bishop****, jdbishop@eiu.edu, who teaches in the Department of Mathematical and Computer Sciences at Eastern Illinois University, Charleston, IL 61920. Each month, this section of the “Problem Solvers” department discusses the classroom results of using problems presented in previous issues of* Teaching Children Mathematics.


**Ryan kept working at folding and refolding to see if he could find more than the four shapes everyone else had discovered.**



The Characteristics of Shapes problem provided students with fun, hands-on practice; the chance to review and clarify mathematical terminology; and the challenge of writing about geometric shapes. The problem also afforded their teachers insight into student misconceptions about angles and shape orientations. Mary Kay Varley presented the problem to her fourth-grade class at the end of the school year. She shares her account of how the lesson "unfolded":

> This looked like a very involved problem at first, but it turned out to be just exactly right to end the year and review concepts we had discussed since early September. Categorizing and recalling names for polygons, angles, and so on can be tiresome as a rote experience. This activity enticed the students, and they never even realized I was assessing their recall of prior knowledge.
>
> I had enlarged the problem directions and chart for all students to have their own copy. I also had a stack of squares for them to fold according to the directions, along with extras in case they made an error. After we completed step 1, I stopped and asked the class to describe what polygon we had created. They all said *triangle*, but they were not sure what I meant by describing its *characteristics*. I had to prompt them to say that this particular triangle had a special name. Madison was the first to identify it as a *right* triangle. We then discussed why it was called that and whether a right triangle has another name. Brian was able to remember the rather unusual word *isosceles*. As we discussed the inside angles, Ryan recognized that the other two angles were *acute* and 45-degree angles. Ross remembered that all the angles in the triangle must add up to 180 degrees.
>
> We then proceeded to do the next two folds. I explained that their job was to now fold only along the creases we had created and see how many different polygons they could produce. I shared with them that I had also tried the problem and was not able to find very many polygons. I asked them to use the back of their paper to keep track of shapes they found and then fill in the necessary information on the chart. Together, we reconsidered the triangle and discussed how to fill in each section of the chart. I then left them on their own to fill in their charts.

The solution results in four shapes: a right isosceles triangle, a rectangle, a square, and a trapezoid. As Varley's students worked on the problem, she used the opportunity to observe, ask questions, and listen to her students' responses:

> Brian was certain he had found six different shapes, but when I asked him to show them to me, we discovered he had made an extra fold in his square. As we continued to investigate and fold, the students began to realize they could only find four unique shapes. A few tried to say that if they rotated the quadrangle, then they had a new shape. At that point, we had to talk again about congruency and whether a rotation


**Madison was thrilled that she remembered the term *right triangle*.**

would actually form a new and unique shape or whether the shape would maintain its same characteristics. The students were ready to fill in their charts but still had difficulty recalling the vocabulary to fully describe the shapes.

I then posed the last question, asking if there was any way to create a five-sided figure. At first, they thought there might be, but because we had tried every possible fold, they agreed it was not possible. Ross said that if we made another fold by taking one corner vertex and folding it to the center, we could make a five-sided shape, but only if we made the extra fold. (See **fig. 1** for an example of student work from Varley's classroom.)

Varley describes the challenge that *writing* in mathematics may present to students:

This problem did not take them long at all to figure out, and I wondered if it was because we had worked with geometric figures all year. The hardest part was filling in the chart. Even at the end of fourth grade, students still have difficulty with vocabulary and creating a clear, concise description of anything. Because I had seen what they did, and they had verbalized what they did, they felt they had finished the assignment. They were very willing to verbalize their thoughts, but they seemed reticent to commit anything to paper. Apparently, mathematics writing still seemed like a foreign language to them.

Jeanette McGee presented the problem to her fifth-grade summer school students at Thornton Elementary School in Arlington, Texas. She describes how one student's misconception emerged during the discussion:

Our theme in summer school this year was "Mysteries," so I presented this problem as a chance for my new fifth graders to find the "hidden" shapes in the square of paper. I was excited from the start with the first answer I received from a very verbal student. I had asked the question about how to describe the shape that my first fold had just formed. This student responded, "It's a triangle, and it has two acute angles and one right angle." So, at that point, we stopped for a short discussion about the three kinds of angles, just in case the students encountered them during the problem solving and needed a reminder about the angle names. When we were about to go on with the Characteristic of Shapes problem, my verbal student spoke up: "If you turn that triangle up (meaning, with the right angle in my hand and the hypotenuse pointing upward), then the angle in your hand is an obtuse angle." At that point, we stopped again and talked about what I had done or not done with that right triangle while I was changing the orientation. I was floored that she thought turning it could change its characteristics.

## Figure 1

**An example of the chart that students completed in Varley's class**

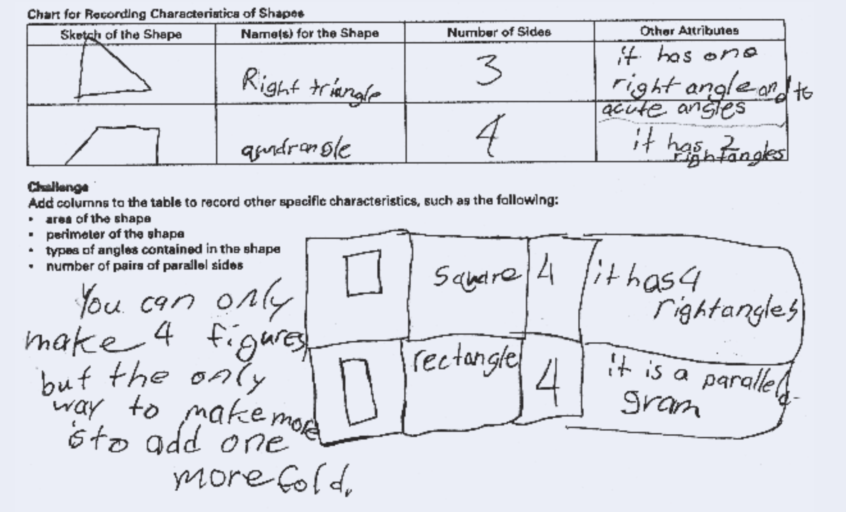


Chart for Recording Characteristics of Shapes

| Sketch of the Shape | Name(s) for the Shape | Number of Sides | Other Attributes |
|---|---|---|---|
| | Right triangle | 3 | it has one right angle and to acute angles |
| | quadranlge | 4 | it has 2 right angles |

Challenge
Add columns to the table to record other specific characteristics, such as the following:
- area of the shape
- perimeter of the shape
- types of angles contained in the shape
- number of pairs of parallel sides

| | | | |
|---|---|---|---|
| | square | 4 | it has 4 right angles |
| | rectangle | 4 | it is a parallelogram |

You can only make 4 figures but the only way to make more is to add one more fold.

McGee guided her students toward identifying specific shape characteristics by adapting the table to include columns for number of sides, perimeter, types of angles, and number of pairs of parallel sides. She also included more rows in the table for listing additional shapes. She describes some of her students' discoveries:

The students gave responses I was expecting, except for the student who found a cone while he was in the midst of making his folds. I looked back over the directions and, sure enough, they never did say we were looking for only 2-D shapes; so, I accepted his answer. I find it interesting that, at this age, students recorded a square and then rotated it and recorded the shape as a rhombus. Some students [informally] recorded it as a diamond. As we discussed our final answers as a class, we considered the [attributes of a] *rhombus* and figured out that a square *does* fall into that category.

When determining the perimeter, most students remembered how to find it, but several were confused with how to add the halves they were encountering, and many chose to round off to the nearest whole number. I decided to save that review lesson for another day when we could take more time to practice perimeters.

McGee describes some benefits of problem-solving lessons such as this one for her and her students:

> I think, once again, I am the one who learns much from this type of lesson requiring students to verbalize what they are thinking. I learn more about how they are applying or not applying what they have been taught, and then I can use my findings to clarify wrong thinking. I think this lesson provides a good review or reminder of terminology the students keep forgetting because they are not getting enough practice time to apply it in hands-on ways throughout the year.

Both Varley and McGee mention the importance of students using mathematical language. Geometry, particularly, requires special terminology. Thus, problems such as the Characteristics of Shapes provide important opportunities for students to explore shapes and describe their observations.

Just as important, such problems afford students and teachers the *joy* of problem solving. As Varley comments, "It was fun to end the year with the students still talking and thinking mathematically." McGee adds, "Thank you for having the 'Problem Solvers' section in your magazine."

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*The editors of the "Problem Solvers" department would like to express their appreciation and admiration for all the teachers who value the process of problem solving enough to help their students learn how to be mathematical problem solvers. A special thanks to the teachers and students who made contributions to this article:*

*Mary Kay Varley and her fourth graders at Fort Worth Country Day School in Fort Worth, Texas*

*Jeanette McGee and the fifth-grade summer school class at Thornton Elementary School in Arlington, Texas* ▲