Horseshoes in Flight

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Purpose	Students analyze the structure of algebraic expressions and a graph to determine what information each expression readily contributes about the flight of a horseshoe. This task is particularly relevant to students who are studying (or have studied) various quadratic expressions (or functions). The task also illustrates a step in the mathematical modeling process that involves interpreting mathematical results in a real-world context.	
Task Overview	The height of a thrown horseshoe depends on the time that has elapsed since its release. Derive information about the flight of a horseshoe from a graph and four given equivalent algebraic expressions that describe its flight. *An activity sheet that gives students the complete task is included.*	
Focus on Reasoning and Sense Making	**Reasoning Habits** ***Focus in High School Mathematics: Reasoning and Sense Making*** Analyzing a problem—identifying relevant representations; looking for hidden structure Reflecting on a solution—interpreting a solution; justifying or validating a solution **Process Standards** ***Principles and Standards for School Mathematics*** Reasoning and proof—recognize reasoning and proof as fundamental aspects of mathematics Connections—recognize and apply mathematics in contexts outside of mathematics Representation—select, apply, and translate among mathematical representations to solve problems	**Standards for Mathematical Practice** **Common Core State Standards for Mathematics** 4. Model with mathematics. 7. Look for and make use of structure.
Focus on Mathematical Content	**Key Elements** ***Focus in High School Mathematics: Reasoning and Sense Making*** Reasoning with algebraic symbols—meaningful use of symbols; linking expressions and functions	**Standards for Mathematical Content** **Common Core State Standards for Mathematics** A-SSE-3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
Materials and Technology	Horseshoes in Flight activity sheet	

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Use in the Classroom

After introducing the task and distributing the activity sheet, you might begin by having students work individually or in groups to come up with their own conclusions.

Asking these types of questions may assist students in realizing that they need to analyze the problem by looking at the structure and various parts of the algebraic expressions.

If students have difficulty getting started on the task, you might ask, "Just by looking at the four expressions, (a)–(d), without doing any computation, can you tell me one thing that you know about the flight of the horseshoe?"

A class discussion of possible responses might follow, encompassing the questions on the activity sheet. Question 1 asks which expression is most useful for finding the maximum height of the horseshoe. You might have different students present their solutions and the reasoning behind them. Question 2 probes other conclusions that students can draw about the horseshoe's flight from the other expressions. If the students are having trouble with question 1, discussing question 2 first might be easier and more productive.

Question 1 requires students to choose an appropriate representation and justify their choice.

In responding to question 2, students move back and forth between the parts of the mathematical models and the real-world context as they analyze expressions, draw mathematical conclusions, and interpret the results in context.

After the students have discussed what they can conclude from the other expressions, ask questions to focus their attention on the relationships among their conclusions about the values of the expressions, where those values show up on the graph, and what those values mean in the context.

Reflecting on the solutions can suggest or strengthen students' belief in the potential of a mathematical reasoning strategy that employs multiple mathematical representations in other situations.

You might conclude by having the class observe that although these expressions are equivalent, each one provides different insights into the situation—an observation that will naturally raise the question, "Are there other situations like this one?"

Focus on Student Thinking

In responding to question 1, students might note that since the term $-16(t - 9/16)^2$ is always negative or zero, expression (d) makes it clear immediately that the height can never go above 100/16, or 6 1/4, feet, and it reaches this height at $t = 9/16$ seconds, which is when that first term has the value 0.

Alternatively, if students look at the graph, they might notice that the maximum height is a little more than 6 feet and is reached after just slightly more than 0.5 seconds have elapsed. They could then use this graphical information to help narrow the group of expressions to determine which one would be most useful for finding the maximum height.

In considering question 2, students might note that substituting in $t = 0$ in expression (a) reveals that the initial height of the horseshoe is 1 3/16 feet, which would be where the thrower held the horseshoe at the start of the toss. This is also the y-intercept of the graph.

Students might note that expressions (b) and (c) are essentially the same, since they are factored forms of the original expression. Such forms are useful in finding the zeros, or x-intercepts, which in this case are –1/16 and 19/16. The negative zero isn't useful in considering the flight of the horseshoe, since the flight begins at $t = 0$. However, the positive zero tells when the height of the horseshoe returns to 0 feet, which is when it hits the ground. In looking at the graph, this will be at about 1.2 seconds, which is close to 19/16 seconds.

In looking at the factored forms, (b) and (c), students might also note that since the graph is a parabola and parabolas are symmetric, the maximum height should occur at the midpoint of the two zeros, which would be 9/16 seconds, the same value as shown in expression (d).

Assessment

To gain insight into issues that you might need to address in the discussion, walk around and observe what the students initially do with the task.

For homework or as a summative assessment, you might assign one of the following tasks:

- Ask students to write up their conclusions about the task. Alternatively, give them an expression with different coefficients and ask them to use that expression to analyze the path of the horseshoe (or some other object).
- Ask students to write an expression giving the profit that Crumbly Cookies will make in the situation described below and then analyze what they can conclude from the original expression or equivalent expressions:

  A survey showed that if Crumbly Cookies sells a dozen cookies for x dollars per dozen, it will sell $1000 – 200x$ dozens.

  Their cost to make a dozen cookies is about $1.
- Give the students an expression and ask them first to come up with a situation that that it might describe and then to find out what they can about that situation by analyzing the expression and equivalent forms of it.

Resources

Common Core State Standards Initiative (CCSSI). *Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math).* Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010. http://www.corestandards.org.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

———. *Focus in High School Mathematics: Reasoning and Sense Making.* Reston, Va.: NCTM, 2009. Example 13, pp. 52–53.

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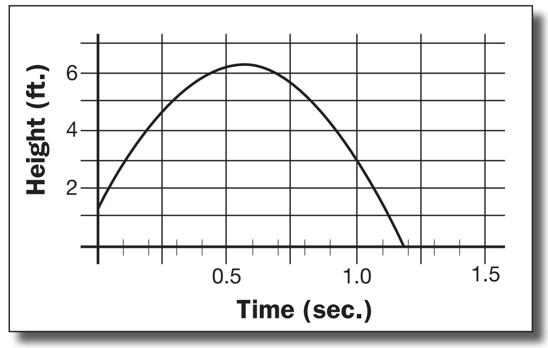
Student Activity Sheet

As shown in the graph, the height of a thrown horseshoe depends on the time that has elapsed since its release. (Note that this graph of the horseshoe's height is parabolic, but it is not the same as the graph of the horseshoe's flight path.)



The height of the horseshoe (measured in feet) as a function of time (measured in seconds and represented by the variable t) from the instant of release is

$$1\,\tfrac{3}{16} + 18t - 16t^2.$$

The expressions (a)–(d) below are equivalent:

(a) $1\,\tfrac{3}{16} + 18t - 16t^2$

(b) $-16(t - \tfrac{19}{16})(t + \tfrac{1}{16})$

(c) $\tfrac{1}{16}(19 - 16t)(16t + 1)$

(d) $-16(t - \tfrac{9}{16})^2 + \tfrac{100}{16}$

1. Which expression is the most useful for finding the maximum height of the horseshoe, and why is it the most useful expression?

2. What information can you conclude about the horseshoe's flight from other equivalent expressions? Explain your answers.