## **Contents**

Foreword	
Preface	ix
Introduction	1
Pedagogical Content Knowledge	1
Model of Teacher Knowledge	3
Characteristics of Tasks	7
Types of Questions	9
Conclusion	10
Chapter 1	
The Meaning of Multiplication	11
Working toward Essential Understandings $1a$ , $1c$ , and $1d$	11
Summarizing Pedagogical Content Knowledge to Support Essential Understandings 1a, 1c, and 1d	31
Knowledge of learners	31
Knowledge of curriculum	32
Knowledge of instructional strategies	32
Knowledge of assessment	33
Conclusion	34
Chapter 2	
Problem Types, Representations, and Strategies	35
Working toward Essential Understanding 1b	36
Problem types for multiplication and division	37
Physical and symbolic models for multiplication and division	40
Student strategies for multiplication and division	45
Summarizing Pedagogical Content Knowledge to Support Essential Understanding 1b.	52

## Contents

Knowledge of learners	52
Knowledge of curriculum	53
Knowledge of instructional strategies	53
Knowledge of assessment	54
Conclusion	54
Chapter 3	
The Meaning of Division	55
Working toward Essential Understandings 1e and 1f	55
Summarizing Pedagogical Content Knowledge to Support Essential Understandings 1e and 1f	69
Knowledge of learners	69
Knowledge of curriculum	
Knowledge of instructional strategies	
Knowledge of assessment	70
Conclusion	71
Chapter 4	
Multiplication and Division Properties	73
Working toward Big Idea 2 through Essential Understandings 2a, 2b, and 2c	74
Addressing misconceptions about multiplication and division	75
Developing understanding of the commutative property	79
Developing understanding of the distributive property	85
Developing understanding of the associative property	92
Developing understanding of multiplication and division with 1 and	d 0 98
Developing understanding of the right distributive property for div	ision 100
Developing understanding of order of operations	102
Summarizing Pedagogical Content Knowledge to Support Big Idea 2 through Essential Understandings 2 <i>a</i> , 2 <i>b</i> , and 2 <i>c</i>	105
Knowledge of learners	
Knowledge of curriculum	
and meage of currentum	

Knowledge of instructional strategies	107
Knowledge of assessment	107
Conclusion	108
Chapter 5	
Algorithms for Multiplication and Division	109
Working toward Essential Understanding 2d	110
Building understanding of algorithms for multiplication	111
Building understanding of algorithms for division	120
Summarizing Pedagogical Content Knowledge to Support Essential Understanding 2d	123
Knowledge of learners	123
Knowledge of curriculum	123
Knowledge of instructional strategies	123
Knowledge of assessment.	124
Conclusion	124
Chapter 6	
Looking Back and Looking Ahead with Multiplication and Division	127
Supporting Knowledge in K-Grade 2 for Multiplication and Division in Grades 3-5	127
Focusing students on the unit	127
Multiplicative thinking	130
Extending Knowledge of Multiplication and Division in Grades 6–8	134
Multiplying and dividing fractions	136
Writing and manipulating algebraic expressions	141
Conclusion	142
Appendix 1	
The Big Ideas and Essential Understandings for Multiplication and Division	143

## Contents

App	endix 2
Reso	urces for Teachers
	endix 3
Tasks	100
Refe	rences 169
Acc	ompanying Materials at More4U
	Appendix 1 The Big Ideas and Essential Understandings for Multiplication and Division
	Appendix 2 Resources for Teachers