Using KenKen to Build Reasoning Skills

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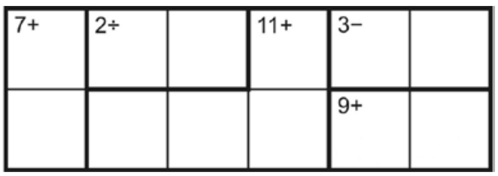
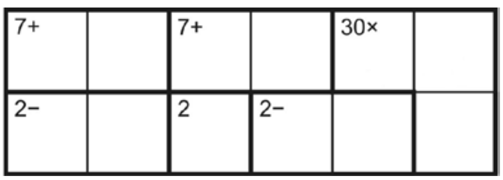
ADDITIONAL PUZZLES AND SOLUTIONS

The following puzzles will provide some practice with the strategies covered in this article as well as demonstrate a few other strategies that may be useful.

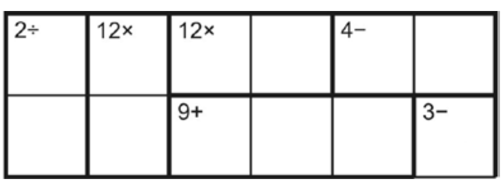
1. The figure below shows two rows from a 6 × 6 puzzle. What numbers go in the [9+] cage?



2. The figure below shows two rows from a 6 × 6 puzzle. What numbers fill the top two cells of the [30×] cage?

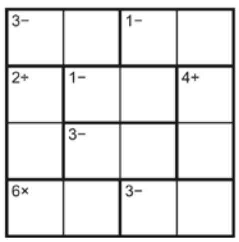


3. The figure below shows two rows from a 6 × 6 puzzle. What number fills the top cell of the [3–] cage? (Only the top cell of the [3–] cage is shown.)

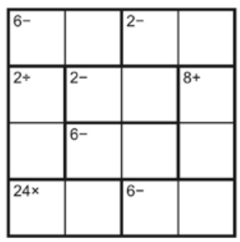


4. The figure below shows two rows from a 6 × 6 puzzle. What is the value of x?

5. KenKen puzzles involve basic arithmetic, but they also offer opportunities to explore advanced mathematics topics, such as isomorphism and mapping functions. Consider the two 4 × 4 puzzles shown below. The first requires distributing the digits 1, 2, 3, and 4, and the second requires distributing the digits 2, 4, 6, and 8. How are these two puzzles related?



(a)



(b)

SOLUTIONS

1. The four cages—[7+], [11+], [3–], and [9+]—are odd. The sum of the two rows is 42, so the number of odd cages must be even. Hence, the [2÷] cage must be even, and the only even candidate set is {4, 2}. It follows that [3–] must be filled with {3, 6}; then the top-left cell must be 1 or 5. Either way, the [9+] cage must be filled with {4, 5}.

2. One of the [2–] cages will have even entries, and the other will have odd entries. The cage with even entries will contain $\{4, 6\}$ because 2 has already been used; and the cage with odd entries will contain either $\{1, 3\}$ or $\{3, 5\}$. Therefore, the bottom-right corner will contain either 1 or 5. If the bottom-right corner were 1, then the two cells of the [30×] cage would be $\{5, 6\}$, and the top row would have a sum of $5 + 6 + 7 + 7 = 25$. However, the sum of the entries in a row of a 6×6 puzzle must be 21, so there is a contradiction. The number in the right-bottom corner must be 5. Consequently, the top two cells of the [30×] cage must be $\{1, 6\}$ because their sum must be 7.

3. One of the three cells in the top row that are part of the [12×] cages must be filled with either 2 or 6. Consequently, the [4–] cage must be filled with $\{1, 5\}$. By orthogonality, one of the [12×] cages is $\{3, 4\}$, and the other is $\{2, 6\}$. Further, there are three candidate sets for the [2÷] cage: $\{1, 2\}$, $\{2, 4\}$, and $\{3, 6\}$. All these pieces lead to an interesting result: The sum of the numbers in each candidate set for [2÷] is a multiple of 3; the sum of the numbers filling the [4–] and two [12×] cages is $(1 + 5) + (3 + 4) + (2 + 6) = 21$; and the sum of the numbers in each candidate set for [2÷] is also a multiple of 3. Consequently, the sum of all eleven numbers other than the top cell of the [3–] cage is a multiple of 3. Thus, the top cell of the [3–] cage must also be a multiple of 3 because the sum of all entries in these two rows must be 42. Because a 3 must occur in the [9+] cage, the top cell of the [3–] cage must be 6.

4. The following idea comes from mathematics professor John Watkins, who credits author Barry Cipra. The [4+] cage must be $\{1, 3\}$, and the [6×] cage must be $\{1, 2, 3\}$. The two cells in the first column of the [6×] cage are parallel to the two cells of the [4+] cage, so the unique solution requirement means that the two numbers in first column cannot be the same as the two numbers in the [4+] cage. Further, the two cells in the bottom row of the [6×] cage are orthogonal to the [4+] cage, so they cannot be filled with $\{1, 3\}$, either. Consequently, $x = 2$.

5. These two puzzles are isomorphic. Look at the relation between the addition and subtraction cages. In the second puzzle, the clues are all twice as big. But the division cage has the same clue, and the multiplication cages are [6×] in the first and [24×] in the second. The doubling function $f(n) = 2n$ maps the solution of the former to that of the latter.