Can We Really Count on Frank?

Use this new framework to select and analyze picture books for their mathematical content, such as the accuracy of the number of cans that stack in a shopping cart.

> By Jennifer L. Marston, Tracey Muir, and Sharyn Livy

he National Council of Teachers of Mathematics (NCTM) and the Australian National Curriculum encourage the integration of literacy and numeracy, and *Teaching Children Mathematics* (*TCM*) regularly includes articles on incorporating picture books into the mathematics program. Marston has developed a new framework (2010) to assist teachers in selecting and evaluating picture books. Using this framework, the authors assess the potential of three different books for mathematical teaching and learning in the primary classroom.

Developing math concepts

A wide range of children's literature may directly or indirectly promote mathematics learning (Marston 2010), and *TCM* readers are likely familiar with articles that have demonstrated how this can occur (e.g., Kuntz 2010). Teacher resource books are available to provide articles and lesson ideas for exploring math through literature (e.g., Thiessen and Matthias 1992), with authors emphasizing the importance of the literary quality of books that are used for their mathematical content (Whitin and Whitin 2004). According to Marston (2010),



teachers require contemporary criteria on which to base their choice of picture books for use in the classroom; without such criteria, teachers may impose mathematics on any book or miss mathematical opportunities inherent in the story. This article describes and implements a framework as a model for teachers to use in selecting and evaluating picture books for mathematical concept development.

Using the framework

Marston (2010) has identified three different types of mathematical picture books:

1. Those with *perceived* content, that is, books written to entertain but which contain

opportunities to use for mathematical concept development (e.g., *Half a World Away*, Gleeson 2007)

- 2. Those with *explicit* content, written specifically to highlight and teach mathematical concepts (e.g., *One Is a Snail, Ten Is a Crab,* Sayre and Sayre 2003)
- 3. Those with *embedded* content that can be read for the "story" alone but with mathematics purposefully embedded throughout the text and visual images (e.g., *Counting on Frank*, Clements 1990)

Frameworks have previously been developed to aid teacher selection and evaluation of picture books for mathematical learning. These include Schiro's (1997) standards that address "trade," or explicit, mathematics books and Whitin and Whitin's (2004) criteria for selecting "Math-Related Books" (pp. 1–25), which are linked to

An expanded version (Marston 2010) **TABLE** of this classification scheme contains

a definition and an example of a

book displaying that element.

Categories	
Mathematical content	
Curriculum content, policies, and principles	
Integration of mathematics content	
Mathematical meaning	
Mathematical problem solving and reasoning	
Affordance for mathematics learning	
Pedagogical Implementation	

TABLE

One Is a Snail, Ten Is a Crab (Sayre and Sayre 2003) is strong in most framework categories, but Marston, Muir, and Livy focus on the book's problem-solving qualities and how the book fulfills elements within the category of mathematical problem solving and reasoning.

Mathematical Problem Solving and Reasoning

Code	Element	Definition
MPS1	Application	Mathematical contexts are applicable to everyday problem-solving situations.
MPS2/6	Problem-solving opportunities	The text and visual images offer opportunities for problem solving, both individual and collaborative.
1PS3/4	Problem posing: text and visual images	The text and visual images afford opportunities for the teacher and students to <i>pose</i> problems.
1PS5	Multiple solutions: text and/or visual images	The text and/or visual images provide opportunities for multiple paths and multiple solutions to problems posed explicitly within the text or derived from the text.

NCTM Standards (2000) and NCTE standards (1996). Marston's (2010) framework incorporates concepts from both these criteria and addresses recent research in young children's development of mathematical concepts, the role of the visual images, the importance of the interaction of text and visual images, and changes in curriculum and pedagogy.

The seven categories of the framework (see table 1) each include a series of elements. An expanded version (Marston 2010) contains a definition and an example of a book displaying that element. Three of these categories are highlighted later in this article. Although a Likert scale can assist teachers, the framework is intended as a guide for teachers wishing to address a particular concept or meet the needs of their program or students. A book does not necessarily have to score highly in every category to be a useful teaching tool, as the three books discussed in this article demonstrate. The authors have used this framework to evaluate three picture books that teachers use in primary classrooms. The categories of mathematical content, integration of mathematics content, and mathematical problem solving and reasoning have been particularly addressed, although each book may contain opportunities from other categories and elements that could provide quality learning experiences.

Problem solving and reasoning

Whitin and Whitin (2009) first introduced readers to One Is a Snail, Ten Is a Crab (Sayre and Sayre 2003) as a powerful method for creating a meaningful and engaging context to initiate problem-solving activities. Although the book is strong in most framework categories, this article focuses on the book's problem-solving qualities and how the book fulfills elements within the category of mathematical problem solving and reasoning (see table 2). The mathematical content in Sayre and Sayre's (2003) story is "explicitly referenced" (Marston 2010, p. 383) in the title, the text, and the visual images. This story, however, goes beyond the format of simple counting books: Both the text and the illustrations demonstrate combinations of numbers using different animals (e.g., "3 is a person and a snail"). Another of its particular strengths is that the numbers do not stop at 10; Sayre and Sayre represent the numerals 1–10 followed by

the decade numbers up to 100 (e.g., "20 is two crabs," "90 is nine crabs or ten spiders and a crab" [see **table 3**]). In their earlier article, Whitin and Whitin (2009) described how they used the story to encourage students to engage in a variety of problem-solving methods, particularly through using the illustrations of creatures from the text. Authors Muir and Livy have used the story similarly with grades 1 and 2 classes, having the children do the following:

TABLE

FIGURE

• Reflect on the story by drawing and writing their own number sentences using the creatures from the story. (See **fig. 1**.)

This activity demonstrates the potential of the story to support the aspect of Marston's (2010) framework that states, "The text and visual images offer opportunities for problem solving, both individual and collaborative" (MPS: Problem Solving).

• Make connections with the "answer" and generate possibilities for it.

For example, when 60 was described as "six crabs or ten insects," children were asked to identify other possibilities using a combination of animals. Scaffolding was used to assist less confident children with generating possibilities:

- * So how many dogs would that be?
- * If we had five crabs, how many insects would we need?
- Generate their own problems for a friend to solve.
- Consider examples of a "good" question.

Discussion occurred around possible answers, with the answers and strategies the children used being recorded on the whiteboard. For example,

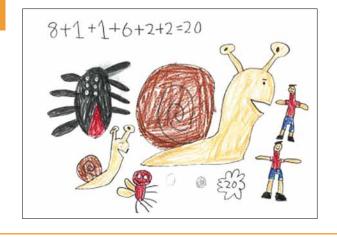
While you were walking home from school one day, you saw 14 animal legs in a yard. You know that 5 pets live in the yard some birds and some rabbits. How many birds and rabbits live there?

These examples demonstrate further elements from the framework, including how the text and visual images provide the potential In One Is a Snail, Ten Is a Crab, Sayre and Sayre (2003) represent the numerals 1–10 followed by the decade numbers up to 100.

Sampling of Combinations Mentioned in the Book

Number	Animal or combination	Corresponding number or number sentence
1	Snail	1
2	Person	2
5	Dog and snail	4 + 1
7	Insect and snail	6 + 1
10	Crab	10
30	Three crabs or ten people and a crab	3 × 10 10 × 2 + 10
100	Ten crabs or one hundred snails	10 × 10 100 × 1

As a powerful method to initiate problem-solving activities, One Is a Snail, Ten Is a Crab (Sayre and Sayre 2003) gives students a context for drawing and writing number sentences.



for the teacher and students to *pose* problems (MPS: Problem Posing) and how the activities "allow children to become problem posers" (Whitin and Whitin 2009, p. 87). Figure 2a and Figure 2b (see p. 444) also show two different approaches students took to writing their own problems. They display the opportunities for using multiple paths and finding multiple solutions (MPS: Multiple Solutions).

A variety of problem-solving skills and opportunities have been evidenced here, and the book would score well for its opportunities Text, visual images, and activities based on storybooks have problem-posing potential for the teacher and students.

(a) Two students took different approaches to writing their own problems.

If there were 10 octopuses and 2 crabs in an aquarium, how many legs would there be?

(b) Notice the opportunities for using multiple paths and finding multiple solutions (MPS: Multiple Solutions).

In a rock pool, I counted 6 heads and 42 legs. If starfish have 5 legs and an octopus has 8, how many starfish and octopuses were in the pool?

> to promote mathematical problem solving and reasoning. However, the use of the framework with *One Is a Snail, Ten Is a Crab* (Sayre and Sayre 2003) would also alert teachers to the first element of this category (MPS: Application). Authors Muir and Livy, as well as Whitin and Whitin (2009), do not address the application of the skills developed here to everyday situations. Students could also be encouraged to transfer this knowledge to another context and to other everyday reasoning and problem-solving activities; for example, finding addition combinations of 10 or solving such problems as the following:

A carton of yogurt holds 4 packs of 6 tubs. How many cartons and packs do I need to give 42 children a tub of yogurt each?

Purposeful math

Counting on Frank (Clements 1990) is a popular, entertaining, and appealing picture book with purposefully embedded mathematical content (Marston 2010). It tells the story of a young boy and his dog, Frank, who—within the context of a narrative—present the reader with a number of amazing mathematical "facts":

I don't mind having a bath—it gives me time to think. For example, I calculate it would take eleven hours and forty-five minutes to fill the entire bathroom with water. That's with both taps running. It would take less time to empty, as long as no one opened the door! (Clements 1990, unpaged)

When applying Marston's framework, the authors found that the picture book is strong in terms of its mathematical problem solving and reasoning and its affordance for mathematics learning (see **table 1**), particularly in the promotion of positive values and attitudes toward mathematics and creative intellectual endeavor. At first glance, it also appears to meet the criteria for mathematical content (see **table 4**), but closer examination reveals that although the content is visible, it is not displayed with accuracy and authenticity. To illustrate this, the authors selected two claims in the book and investigated their mathematical accuracy.

Accurate math

Marston (2010) emphasizes that a book's mathematics should be correct and that this accuracy should extend to such aspects as calculations and illustrations. Schiro (1997) cites examples of picture books with inaccurate mathematical content: The illustrations in *The Doorbell Rang* (Hutchins 1986) show 58 cookies on a tray on one page; the same tray on the next page shows 68 cookies. Similarly, Schiro objects to the doubling of "over half a ton" in *The King's Chessboard* (Birch 1988), criticizing the author's treatment of fractional amounts as not making a difference, which in fact they do.

In the authors' experiences, *Counting on Frank* (Clements 1990) has been used widely in primary school classrooms to promote mathematical thinking and investigating. The *Illuminations* website (NCTM 2010) includes activities for using this book to investigate size **ABLE** 4

Applying Marston's framework to *Counting on Frank* (Clements 1990) reveals visible but inaccurate content.

A	Mathematical Content		
•	Code	Element	Definition
	MC1	Mathematical representation: visibility	Mathematics is easily seen/recognized by the reader in the title, text, or representations (symbols, diagrams, pictures, actions).
	MC2/3	Mathematical content: accuracy	Mathematical content, linguistic terms, concepts, calculations, and representations are accurate and consistent with school mathematics curricula.
	MC4	Mathematical contexts: authenticity	The mathematical content (text and/or visual images) is presented in authentic everyday contexts.
	MC5/6	Mathematical language	The text and visual images provide opportunities for the child to verbalize mathematics ideas using mathematical language.

and dimensions, and other potential uses would include the investigation of larger numbers, measurement concepts, and estimation. Although many teachers reading this article may be familiar with the picture book and have read it to their class, how many would have stopped to consider whether the mathematical calculations and claims made in the book are accurate? Consider the two following examples and you may come to look at this picture book in a new light.

About peas for dinner, Frank says,

I enjoy dinner, not because of the delicious grill Mum cooks EVERY night, or the thrilling conversation. It's the peas. If I had accidentally knocked fifteen peas off my plate every night for the last eight years, they would now be level with the table top. Maybe, then, Mum would understand that her son does not like peas. (Clements 1990, unpaged)

To test the validity of this statement, the problem was roughly calculated to gauge whether dropping 15 peas off a plate every night for eight years would reach the level of the tabletop. On the basis of some assumptions [a table height of 75 cm or 29.5 in.; the volume of a pea; 1 cm³ or 0.4 in.³ (the volume referring to that of a square-base prism), and some calculations (the number of days in eight years is 2,922, and 15 peas multiplied by 2,922 days would equal 43,830 peas)], the teacher concluded that the area of the floor covered by peas would be only 584 cm² (90.25 in.²), or in other words, the floor area would be a square of 24.2 cm by 24.2 cm $(9.5 \text{ in.} \times 9.5 \text{ in.})$. The calculations in the book, therefore, are completely misrepresentative of the average-size dining room and dining room table. To more accurately predict the number of peas that would have to drop from the plate, the dimensions of a small table (140 cm × 80 cm or 55 in. × 31.5 in.) and height of 75 cm (29.5 in.) were used. Rounding off a pea volume previously stated, it was calculated that during an eight-year period, the number of peas dropped each day would have to be 256.

Later in the book, Frank talks about grocery shopping and buying cans of dog food:

Going shopping with Mum is a big event. She is lucky to have such an intelligent trolleypusher. It takes forty-seven cans of dog food to fill one trolley, but only one to knock over one hundred and ten! (unpaged)

Does it really take 47 cans of dog food to fill one shopping cart? Even allowing for different sizes of carts and cans of dog food, this seems a gross underestimation. To test this claim, a twelve-year-old student, using real cans of dog food (680 g or 23.94 oz. tins), found that it took about 47 cans of dog food just to fill the base of a shopping cart and that it would take closer to 216 cans to fill an average shopping cart.

Becoming involved with the mathematics

The math of Counting on Frank (Clements 1990) is visible, is authentic, and provides opportunities for the use of mathematical language but has been found to lack mathematical credibility in terms of accuracy of mathematical content. Does this mean that teachers should avoid using it in the classroom? The picture book (text and illustrations) is strong in other elements of Marston's framework, which would also justify its selection for reading aloud and sharing with a class. It has the potential to "involve the reader in its mathematics" (Schiro 1997, p. 96) and promote positive attitudes toward math (affordances for mathematics learning [see table 1]). Schiro (1997) makes suggestions for enhancing books that are mathematically flawed and even goes so far as to propse altering the text of some books to either clarify or enhance some of their mathematical ideas. The authors of this article would not like to see Counting on Frank altered but would advocate that the framework provide teachers with a tool for evaluating the appropriateness of mathematical literature and that the inconsistencies with content or mathematical language be used as valuable teaching points and areas for further investigation.

Integration of math content

Half a World Away, by Australian writer Libby Gleeson (2007), tells the story of two friends,

Amy and Louie, who live next door to each other. They do everything together; they build towers, play dress up, and dig holes until the day Amy moves "half the world away" (unpaged). Louie then "discovers how to rebuild their special bond" (back cover). The title may suggest some mathematical content, but the original title of Amy and Louis (Gleeson 2006) published in Australia does not suggest any. By using the framework, the many opportunities for both teachers and young readers to explore the wonderful mathematical ideas throughout the book become evident. Table 5 shows some of the diversity of mathematical activities that are possible when using this picture book. All elements within the category of integration of mathematical content (see table 6) are therefore well met here. Although this book targets younger school-age readers, the included mathematical concepts and activities can be adapted for a range of age levels.

Implications for teachers

Picture books are wonderful resources to aid the development of mathematical concepts. They provide content, skills, and ideas for creative learning; and they address the benefits of and need for integrated learning. However, using the framework to evaluate different aspects of these three picture books highlights the need for closer examination of each book before incorporating its use. A variety of available resources identify picture books that can be used in the math classroom (e.g., Young and Marroquin 2006). However, teachers must be aware of the elements within a book that can affect children's

All elements within the integration of mathematical content are well met in *Half a World Away* (Gleeson 2007).

TAB	Integration of Mathematics Content		
÷	Code	Element	Definition
	IMC1/2	Integration within and across the mathematics curriculum	Evidence of connected concepts within the same content or process strand (e.g., area and length within measurement) or across content and/or process strands (e.g., number and measurement).
	IMC3/4	Integration across other disciplines and with ICT	Evidence of integration links across disciplines (e.g., mathematics and science) and incorporation of information and communication technology (ICT) into mathematical contexts.

BL

Diverse mathematical activities are possible using *Half a World Away* (Gleeson 2007). *Note:* * indicates cross-curricular links.

Teaching and Learning Experiences for Half a World Away (Gleeson 2007)

leaching and Learning Experiences in	of Hall a world Away (Gleesoff 2007)
Within the measurement strand	Across mathematics
Length What is "a long, long way away"? Where is "too far away"? How far can your cry be heard? (SS) Can you "build towers as high as the sky"? How tall can you build? Can you dig "holes deep enough to bury bears"? How far is the "other side of the world"? (GG)	 2D and 3D Free play with three-dimensional objects; name them. Use three-dimensional objects to make the city that is portrayed toward the back of the book. Name the two-dimensional shapes that make up the three-dimensional objects. What are the properties of two-dimensional shapes and three-dimensional objects in the book?
Area What is the area of the clouds in the book? (Use informal/formal units.) (SC)	Position Where is the "other side of the world"? (GG) Map your classroom, town, and so on in a manner similar to the map of Louie's town.
Time How long would it take to get to the other side of the world? (*T) Why is Amy awake when Louie is asleep? (*GG)	Proportion Amy is sitting beside a packing box. Make the box in the illustration to the correct size.
Volume Packing boxes: What size do you need to pack specified objects?	
Across other	earning areas
Science of sound/sound travel (SS): What makes sound? How fast is sound? How far can sound travel?	Communication (C): How do we communicate with people close by and those far away?
Interpersonal relationships (IR): Friendships; what makes a good friend? Can friends live across the world? How can friends communicate?	Transport (T): How do we travel around the world? What is the fastest way? Was this always the case?
Global geography (GG): Where is the other side of the world?	Science of clouds (SC): How big are they? How do they form? What are the different types of clouds?

learning of mathematical concepts. This framework allows teachers to select and evaluate any book for its potential to develop mathematical concepts and meet the needs of their program and the children in their classroom.

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