Mathematics Education in the United States 2008

A Capsule Summary Fact Book

written for

The Eleventh International Congress on Mathematical Education (ICME-11)

Monterrey, Mexico, July 2008

by

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under the auspices of the

National Council of Teachers of Mathematics

and the

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Table of Contents

Table of Contents—*Continued*

Preface

On 1 January 2008 the population of the United States was approximately 303,140,000. Approximately 19% of these individuals were formally enrolled in an elementary or secondary school, while another 6% were enrolled as students in a degree-granting postsecondary institution. In the entire population, about 63% were age twenty-five or older, and, of those adults, 86% had completed high school or its equivalent and 28% had at least a college bachelor's degree, both record highs. In fact, over the period from 1986 to 2006, the percentage completing high school had increased by 2% and the college graduation rate had increased by 4% (Snyder, Dillow, and Hoffman 2007; U.S. Census Bureau 2008a, 2008b).

No single government agency controls public education in grades K–12 in the United States. Rather, authority for most educational decisions lies with education agencies in the fifty individual states, which in turn share decision making with the nearly 14,000 individual school districts within them. In addition, 11% of students in grades K–12 were enrolled in private (not government controlled) schools in 2005 (Snyder, Dillow, and Hoffman 2007).

Similarly, both public and private institutions exist at the college and university level, with ultimate authority at the state level for public institutions and at the institutional level for most private institutions. Of the 4,276 colleges and universities existing in 2005–06, 2,582 were four-year colleges (those that award a bachelor's degree) and 1,694 were two-year colleges. Two hundred eighty of the four-year colleges had enrollments of more than fifteen thousand students, and the fifteen largest had enrollments between forty thousand and fifty-one thousand (Snyder, Dillow, and Hoffman 2007).

Determining what is happening in such a large and complex arena is quite difficult even for those in the United States and others familiar with education in the United States. Furthermore, many at International Congress on Mathematical Education (ICME) conferences lack familiarity with education in the United States. Consequently, in 1999 the U.S. National Commission on Mathematics Instruction recommended that the National Council of Teachers of Mathematics (NCTM) request funds from the National Science Foundation (NSF) to bring together available data about mathematics education in the United States for a document to be distributed at the Ninth International Congress on Mathematical Education (ICME 9) to provide mathematics educators throughout the world with information about this complex system. This process was repeated in 2004 for ICME 10, and this latest version for ICME 11 in 2008 extends the series with information available as of early 2008.

This document begins with some general information about education in the United States. The three kinds of curriculum identified in the Second International Mathematics Study—intended, implemented, and attained—are then described. A special focus is given to developments in statistical education because of the recent growth of interest in this area from elementary school through undergraduate study. As in earlier editions, this edition has sections dealing with programs for high-achieving students, programs for mathematics teacher education, and doctoral programs in mathematics education. One message that comes through repeatedly in these descriptions is the variety of available programs and thus the inability to characterize them adequately in a brief document like this one. Another message is that a great flux exists at all levels of the educational system, and even though we have attempted to provide the latest available information, we realize the information presented here will quickly become dated. By listing our sources, we hope we enable the interested reader to obtain updated information.

We would like to acknowledge the efforts of Gail Burrill, who wrote the grant under which the funding for this publication was obtained; the insightful and constructive advice of Sadie Bragg, David M. Bressoud, Francis (Skip) Fennell, Kathy Mowers, Joseph G. Rosenstein, Patrick (Rick) Scott, Ester Sztein, and the fine work of Nancy Busse, Ann Butterfield, Ramona Grewal, Evonne Tvardek, David Webb, and Randy White at NCTM in editing and producing this document. We have tried to be as accurate as we could and apologize for any errors.

John A. Dossey
Katherine Taylor Halvorsen
Sharon Soucy McCrone
January 2008

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PART I: General Information

Overall Organization of Education in the United States

Figure 1 shows the general framework of education in U.S. schools. This system can be thought of as having four broadly defined levels: elementary school (grades K–5 or K–6, corresponding to ages 5–10); middle school or junior high school (grades 6–8 or 7–8, ages 11–13 or 12–13); senior high school (grades 9–12, ages 14–17); and postsecondary, or tertiary, education (grades 13 and above, ages 18 and older).

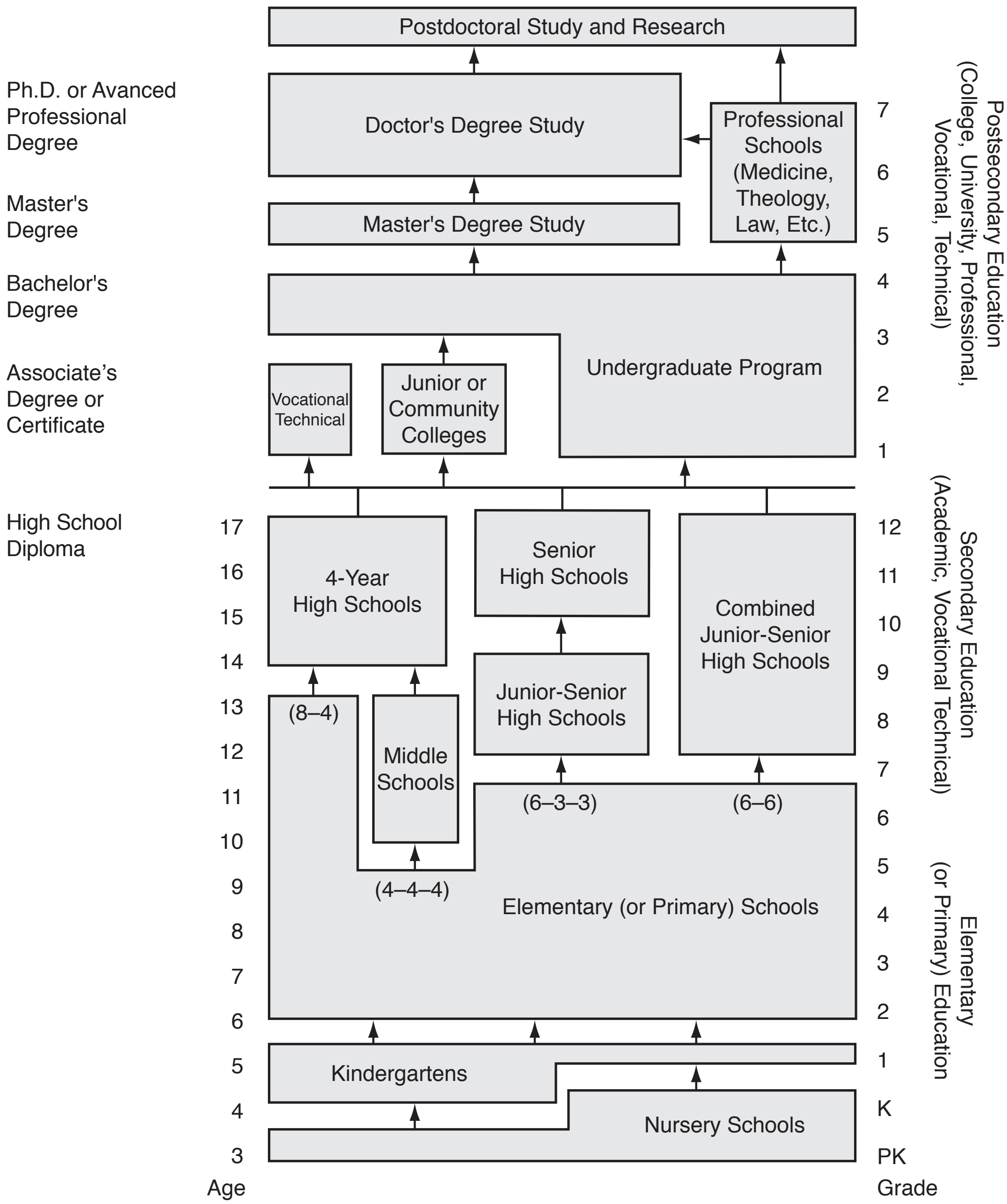

Fig. 1. The structure of education in the United States (Snyder, Dillow, and Hoffman 2007)

After graduating from high school, students may leave the educational system; attend a nonuniversity, tertiary institution focusing on technical or vocational education, such as a two-year or community college; or attend a four-year college or university. The two-year and community colleges usually offer a mixture of the first two years of a four-year college's curriculum along with a number of courses also found in the technical colleges and high schools. In these two-year or community colleges, an Associate of Arts (A.A.), Sciences (A.S.), or Applied Sciences (A.A.S.) degree can usually be earned through the equivalent of two years of full-time study. The four-year colleges and universities offer Bachelor of Science (B.S.) and Bachelor of Arts (B.A) degrees, which can usually be completed in four years of full-time study. In addition, the universities offer graduate programs leading to masters (M.S. and M.A.) and doctoral (Ed.D. and Ph.D.) degrees. Programs leading to professional degrees (law, medicine, business, etc.) exist both in universities and in institutions that offer no other degree programs. Time to complete post-bachelor degrees varies with the field and institution.

Educational Enterprise

The delivery of elementary and secondary education through the structures shown in figure 1 is a complex enterprise. In the 2005–06 school year, 97,382 public elementary or secondary schools were in operation. The majority of these schools (87,585) were focused on the standard curriculum, whereas 1,221 provided targeted vocational or technical education, 2,128 provided special education services, and 6,448 offered some alternative form of education (Hoffman 2007). Included in this total number of schools based on public funding were 3,780 charter schools (Hoffman 2007). Charter schools are public schools that are allowed to operate with freedom from many of the regulations that apply to traditional public schools. In addition to public schools at the elementary and secondary levels, approximately twenty-nine thousand private schools were in operation, allocated 45:35:20 to Catholic, other religious, and nonsectarian schools, respectively (Snyder, Dillow, and Hoffman 2007; U.S Department of Education 2008b).

In 2005–06, U.S. public schools accounted for 49,113,474 students (Sable and Garofano 2007), private elementary and secondary schools contributed another 6,062,000 students (Snyder, Dillow, and Hoffman 2007), and home schooling accounted for approximately 1,200,000 students (U.S Department of Education 2008b). As table 1 suggests, the number of students in U.S. schools has risen steadily since 1985 (Hussar and Bailey 2007).

Education is compulsory by law in all states from age six or seven through at least age sixteen (Snyder, Dillow, and Hoffman 2007). Although the law requires compulsory education, it also allows for home schooling of students by their parents. The percent of students completing a public school education can be quantified many ways. The *average freshman graduation rate* provides an estimate of the proportion of public high school students who graduate from high school four years after having entered the ninth grade. As such, it provides a picture of the percentage of students completing the secondary school program on schedule. Of those who entered high school as ninth graders in the fall of 2000, 74.3% graduated (finished twelfth grade) in the spring of 2004 (Laird et al. 2007), an improvement of 2.4% since the spring of 2002. Many who do not complete high school with their class earn equivalent diplomas later. The *status completion rate*, another completion ratio, provides data on the percent of people by age ranges who are not attending a secondary school but who have a high school diploma or have completed a high school equivalency program, irrespective of when either path to completion was accomplished. In the twenty-two- to twenty-four-year-old age group in 2005, 89.8% of females and 85.4% of males had a high school diploma or its equivalent, with major differences among racial or ethnic subgroups: 92.3% of whites, 85.9% of blacks, and 70.2% of Hispanics (Laird et al. 2007).

Table 1
School and College Enrollments and Projections over Time (in Millions)

	Year						
Level	1985**	1990**	1995**	2000**	2005***	2010***	2015***
K–12 Public	39.4	41.2	44.8	47.2	48.7	49.4	51.2
K–12 Private*	5.6	5.6	5.9	6.2	6.4	6.6	6.9
Postsecondary	12.2	13.8	14.3	15.3	17.4	18.7	19.9

*Nongovernmental, including parochial schools (governed by religious bodies)
Sources: **Snyder, Dillow, and Hoffman 2007; Sable and Garofano 2007; Hussar and Bailey 2007.
***Projections are required in last three columns as official statistics trail calendar by four years).

As table 1 suggests, the number of students in U.S. schools has risen steadily since 1985. Projections through 2015 show the total number of students in public grades K–12 schools continuing to increase. Elementary and middle grades enrollments are projected to increase at a decreased rate through 2015, whereas secondary school enrollments are projected to peak in 2008 and then start to decrease slowly (Hussar and Bailey 2007). The proportion of grades K–12 students in private education has changed very little since 1985, varying between 11 and 12% of the total grades K–12 enrollment. At the same time, the percent of students continuing after secondary school to some form of postsecondary education has increased dramatically, 29% between 1990 and 2005. Projections see this increasing to a 44% growth from 1990 to 2015 (Hussar and Bailey 2007).

College Admission

Graduates of public or private senior high schools may matriculate into the nation's colleges, but they must apply to individual schools to be considered for admission. Many two-year colleges will accept any secondary school graduate from the geographic area they serve. Other two-year colleges and most four-year colleges require students to have taken certain numbers of courses in English, mathematics, science, social studies, and foreign language for admission. Many state-supported institutions have formulas for admission that may take into consideration the intended field of study, secondary school course grades, percentile rank in class, scores on college entrance examinations, letters of recommendation, sports and other extracurricular activities, and other information supplied by the high school or the students. Private colleges use some of the same criteria as public institutions but may take other factors, such as family variables, into consideration. More selective schools also consider the difficulty of courses taken in high school and scores on Advanced Placement examinations offered by the College Board.

The mean costs of college attendance, including tuition, fees, room, and food at four-year public and private colleges in 2006–07 were $10,530 and $26,057, respectively (College Board 2006a). Many students receive scholarships and other types of financial aid from various sources, including the college they attend, government sources, or private foundations. The College Board scholarship database has more than 2,100 programs that award more than $3 billion in support of 1.7 million students annually (College Board 2007d). In 2003−2004, 63% of all undergraduate postsecondary students received financial aid (grants, loans, or student work programs). The federal government offers tax credits for tuition and certain other education expenses (Berkner and Wei 2006).

Matriculation to college does not have to occur immediately after high school graduation, nor does attendance in college need to be full-time. For financial and other reasons, many students delay college work, either altogether or by attending only part-time. From 1990 to 2005 (the latest data available), proportionally more college students became enrolled full-time on two-year and four-year campuses (57% in 1990 versus 62% in 2005). Between 1990 and 2005, the percent of full-time enrollments on four-year campuses increased from 69% to 74%. A similar phenomenon occurred on two-year campuses where the percent of full-time enrollments increased from 37% to 41% (Horn, Cataldi, and Sikora 2005; Snyder, Dillow, and Hoffman 2007). Women overtook men as the majority of college attendees in 1979 and accounted for 57.4% of postsecondary enrollments in 2005. In 2005, 61.3% of postsecondary students were twenty-four years of age or younger (Snyder, Dillow, and Hoffman 2007). Recent data suggest that the percentage of underrepresented groups enrolling or intending to enroll is increasing (College Board 2007a).

The Governance of Education in Mathematics: "No Child Left Behind" Legislation

Because the Constitution of the United States does not claim education as a responsibility of the federal government, individual states are considered to have responsibility for the education of their citizens. Legislation at the state level provides most of the structure for schooling. State laws define the ages for compulsory education for all students; outline the general framework for required studies in reading, writing, mathematics, science, social science, physical education, and other subjects; define the minimum number of days of school attendance per year; and define requirements for teacher certification. These laws, however, present little or no regulation or monitoring of home schooling. State laws also provide the mechanisms by which local schools are recognized by the state government and provide statutes for the founding and accreditation of private schools.

Nevertheless, the U.S. government has a Department of Education that sets certain standards for schooling and provides funding for special programs, such as school lunch programs for students in poverty and compensatory programs for students needing special educational assistance. The role of the federal government has increased markedly in the past few years as a result of the No Child Left Behind Act (NCLB Act) passed by Congress in 2001. The NCLB Act authorizes the Department of Education to undertake a program with financial incentives for schools with good performance and penalties for schools with poor performance that is unprecedented in the nation's history (U.S. Department of Education 2008a).

Three days after taking office in January 2001, President Bush announced No Child Left Behind, his framework for education reform that he described as "the cornerstone of my Administration" (Bush 2001). Less than a year later, Congress passed the NCLB Act of 2001.

The NCLB Act has four main thrusts: increased accountability for states, school districts, and schools; greater choice for parents and students, particularly those attending low-performing schools; more flexibility for states and local educational agencies (LEAs) in the use of federal education dollars; and a stronger emphasis on reading, especially for the youngest children. The disaggregation of state and local data required by the NCLB Act mandates that all students, and in particular special education students of various types, receive a high quality mathematics education. In short, the success of *all students* does truly mean a focus on *all*. The summary here is taken from the Executive Summary written by the Department of Education (U.S. Department of Education 2008a).

Increased accountability

The NCLB Act requires all fifty states to implement statewide accountability systems covering all public schools and students. These systems must be based on challenging state standards in reading and mathematics, annual testing for all students in grades 3–8, and annual statewide progress objectives ensuring that all groups of students reach proficiency within twelve years. Assessment results and state progress objectives must be broken out by poverty levels, race, ethnicity, disability, and limited English proficiency to ensure that no group is left behind. School districts and schools that fail to make adequate yearly progress (AYP) toward statewide proficiency goals will, over time, be subject to improvement measures, corrective action, or restructuring measures aimed at getting them back on course to meet state standards. Schools that meet or exceed AYP objectives or close achievement gaps will be eligible for state Academic Achievement Awards.

More choices for parents and students

Districts must give students attending schools identified for improvement measures, corrective action, or restructuring the opportunity to attend a better public school, which may include a public charter school, within the school district. The district also must provide transportation to the new school. Districts must permit low-income students attending persistently failing schools (those that have failed to meet state standards for at least three of the four preceding years) to use Title I funds (a federal program that provides financial assistance through state and local school districts to schools with high numbers or high percentages of poor children to help ensure that all students succeed) to obtain supplemental educational services from the public- or private-sector provider selected by the students and their parents. Providers must meet state standards and offer services tailored to help participating students meet challenging state academic standards. Schools that want to avoid losing students—along with the portion of their annual budgets typically associated with those students—will have to improve or, if they fail to make AYP objectives for five years, run the risk of reconstitution under a restructuring plan.

Greater flexibility for states, school districts, and schools

New flexibility provisions in the NCLB Act include authority for states and local districts to transfer up to 50% of the funding they receive under four major state grant programs to any one of the programs or to Title I. The covered programs include Teacher Quality State Grants, Educational Technology, Innovative Programs, and Safe and Drug-Free Schools.

Putting Reading First

The NCLB Act stated President Bush's unequivocal commitment to ensuring that every child can read by the end of third grade. To accomplish this goal, the new Reading First initiative significantly increases the federal investment in scientifically based reading instruction programs in the early grades. This new initiative makes competitive six-year awards to LEAs to support early language, literacy, and prereading development of preschool-age children, particularly those from low-income families. Recipients are to use instructional strategies and professional development drawn from scientifically based reading research to help young children to attain the fundamental knowledge and skills they will need for optimal reading development in kindergarten and beyond.

Other major program changes

The NCLB Act of 2001 combines the Eisenhower Professional Development and Class Size Reduction programs into a new Improving Teacher Quality State Grants program that focuses on using practices grounded in scientifically based research to prepare, train, and recruit high-quality teachers. The new program gives states and LEAs flexibility to select the strategies that best meet their particular needs for improved teaching that will help them raise students' achievement in the core academic subjects. In return for this flexibility, LEAs are required to demonstrate annual progress in ensuring that all teachers of core academic subjects within the state are highly qualified.

Reactions to the NCLB Act

The NCLB Act has, perhaps, been the most discussed piece of federal legislation dealing with education in the history of the United States. Although the need to develop some form of accountability system that monitors whether all children enrolled in grades K–12 public education have access to programs aimed at their success is generally accepted, neither the public nor those involved in the operations of grades K–12 systems believe that the present formulation of NCLB is the best possible. Opponents point to the facts that the measurement of students' capabilities in an academic area is based on a single assessment and that the sanctions implemented by the program do not directly address the improvement of and support of the educational programs that provide instruction for the affected children.

As Congress struggles in 2007–08 to improve the supports and limit the restrictions associated with the NCLB Act, schools report that they are struggling to comply with the regulations amidst the many unintended consequences the monitoring system has imposed on states, school districts, and individual schools. In many instances, schools report that they have to spend more time dealing with responding to queries than they do in educating the students they serve (National School Boards Association 2008).

One hopeful sign amidst the struggles over the direction of federal support was the signing in 2007 of the *America Competes Act*. This act, titled America Creating Opportunities to Meaningfully Promote Excellence in Technology, Education, and Science, is focused on strengthening education and research related to science, technology, engineering, and mathematics (STEM). In particular, it created grant programs for states to remove barriers to excellence in STEM and develop better alignment of school curricula in these areas with the ongoing demands and needs of higher education, the workforce, and the nation's armed forces. In addition, the act provides for increased support for professional development programs for teachers of STEM-related subjects in the nation's schools. These issues were also firmly supported by the Business Roundtable's report *Tapping America's Potential: The Education for Innovation Initiative* (2005). Many of the same recommendations were made in the National Academy of Sciences Committee on Science, Engineering, and Public Policy's report *Rising above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future* (2007).

The results of the *39th Annual Phi Delta Kappa/Gallup Poll of the Public's Attitudes toward the Public Schools* reflect a similar dissatisfaction with the outcomes associated with the legislations. Globally, the June 2007 respondents reported the following:

> 26% believe NCLB is ‘helping’; 27% believe it is ‘hurting’; 41% say it is making no difference at all. This is a total of 68% who believe the law is hurting the performance of schools or making no difference. Of those respondents who claim to know a great deal or a fair amount about NCLB, 37% believe it is ‘hurting’ as compared to 28% believing it is ‘helping.’ (Rose and Gallup 2007; p. 34)

Mathematics Education in U.S. Schools

The description of mathematics education as an enterprise in U.S. schools is more difficult than the description of education as a whole, because education in mathematics is, in most instances, left to the control of a locally elected board of education in each school district. Each district, operating under its own authority and various state laws, sets standards for, designs delivery programs for, and provides financial support for its own mathematics education program. Given that the United States had 14,205 public school districts in 2005, the views of mathematics and its goals—and the amount of resources expended toward mathematics—are quite variable. The growing number of students who are home schooled for all or a portion of their grades K–12 education only compounds this diversity (Snyder, Dillow, and Hoffman 2007).

The guidance that states provide to schools in their jurisdictions also varies. All fifty states have standards or curricular frameworks for mathematics as part of their required programs of study. These standards or frameworks for mathematics take a variety of forms in outlining what students should know and be able to do as a result of their study of school mathematics. In most states, the curricular programs of study are not binding, but they may define the boundaries for state assessment programs in mathematics, outlining what mathematical knowledge is expected by a certain grade level. At least forty-six states also employ resource individuals (state mathematics supervisors or consultants) who consult with schools on questions concerning the classroom teaching and learning of mathematics and on issues concerning statewide assessment programs. In the end, however, the decisions made in a local school district determine the actual content of the school mathematics program within that district. Research studies have shown that school curricular decisions are influenced by a number of factors, which include the adopted textbook series (Porter et al. 1988) and state and national standards (Reys 2006).

The picture is much the same at the college and university level. The programs of study U.S. students complete under the name of mathematics vary greatly. Yet a great deal of similarity exists in the mathematics curricula offered by schools and universities. Part of this similarity is the result of core recommendations for study in mathematics issued explicitly by state governments and professional societies and implicitly by commercial textbooks and examinations.

PART II: Intended Curriculum

Historical List of Major Documents—Grades K–12

As in the 1990s, the 1890s also saw major increases in enrollments and pressures on schools to adjust to these increases. Thus began more than a century of meetings and recommendations of professional groups of national scope attempting to define, in varying amounts of detail, a strong mathematics program. Major recommendations and reports include the following:

1894: Committee of Ten on the Secondary School Syllabus

1899: NEA Committee on College Entrance Requirements

1911–18: International Commission on the Teaching of Mathematics

1916–23: National Committee on Mathematical Requirements

1933–40: Joint Mathematical Association of America MAA-NCTM Committee to Study Place of Mathematics in Secondary Education

1938–40: Progressive Education Association, *Mathematics in General Education*

1943: *Pre-Induction Courses: Essential Mathematics for Minimum Army Needs*

1944–47: Commission on Post War Plans

1959–60: Commission on Mathematics, College Entrance Examination Board

1963: Cambridge Conference on School Mathematics, *Goals for School Mathematics*

1964: First International Study of Mathematics Achievement by the International Association for the Evaluation of Educational Achievement IEA (report in 1967)

1975: National Advisory Committee on Mathematical Education (NACOME), *Overview and Analysis of School Mathematics K–12*

1975: National Institute of Education Conference on Basic Mathematics Skills and Learning

1976: National Council of Supervisors of Mathematics, *Position Paper on Basic Mathematical Skills*

1978: NCTM-MAA, *Recommendations for the Preparation of High School Students for College Mathematics Courses*

1980: NCTM, *An Agenda for Action*

1982: Conference Board of Mathematical Sciences, *The Mathematical Sciences Curriculum K–12: What Is Still Fundamental and What Is Not*

1983: National Commission on Excellence in Education, *A Nation At Risk*

1983: College Board, *Academic Preparation for College*

1987: SIMS, *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective* (using data collected in 1981–82)

1989: National Research Council (NRC), *Everybody Counts*

1989: NCTM, *Curriculum and Evaluation Standards for School Mathematics*

1990: National Research Council, *Reshaping School Mathematics*

1991: NCTM, *Professional Standards for Teaching Mathematics*

1995: NCTM, *Assessment Standards for School Mathematics*

1995: American Mathematical Association of Two–Year Colleges AMATYC, *Crossroads in Mathematics: Standards for Introductory College Mathematics*

1995–96: TIMSS Third International Mathematics and Science Study (follow-up reports in 2000, 2004, 2008, …)

2000: NCTM, *Principles and Standards for School Mathematics*

2001: Conference Board of the Mathematical Sciences CBMS, *The Mathematical Education of Teachers*

2001: NRC, *Adding It Up: Helping Children Learn Mathematics*

2004: Organization for Economic Cooperation and Development OECD, *Learning for Tomorrow's World: First Results from PISA 2003* (follow-up reports in 2007, 2010, 2013, …)

2004: OECD, *Problem Solving for Tomorrow's World: First Measures of Cross-Curricular Competencies from PISA 2003*

2004: MAA Committee on the Undergraduate Program in Mathematics CUPM, *Undergraduate Programs and Courses in the Mathematical Sciences*

2005: ASA: *Guidelines for Assessment and Instruction in Statistics Education: A Pre-K–12 Curriculum Framework* (GAISE Report)

2006: AMATYC, *Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College*

2006: College Board, *College Board Standards for College Success: Mathematics and Statistics*

2006: NCTM, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*

2007: NCTM, *Mathematics Teaching Today: Improving Practice, Improving Student Learning*

National Council of Teachers of Mathematics *Standards* and *Focal Points*

In 1985, building on recommendations within the mathematics education community starting in 1975 and the wider consensus of a need for raising expectations in a contemporary mathematics curriculum, NCTM began the process that led to standards documents on curriculum and evaluation (National Council of Teachers of Mathematics [NCTM] 1989), teaching (NCTM 1991), and assessment (NCTM 1995).

Although throughout the twentieth century various groups had made suggestions relative to what the U.S. school mathematics curriculum should be, none combined the scope and detail of these three documents. The *Curriculum and Evaluation Standards* (NCTM 1989) provided a listing, by grade-level bands (K–4, 5–8, and 9–12), of the mathematics that students should know with respect to problem solving, reasoning, communication, connections, and various content aspects of mathematics relevant to those grade levels.

No one had described mathematics teaching in as much detail as the *Professional Standards for Teaching Mathematics* (NCTM 1991). The teaching standards took as a basic principle that what students know about mathematics is a product of how they learned it. They asserted that concepts, procedures, and relationships are often best developed in contexts that ask students to develop knowledge themselves under the guidance and watchful eye of a teacher and to engage in discourse about that knowledge. They called on teachers to create conditions that would allow learners to focus on important aspects of content and on the connections between mathematics and other subject areas and between various areas within mathematics. The *Assessment Standards for School Mathematics* (NCTM 1995) asked for assessments that reflected the deeper understanding called for in the curriculum and teaching standards.

The existence of the NCTM *Standards* documents also influenced other school subjects. By the end of the 1990s, professional organizations in virtually every other major school discipline had published their own standards. The visibility of the *Standards* also led to a greater interest in the revision that began in 1995 and culminated in the release of *Principles and Standards for School Mathematics* in April 2000 (NCTM 2000). Recognizing that not everyone agreed with the recommendations in the *Standards*, NCTM took the approach of developing a draft of *Principles and Standards* and then holding a nationwide year of discussion on that draft that allowed individuals and organizations to comment on them, so they might be revised on the basis of that feedback.

Principles and Standards is guided by six principles: (1) Equity: Excellence in mathematics education requires equity—high expectations and strong support for all students; (2) Curriculum: A curriculum is more than a collection of activities—it must be coherent, focused on important mathematics, and well articulated across the grades; (3) Teaching: Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well; (4) Learning: Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge; (5) Assessment: Assessment should support the learning of important mathematics and furnish useful information to both teachers and students; (6) Technology: Technology is essential in teaching and learning mathematics—it influences the mathematics that is taught and enhances students' learning (NCTM 2000).

At each of four grade-level intervals—pre-K–2, 3–5, 6–8, and 9–12—*Principles and Standards* contains a comprehensive body of mathematical understandings and competencies organized under five content areas—number and operations, algebra, geometry, measurement, and data analysis and probability—and five ways of acquiring and using that content—problem solving, reasoning and proof, communication, connections, and representation. Despite its potential influence, *Principles and Standards* is only a resource and guide in the U.S. educational system; it carries no legal weight.

Following NCTM's release of *Principles and Standards*, mathematics educators clamored for more guidance for what to teach at various grade levels. Although state curricular frameworks had fleshed out sets of expectations, they frequently contained more objectives than instructional time would accommodate (Reys et al. 2005; Reys et al. 2006). This packing of the grade-level curricula was compounded with the development of instructional lists to meet mandates imposed by state and NCLB Act demands. Teachers were asking for a listing of the "big ideas" that would form the core objectives for each grade level and allow them to better connect the topics they taught in a given year. In response to this demand and with a desire to better articulate the expectations of the grade-level interval curriculum statements in *Principles and Standards* with a set of coherent concise statements about appropriate content grade-by-grade, NCTM released *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* in 2006. The *Curriculum Focal Points* listed three focal points, or "big ideas," per grade and made statements about related connections to these big ideas. *Curriculum Focal Points* was immediately met with acclaim from both sides of the struggle between concepts and procedural skills.

The curricular focal points do not specify instructional delivery, materials to be used, or professional development required for teachers; they do focus on the communication of a linked listing of content that might be found in a coherent grades K–8 curriculum focused on developing a basis for students' deep understanding of mathematics and its applications. For example, the focal points for grade 6 are as follows:

- Number and Operations: Developing an understanding of, and fluency with, multiplying and dividing fractions and decimals
- Number and Operations: Connecting ratio and rate to multiplication and division
- Algebra: Writing, interpreting, and using mathematical expressions and equations

The first portion of each focal point ties the point to one of the *Principles and Standards* Content Standards. The accompanying descriptive phrase identifies the exact learning target for the grade level. Each of these three focal points then has an accompanying paragraph describing the nature of the learning expected for students in that grade. Additional material is presented connecting these focal points to other content of lesser importance that will either be extended or developed during the grade level. In essence, the curricular focal points provide a crucial path for teachers, supervisors, curricular developers, and measurement specialists as they look at students' progress through their education.

Two projects have established national level expectations for grades 9–12 mathematics classrooms. These projects are the *College Board Standards for College Success: Mathematics and Statistics* (College Board 2006b) and the exit requirement competency list developed by the American Diploma Project (ADP) of Achieve (Achieve 2007). The first of these documents provides an in-depth look at the expectations in mathematics for students in grades 6–12. The document places specific emphasis on preparing students to enter college or the workplace in a competitive stance. The standards cover both mathematics and statistics in a connected and complementary form. Each grade level's requirements derive from four to six standards, based on the Board's big ideas, which a statement of three to four enabling objectives then further describes. Still further explication comes from the statement of performance expectations that would assist assessment specialists in designing instruments to measure students' proficiency associated with the grade-level goals. Thus the standard-objective-enabling performance triads are aimed at administrators and the public, curricular leaders and teachers, and assessment developers, respectively.

The ADP's benchmarks provide a listing of mathematical content and skills in which high school students must be proficient when they graduate if they intend to continue to college or enter the workforce in a position that provides them the possibility of continued advancement. Thirty states currently belong to the ADP Network. The ADP benchmarks are divided into four general content areas, but no specific grade-level expectations are used to subdivide the benchmarks further. The four content areas are the following:

- Number Sense and Numerical Operations
- Algebra
- Geometry
- Data Interpretation, Statistics, and Probability

State Standards and Textbook Adoption Guidelines

Because the requirements of the NCLB legislation include test standards, all fifty states have now created and adopted state curricular frameworks for mathematics. Most of the states (forty-six) had some form of grades K–12 mathematical expectations before the release of the initial NCTM *Standards* document in 1989. Many of these lists stemmed from the mid-1980s following the release of the *A Nation at Risk* report (National Commission on Excellence in Education 1983). These documents served as a basis for the renewed focus on the mathematics curriculum in the early 1990s following the release of the original NCTM *Standards* (1989) and provided a foundation for revision and extension of state standards following the release of the recommendations of *Principles and Standards* (CCSSO 2003).

Within states, local school districts make the decisions regarding which materials to use. For textbooks, this decision process is called *textbook adoption* and may be subject to formal regulations. In twenty-two states, mainly in the south and west, certain state funds are earmarked for textbooks selected or recommended by a statewide committee for use in that state in accordance with the state's content standards. In these states, all adoptions for a given course or level may take place in the same school year. State textbook adoption practices have tended to become more lenient over the years, allowing schools a greater selection of materials. In the other twenty-eight states, school districts typically adopt under more relaxed rules; they may have to adopt every five years but they may also select from any available materials (Association of American Publishers 2007; National Association of School Textbook Administrators 2007).

During the mid-1980s many of the states raised their high school graduation requirements in mathematics in reaction to the findings of the National Commission on Excellence in Education's report *A Nation at Risk* (1983). As of 2007, forty-three states have statewide credit requirements for high school graduation, whereas seven transfer this power to local school districts. Two of the forty-three with statewide credit requirements require four credits (years) in mathematics, twenty-six require three credits, and fifteen require two credits. Twenty of these states identify one or more specific mathematics courses that must be taken for graduation, typically first-year algebra. In the next five years, graduates in several of these states will face increased requirements. Only two states essentially required Algebra 1, Geometry, and Algebra 2 or its equivalent in 2007; at least eleven states will have that requirement in place by 2012. Michigan and Kentucky will require all students to take a mathematics course during their final year of secondary school education, starting in 2011 and 2012, respectively (Zinth 2006; Zinth and Dounay 2006, 2007). A consortium of nine states has joined forces to launch a new, common student assessment in Algebra 2 in conjunction with the ADP network (Achieve 2007).

College-Level Documents

Although public schools receive some guidance in terms of mandated subjects, time allotments, and graduation requirements, college or university programs, be they public or private, have few restrictions.

At the community college level, the American Mathematical Association of Two-Year Colleges (AMATYC) developed and released *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus* (1995), a document consistent with the spirit and content of the NCTM *Standards* documents. This document gave recommendations from the AMATYC regarding the teaching and learning of collegiate courses at the precalculus level. Like NCTM, AMATYC realized that it is not just the content that shapes what students learn as a result of their enrollment in collegiate courses in mathematics. As a result, AMATYC extended the reach of their recommendations for the first two years of the collegiate curriculum in 2006 with the publication of *Beyond Crossroads: Implementing Standards in the First Two Years of College*. This document clearly sets out the systemic relationships that exist among content; the formation of content into a curriculum; and the way knowledge of students' learning, classroom pedagogy, and assessment strategies, along with continued professional development of the faculty itself, plays into helping programs reach their potential in serving their students. This document focuses on the fact that what students learn is a function of "how they learn it" (Lutzer et al. 2007).

Recommended programs for the composition and delivery of collegiate courses taken by prospective teachers of mathematics in any state are part of the state's own certification requirements. *The Mathematical Education of Teachers* (CBMS 2001), a set of national recommendations for the content and format of teacher education programs, was developed with broad input from the mathematical sciences community working in conjunction with professions in mathematics teacher education. The recommendations built on previous sets of recommendations developed first in the 1960s by the Mathematical Association of America's (MAA's) Committee on the Undergraduate Program in Mathematics (CUPM 1968) and then supported in the early 1970s by NCTM (Commission on Preservice Education of Teachers of Mathematics of the NCTM 1972). These, too, were both extended in the early 1980s (Commission on Preservice Education of Teachers of Mathematics of the NCTM 1981; MAA 1983). The release of NCTM's *Professional Teaching Standards* in the early 1990s (Leitzel 1991; NCTM 1991) moved the education of teachers of mathematics to center stage. Mathematics teacher education became a major research activity, and the *Journal of Mathematics Teacher Education* appeared on the scene in 1998. In 2007, NCTM published *Mathematics Teaching Today: Improving Practice, Improving Student Learning*, a revised and updated edition of their 1991 professional teaching standards.

Although previous initiatives of the mathematics community in recommending courses of study for preservice teachers of mathematics had been successful in helping shape state requirements for certification, NCTM's *Professional Standards for Teaching Mathematics* (1991) and CBMS's *The Mathematical Education of Teachers* (2001) moved the recommendations and state certification guidelines to a new level. Although many states have significantly improved their certification requirements, some states still allow individuals to teach mathematics with less than the full requirements for certification if they are teaching mathematics for less than one-half of their teaching load or if they are hired in a region experiencing a shortage of mathematics teachers. These loopholes still subject many of the nation's students in the middle school and high school levels to learning mathematics from a teacher who fails to meet the stated certification requirements for teaching mathematics.

Relating to collegiate programs more broadly are the recommendations of CUPM of the MAA. In the 1990s, CUPM provided recommendations on the general program for undergraduate mathematics majors and for the teaching of calculus (CUPM 1991; Tucker 1995; Roberts 1996). In 2004, CUPM issued *Undergraduate Programs and Courses in* *the Mathematical Sciences: CUPM Curriculum Guide 2004*. This set of recommendations resulted from a four-year developmental cycle in which the committee consulted widely with other professional bodies in mathematics and groups that employ graduates of mathematical sciences programs. Similar to recommendations from NCTM, AMATYC, and the American Statistical Association (ASA), the CUPM document addresses the content and delivery of the program most directly linked to the authoring group—undergraduate mathematics. Like the trend noted in other reports of the same type appearing in the past ten years, however, the CUPM document also reaches out to talk about mathematics for all students. Although part of this document features comments for a broadened major, other comments focus on lifting the overall mathematical literacy of students studying at the collegiate level.

The CUPM document focuses on six major recommendations for collegiate mathematics programs:

- Departments should understand both students' capabilities and their aspirations and see that their programs are designed and delivered in a fashion that optimizes student opportunities to reach their goals.
- Coursework in undergraduate mathematics programs should work toward developing crucial reasoning skills and mathematical habits of mind. In particular, students should learn to formulate and state problems carefully; attack them with reflection and persistence, aggressively exploring examples and testing conjectures; and clearly communicate their findings orally and in writing.
- Instruction should focus on presenting primary concepts from a variety of perspectives, employing examples and applications to facilitate and illustrate ideas and procedures. Connections to other subjects and links to contemporary topics from the mathematical sciences should be made to illustrate the vitality and importance of mathematics in contemporary society.
- Faculty from the mathematical sciences should reach out and work with colleagues from other departments in designing curricular projects, undergraduate research projects, and new courses or majors reflecting the links between the mathematical sciences and other disciplines.
- Courses should incorporate activities that assist students appropriately and use technology effectively as a tool in solving problems and understanding mathematical concepts, relationships, and algorithms.
- Department and university administrators should create an environment that fosters professional development and the growth of faculty in the mathematical sciences and that supports and rewards efforts by faculty to improve the teaching of mathematics and the health of the undergraduate curriculum in the mathematical sciences (CUPM 2004a).

In addition, the report provided specific suggestions on programs for liberal arts teachers; preservice elementary, middle, and high school teachers; and undergraduate mathematics majors. Both the report itself and an accompanying guide with numerous examples of programs already implementing the suggestions (*Illustrative Resources for CUPM Guide 2004*[*b*]) can be downloaded free of charge at www.maa.org/CUPM/.

The previous report for ICME-10 focused on reform in the calculus curriculum for undergraduates. Although efforts continue to shift instruction in directions that emphasize the connections between calculus and its applications in modern settings and that use a wide variety of linked representations in promoting deeper understanding (text, graphical, data and numerical, and symbolic), no new major projects or initiatives have taken place since 2004. Data elsewhere in this volume illustrate the increases in enrollments in calculus courses. The faculty members involved in the 2005 CBMS survey on undergraduate instruction ceased comparing the teaching of reform-based sections versus that of traditional sections of calculus, because they were no longer able to ascertain with certainty whether the data continued to be meaningful (Lutzer et al. 2007). Most textbooks have adopted portions of the reform agenda in their design, and programs seem to have incorporated the major features into their curricula and are no longer offering special sections to accommodate reform recommendations.

The AMATYC, MAA, and ASA have been diligently working with NCTM and other partner groups in the mathematical sciences to upgrade the transition from secondary school education to the first two years of undergraduate mathematics with new offerings dealing with mathematical literacy, precalculus, and new forays into data analysis and interpretation. AMATYC has published *Beyond Crossroads* (2006) and ASA has published recommendations from their *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) (Franklin el al. 2007). The GAISE reports are available free of charge online at www.amstat.org/education/gaise. This project has produced a framework for teaching statistics in prekindergarten through grade 12 (Franklin et al. 2007) and recommendations for teaching statistics in introductory college courses (ASA 2007). Both these reports are discussed in detail elsewhere in this volume.

In addition to the CUPM guidelines, the MAA has published several volumes addressing aspects of the undergraduate curriculum in the past decade. These volumes include materials addressing the reform in college algebra and precalculus (CUPM 2007; Hastings 2006; Lenker 1998). Considerable discussion has taken place in the past decade over changes in the core undergraduate curriculum, beginning with linear algebra and continuing through the modern algebra, real variables, and other courses typically found in or recommended for the bachelor's degree in mathematics (Dossey 1998; Arney and Small 2002). At the same time, there has been a growing awareness of the role played by quantitative literacy for all students. Increasing the percentage of the population highly trained in mathematics is not enough if a majority cannot deal with the concepts and problem-solving skills required for active, enfranchised citizenship. Many of these efforts have grown out of work that began at the College Board two decades ago (Steen 2004; Gillman 2006). The one content area receiving the greatest attention in the mathematical sciences from a curricular standpoint in the last decade, however, has been statistics. The change in enrollment patterns noted elsewhere in this volume shows both the need for and the results of changes taking place in this area. Although the work of the ASA in this regard is covered elsewhere in this volume, considerable attention has been given to the topic of statistical education reform by the MAA with the publication of the works by Garfield (2005) and Moore (2000).

These volumes, as well as others dealing with special courses, such as linear algebra and modern algebra, provide teachers and curriculum committees with a quick view of the current thinking of discipline leaders, as well as samples of current programs in those areas. In addition, these organizations sponsor special-interest groups focusing on topics germane to curricular topics in the mathematical sciences. Mathematics has the Research in Undergraduate Mathematics Education (RUME) special-interest group of the MAA.

The American Statistical Association has the Section on Statistical Education for its members with a special interest in education.

Undergraduate mathematics departments have become aware that quality content is not sufficient to produce quality learning. A major portion of what students learn and are able to use resides in the interaction of that content and the ways in which they learned it. Departments have realized that they must make both quality curriculum and grounded majors department goals. Further, national scientific organizations, state boards of higher education, and professional groups are stepping forward to assure that resources are available to provide both ongoing professional development for collegiate faculty and support for continued curricular development work (Snook 2004).

Mathematicians and Mathematics Education

With the advent of NCTM's *Curriculum and Evaluation Standards* in the late 1980s, many individuals in the pure mathematics community became active in programs focused on helping teachers and schools update their curricula and improve their delivery of quality mathematics to their students. These activities included consulting, leading professional development activities, and developing policy at the local, state, and national level to assist schools in garnering more resources and support for or against the changes taking place.

Foremost among the early efforts of mathematicians in support of grades K–12 mathematics education activities was the founding of the Mathematicians and Education Reform Network (www.math.uic.edu/~mer/pages), built on earlier activities that individual members of the pure mathematics community had been involved in over the years. More recently, the Mathematical Science Research Institute at the University of California at Berkeley has hosted a series of workshops on *Critical Issues in Education* (www.msri.org/calendar/index_workshops; MSRI 2007) on the following topics central to mathematics education:

- 2004: Assessing Students' Mathematical Learning Issues, Costs and Benefits
- 2005: The Mathematical Knowledge for Teaching (K–8): Why, What & How?
- 2006: Raising the Floor: Progress and Setbacks in the Struggle for Quality Mathematics Education for All
- 2007: Critical Issues in Education: Teaching Teachers Mathematics
- 2008: Critical Issues in Education Workshop: Teaching and Learning Algebra.

Other special-topic workshops are scheduled to follow. These workshops bring mathematicians and mathematics educators together to discuss and to further the work of providing quality mathematics education to all children. Special sessions focusing on mathematics education have also brought teachers, mathematicians, and mathematics educators together at the Park City Institutes conducted by the Institute for Advanced Study and the National Science Foundation (Institute for Advanced Study 2008).

PART III: Implemented Curriculum

Because the United States has no commonly accepted mathematics curriculum for the public schooling of its youth, the collection of data on the implemented curriculum is difficult, if not impossible, to obtain. National data are not routinely collected regarding topic coverage by grade or commonly available listings of the specific textbooks sold or in use in the United States in grades K–12. Work currently under way at the Center for the Study of Mathematics Curriculum has resulted in one look at the diversity of curricula available to students at the upper intermediate and middle school levels (Reys 2006)

Mathematics Study in Elementary (Grades K–5) Schools

A teacher who teaches reading, science, and social studies in elementary school and is with the same students almost the entire day almost always also teaches mathematics. These teachers do not have the time to create lessons for all these subjects and, as a result, tend to rely on the mathematics instructional materials purchased by their school district. According to an industry survey, more than 93% of schools in the country reported in 2004-05 using a basal or core mathematics series that they either follow very closely or from which they pick and choose as needed. In the past, most publishers would market a coordinated elementary school curricular series to cover all grades K–8. Beginning in the mid-80s, these curricula were split into two parts, a grade K–5 or K–6 elementary school series and a 6–8 middle school series. More recently, the grades K–5 portions of these programs have been segmented into blocks covering grades K–2 and 3–5 for marketing purposes.

Any discussion of most commonly used texts must be written and read with great care, because textbooks remain in use for several years after their initial purchases. Much of the everyday conversation about what is most popular focuses on what texts are currently selling. As a result, two distinct perceptions emerge of which textbooks U.S. students use. We will discuss two sets of curricular materials—books currently in use and books written with National Science Foundation or other support in response to the NCTM *Standards*.

Data on textbook use collected in 2004–05 indicated that 24% of elementary schools were using materials that had been purchased more than five years before, 43% were using materials that had been purchased three to five years previously, and the remaining 33% were using materials purchased within the previous two years. The three most frequently used series reported in use in grades K–2 in 2004-2005 were McGraw-Hill's *Everyday Mathematics*, Scott Foresman–Addison Wesley's *Mathematics,* and Harcourt's *Mathematics.* Collectively, these three series accounted for slightly more than 40% of the books in use in grades K–2 in 2004–05. Examining the acceptance of texts written in response to the *Standards*, the top two currently in use in grades K–5 are the *Everyday Mathematics* mentioned above and Pearson/Scott Foresman's *Investigations in Number, Data, and Space*. Collectively, these two series account for 26% of the textbooks in use in grades K–2 in 2004–05.

Examining the data on textbooks in grades 3–5, the publisher results for both books in use and textbooks written in response to the *Standards* that were in use gave the same results as those found in grades K–2. The overall frequency-of-usage results for 2004–05 showed that the most used series accounted for again a little more than 40% of the books in use. Reexamining the data from the perspective of the top two texts written in response to the *Standards*, *Everyday Mathematics* and *Investigations* were again the two series most in use. These two series collectively accounted for 18% of the texts in use in grades 3–5.

Mathematics Study in Middle or Junior High School (Grades 6–8)

When the data for grades 6–8 are analyzed, the patterns of usage are much more difficult to summarize because many districts use a mixture of basal series that cover the entire span of content topics for some of the grades, but select other textbooks covering algebraic content at either an introductory level or the level that would typically be found in the first year of high school (grade 9). Examining curricular materials that were written as sequential materials for grades 6–8, the three series most frequently reported as in use in 2004–05 were Glencoe's *Math Applications and Connections*, Prentice Hall's *Prentice Hall Middle Grades Math,* and Pearson/Prentice Hall's *Connected Mathematics.* Collectively these texts were reported as being used in approximately 28% of the schools containing grades 6–8 in 2004–05. *Connected Mathematics* was one of the series developed through support from the NSF in response to the *Standards* documents. A second series developed in the same thrust was McDougal Littell's *Math Thematics.* These two texts, the first of which was one of the three most in-use texts, accounted for nearly 12% of the grade 6–8 series in use in 2004–05.

The second classification of textbooks in use in grades 6–9 includes textbooks with titles such as *Prealgebra, Introductory Algebra* (Part 1), and *Algebra 1*. These courses cover, respectively, a mixture of materials found in the grade 8 basal series plus a heavier emphasis on linear equations, the first half of the mainstream algebra course normally found in grade 9, and a full year of mainstream Algebra 1. An analysis of the data indicate that almost 16% of the schools report using a prealgebra textbook in one or more of their courses, almost 7% report using a textbook covering the first half of a first-year course in algebra, and 28% indicate using a textbook that is the equivalent of what would be found in a first year of high school (grade 9) classroom of Algebra 1. Although a listing of individual textbooks is difficult to accomplish for this portion of the curriculum because of varied editions of the same text and overlapping content, McGraw-Hill/Glencoe, Houghton-Mifflin, and Pearson/Prentice Hall/Addison-Wesley were the three leading publishing houses in this market. Additional information concerning textbooks can be found on websites provided in Part IX.

Mathematics Study in High School (Grades 9–12)

At the high school level, the mainstream curriculum currently found in U.S. secondary school classrooms is built around a sequence of three full-year courses, Algebra 1-Geometry-Algebra 2 or Algebra 1-Algebra 2-Geometry, beginning either in eighth, ninth, or tenth grade, followed by a fourth year of precalculus, usually giving strong attention to functions and trigonometry. Since the mid-1950s, an increasing percent of students have completed a year of calculus at the high school level. This latter course usually covers the content normally found in the first semester of university-level calculus. In about 20% of the cases, this course covers the equivalent of the full first year of university-level calculus.

More than 90% of the secondary schools in the country follow this curriculum for their students and present it at a slower pace for lower-performing students. Since 1990, the development of integrated secondary school programs has grown slowly where the content of algebra, geometry, functions, and data analysis have been presented in integrated programs with all topics being part of the curriculum each year of the secondary school program

(Hirsch 2007). During the past twenty-five years, high school graduation requirements and college admission requirements have increased; many four-year colleges and universities now require two years of algebra and a year of geometry for admission. Also, as a result of the impact of technology on everyday lives and worries about students' lackluster performance in international studies, the public appears to have become more aware of the role mathematics can play in the future lives and careers of secondary school students. Furthermore, as the data in table 2 indicate when combined with course-taking data from the middle grades, more students are taking courses in algebra before high school. These factors have contributed to a steady and significant increase over time in the percents of students taking higher-level mathematics courses in high school.

Table 2
Trends in Percent of Students by Most Advanced Mathematics Course Taken in High School

Percent of students by most advanced course enrolled in through final year of secondary school in the United States, from NAEP Long Term Trend studies in years shown (Campbell et al. 1997; Campbell et al. 1996; Campbell, et al. 2000; Mullis et al. 1991a; Perie, Moran, and Lutkus 2005; Shettle et al. 2007).

Course*	1978	1990	1994	1996	1999	2004
Prealgebra or General Mathematics	20	15	9	8	7	4
Algebra 1	17	15	15	12	11	9
Geometry	16	15	15	16	16	16
Algebra 2	37	44	47	50	51	53
Precalculus or Calculus	6	8	13	13	13	17

Breakdown of percentage data for students enrolled in precalculus or above in data from NAEP National studies in years shown: 1990, 2000, 2005 (Mullis et al. 1991b; Mitchell et al. 1999; Braswell et al. 2001; Grigg, Donahue, and Dion 2007).

Course*	1990	1996	2000	2005
Precalculus	9	14	17	21
Calculus	4	7	16	18

* These studies considered more descriptors of the courses than the most commonly used descriptors named here.

The trend data in table 2 indicate that fewer students are discontinuing their studies of mathematics following the completion of either Algebra 1 or Algebra 2. Confirming these data, another source (Blank and Langesen 2005) reports that the percent of high school students enrolled in mathematics significantly increased in higher-level mathematics courses between 1996 and 2004. During this period, the percent of graduating students completing four years of high school mathematics increased thirteen percent points. Data from the 2005 CBMS study of mathematics programs in two- and four-year colleges also indicates a decrease in the number of students enrolling in remedial courses in mathematics covering high school content prior to precalculus. Concurrent with these changes has been an increase in the numbers of students taking advanced placement courses in mathematics (Lutzer et al. 2007; College Board 2007b).

Much like at the grades 6–8 level, describing the actual textbooks in use in high school classrooms is very difficult beyond general comments about the leading publishers. In 2005, the three major publishing houses responsible for the majority of secondary school mathematics textbooks were Glencoe-McGraw Hill, Houghton Mifflin-McDougal Littell-Heath, and Pearson-Prentice Hall-Addison Wesley-Scott Foresman. The hyphenated sequences reflect the number of acquisitions and mergers that have taken place in this market during the past decade, which has had the effect of the merging of some product lines and the discontinuing of other long-running series. Like those interested in the middle school materials, those interested in particular products for high school would be most successful referring to the Web sites of the individual publishers as contained in Part IX of this fact book.

None of the high school series developed with initial support from the NSF has the breadth of usage that is enjoyed by their elementary and middle school counterparts. Together with the NCTM *Standards* documents, these NSF project-related series have influenced mainstream texts to include more applications and more work with technology. At the same time, pressure from colleges has influenced these texts to maintain, if not increase, skill work with algebra and functions.

Pedagogical Patterns in Grades K–12

Prior to the shift of the use of the National Assessment of Educational Progress (NAEP) to collect information for state comparisons and for demographics-based monitoring of students' achievement in the nation's schools, a larger focus was given to the collection of information on curriculum and instructional variables in the nation's classrooms. Starting with the 2005 assessment, this data was severely reduced and many of the long-term lines of data were truncated.

NAEP has tracked the usage of calculators at the classroom level since the 1980s. Initial data showed that schools owned sets for instructional purposes. But, by 2005, the data reflect that 76% of the nation's grade 4 students report owning a regular calculator and 6% report owning a graphing calculator (NCES 2008). Comparable data for grade 8 students is shown in table 3. They show that calculators are used both for learning and assessment purposes in the majority of grade 8 classrooms.

The most recent NAEP data on grade 12 calculator use were collected in 2000, when 69% of students reported using a calculator every day and another 14% reported using a calculator one to two times a week. More than 90% of students at this level have access to a calculator for their mathematics school work, a level that has remained constant since 1992. The frequency of calculator use varies by the type of course, with students in algebra and higher courses more likely to use graphing and other calculators than students in pre-algebra or regular courses (Braswell et al. 2001).

Table 3

Percents of Eigthth-Grade Students in NAEP Studies Reporting Types of Calculator Use in Mathematics Class (Braswell et al. 2001; NCES 2003; NCES 2008)

Use Calculator for Math Tests	**Always**	**Sometimes**	**Never**
2000	23	45	32
2003	22	48	30
2005	19	51	30
2007	20	54	26
Use Scientific Calculators in Math Class	**Yes**	**No**	
1996	60	40	
2000	67	33	
2003	64	36	
	Usually	**Sometimes, Not Often**	**No**
2005	34	30	36
2007	36	32	32
Use Graphing Calculators in Math Class	**Yes**	**No**	
1996	11	89	
2000	18	82	
2003	26	74	
	Usually	**Sometimes, Not Often**	**No**
2005	19	27	55
2007	21	29	49

Mathematics Study at the Postsecondary Level

At the postsecondary level, students have a wide variety of options for studying mathematics. Coursework is available through community colleges, universities, and a variety of vocational schools, work-based educational programs, and commercial outlets. The data collected every five years by the CBMS provide the best trend data for curricular programs and enrollments in two- and four-year colleges.

Mathematics courses at these institutions range from arithmetic and pre-algebra, particularly at vocational and two-year colleges, through advanced courses beyond introductory linear algebra at four-year institutions. Tables 4 and 5 demonstrate this wide range and the change in enrollments over time at two- and four-year colleges, respectively. In these tables, remedial courses include arithmetic, prealgebra, and elementary and intermediate algebra. Precalculus courses include college algebra and trigonometry as well as finite mathematics, noncalculus-based business mathematics, mathematics for prospective elementary school teachers, and other courses for nonscience majors. Calculus includes both mainstream and nonmainstream courses (e.g., calculus courses tailored to students in other majors, such as life sciences or business). These tables do not include mathematics courses taught outside mathematics and statistics departments. Enrollments are for the fall quarter or semester of the year (Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007).

Two-year college enrollments increased from about 5.95 million in 2000 to nearly 6.55 million in 2005, an increase of slightly more than 10%. During this time period, the percent of full-time students also increased by slightly more than 10%. An examination of the data in table 4 shows that this same period saw an increase of more than 25% in the number of students enrolled in mathematics. Not only was there this increase but the increase also occurred across the full range of the two-year college offerings. Remedial enrollments were up by 26%, precalculus enrollments were up by 17%, calculus enrollments were up by 2%, statistics enrollments were up by an amazing 58%, and enrollments in other courses (liberal arts, math for elementary teachers, and so on) were up by 44%. This pattern contrasts with four-year college data over the same time period, as the data in table 5 will show.

Table 4
Estimated Enrollment (in Thousands) in Mathematics Courses in Two-Year Colleges

	Year						
Course	1970	1980	1985	1990	1995	2000	2005*
Remedial	191	441	482	724	800	763	965
Precalculus	134	180	188	245	295	274	321
Calculus	59	86	97	128	129	106	108
Statistics	16	28	36	54	72	74	117
Other	171	218	133	144	160	130	187
Total	555	953	936	1295	1456	1347	1698

*Data account for shift in 2005 to report by percent of sections rather than by percent of students.

Table 4 shows that since 1985, more than half of the mathematics enrollments in two-year colleges have been at the remedial level. The overall increase in number of mathematics courses in two-year colleges is partially a function of the overall increase in enrollments at these institutions. The increase is also, however, partially a function of the increased realization of mathematics as enabling knowledge. The number of students in remedial courses at four-year colleges in 2005 reflects a 23% decrease since 1990. In this same period, two-year colleges have shown an increase of 33% in their enrollments for remedial mathematics offerings.

The data in Table 5 reflect a more static picture for course enrollments in mathematics and statistics than that observed for the two-year colleges. Enrollments in these two areas remained essentially level with only a 0.2% increase over the period from 2000 to 2005. Statistics as a course area showed a 6% gain, while mathematics' enrollment showed a 0.5% decrease over the same time span (Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007).

Examining the 2005 enrollments in the course groupings, one sees the continued decrease in remedial enrollments continuing with an 18% decrease since 2000. Precalculus enrollments decreased by 2% over the same time period. These decreases may partially be due to the shifting of this teaching to two-year colleges supporting four-year institutions and partially due to increased levels of students completing three and four years of mathematics study at the secondary school level prior to entry.

Table 5
Estimated Enrollment (in Thousands) in Undergraduate Mathematics and Statistics Courses in Four-Year Colleges

	Year						
Course	1970	1980	1985	1990	1995	2000	2005*
Remedial	101	242	251	261	222	219	201
Precalculus	538	602	593	592	613	723	706
Calculus	414	590	637	647	538	570	587
Statistics	na	na	na	125	143	171	182
Advanced	135	91	138	119	96	102	112
Total	1188	1525	1619	1744	1612	1785	1788

*The methodology of counting enrollments changed from students to sections by average size in 2005.

Enrollments in calculus grew between 2000 and 2005 by 3%, continuing the recovery from the drop in this area in the early part of the 1990s. Advanced courses in mathematics attracted almost 10% more students in the fall of 2005 than they did in the fall of 2000. Looking deeper into the advanced courses shows some increase in proof writing, combinatorics, discrete structures, advanced linear algebra, and courses historically supporting engineering and the sciences. Advanced courses showing decreased enrollments were number theory, logic, history of mathematics, and topology. The growth in statistics came in the first-year sequence in statistics and probability, applied analysis methods, and regression and correlation, whereas other areas were relatively even (Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007).

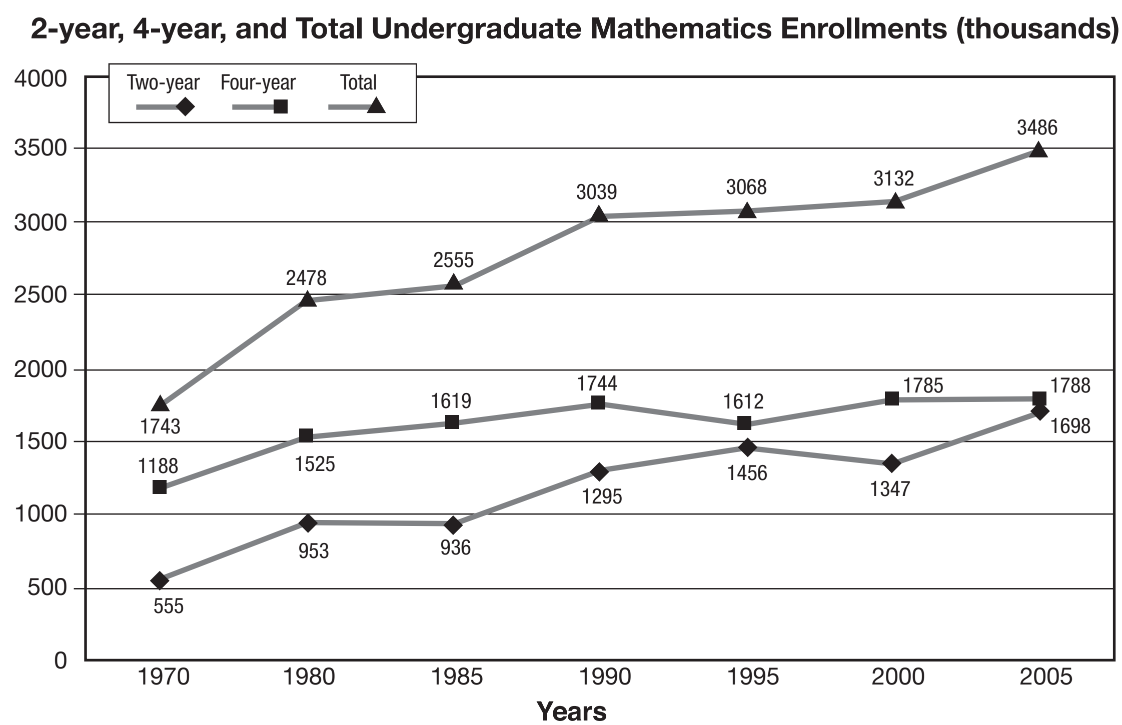

Fig. 2. Undergraduate enrollments in mathematics: 1970–2005

When the enrollments in mathematics courses in two- and four-year college mathematics departments are added (as shown in figure 2), the sums show a consistent increase in total enrollment over the time periods reported in CBMS studies. Trends for the various types of courses are mixed, however, as discussed above.

The next series of tables provide some understanding of the enrollment in big-volume course sequences. Table 6 contains data on enrollments from 1990 to 2005 in the first two semesters of calculus and the two beginning noncalculus-based statistics courses.

Table 6

Estimated Enrollments (in Thousands) in Calculus and Statistics Courses by Level and by Department Teaching Course (Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007)

	Enrollments by Years			
CALCULUS	1990	1995	2000	2005
Four-Year Institutions				
Mainstream Calculus 1	201	192	192	201
Mainstream Calculus 2	88	83	87	85
Two-Year Institutions				
Mainstream Calculus 1	53	58	53	51
Mainstream Calculus 2	23	23	20	19
Total Mainstream Calculus 1	**254**	**250**	**245**	**252**
Total Mainstream Calculus 2	**111**	**106**	**107**	**104**
Four-Year Institutions				
Nonmainstream Calculus 1	148	98	105	108
Nonmainstream Calculus 2	15	14	10	11
Two-Year Institutions				
Nonmainstream Calculus 1	31	26	16	21
Nonmainstream Calculus 2	3	1	1	1
Total Nonmainstream Calculus 1	**179**	**124**	**121**	**129**
Total Nonmainstream Calculus 2	**18**	**15**	**11**	**12**
STATISTICS	1990	1995	2000	2005
Four-Year Institutions Mathematics Departments				
Elementary Statistics	61	97	114	124
Probability and Statistics	25	18	13	19
Departments of Statistics				
Elementary Statistics	22	35	40	43
Probability and Statistics	10	8	4	5
Two-Year Institutions				
Elementary Statistics	47	69	71	111
Probability and Statistics	7	3	3	7
Total Elementary Statistics	**130**	**201**	**225**	**278**
Total Probability and Statistics	**42**	**29**	**20**	**31**
Total Calculus 1 and Statistics 1	**563**	**575**	**591**	**659**

The data in table 6 for enrollments in mainstream calculus (i.e., calculus for mathematics, engineering, and physical science students) show some short-term growth for Calculus 1. Enrollments in nonmainstream calculus show a similar increase. The longer trend picture for the total enrollment in calculus, however, is stable at best. Enrollments in first-year, noncalculus-based statistics courses were quite different. These courses, nationally, had a net gain of 26% over the ten-year period ending in 2005 for the Elementary Statistics and the Probability and Statistics enrollments combined. These data indicate that there continues to be a significant focus in both the offerings within two- and four-year institutions on an introductory course that focuses on statistics, data analysis, and chance.

Combining the enrollments of mainstream and nonmainstream Calculus 1 with those of the two statistics courses over the ten-year period from 1995 to 2005 shows an overall 15% growth in first-course enrollment in the decade. This growth was based on growth in both course areas, which is somewhat different than the picture in 2000, when the change was almost totally due to the growth in statistics enrollments. This shift was most prominent in the shift of students from nonmainstream calculus courses to courses in probability and statistics (Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007).

The data in table 7 show the fall enrollments in several calculus-level courses and a few advanced mathematics courses. These particular courses are often found in the core of both the undergraduate major and the secondary mathematics education major. Collectively, they provide a picture of some stability within "major-related" course enrollments. One could add to this list the enrollment in the first-year sequence in mathematical statistics based on a calculus prerequisite. Enrollment in this sequence moved from thirty-five thousand to thirty-eight thousand in the five years from 2000 to 2005, an increase of more than 8% (Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007).

Table 7

Fall Enrollments (in Thousands) in Selected Undergraduate Courses, 1980–2005 (Loftsgaarden, Rung, and Watkins 1997; Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007)

	Year					
Course	1980	1985	1990	1995	2000	2005
Total Calculus-Level Including	590	637	647	539	570	586
Linear Algebra	37	47	44	33	41	37
Discrete Math	14	17	16	20	17	
Differential Equations	44	45	41	33	34	36
Total Advanced Mathematics Including	91	138	120	96	102	112
Geometry	4	7	8	6	6	8
Modern Algebra 1, 2	10	13	12	13	11	11
Advanced Calculus or Real Analysis 1 & 2	15	19	16	11	10	15
Total All Mathematics	1,525	1,619	1,621	1,471	1,614	1,606

The return of enrollments in geometry and real analysis combined with the stable, or minor, growth of enrollments in the other core courses indicates that the pattern of recovery from the decrease in enrollments in these courses in the early 1990s is still continuing.

According to the 2005 CBMS data, the total enrollment in calculus-based statistics courses in four-year colleges and universities increased from fifty-five-thousand in 2000 to fifty-seven thousand in 2005. This growth in advanced statistics courses, when analyzed more closely, also reflects a shifting of approximately one thousand students out of the mathematics departments' enrollments and adding three thousand students to statistics departments' enrollments. This probably reflects the growth of statistics departments and the increased importance of statistics in a variety of fields.

Bachelor's Degrees Awarded

Table 8 shows the number of bachelor's degrees in the mathematical sciences awarded from mathematics or statistics departments in five-year increments since 1975. Discussing the change in these data is somewhat difficult, as secondary mathematics education majors and statistics majors may or may not be included at colleges on the basis of the location of the mathematics education program within the mathematics department or within the College of Education and on whether the campus has a freestanding statistics department.

Data from computer science graduates is added for the information of those departments where earning degrees in computer science still falls within the auspices of the mathematics department. The latter totals, however, are not included in the total of degrees in the mathematical sciences.

Table 8
Bachelor's Degrees Awarded in the Mathematical Sciences and Computer Science, 1975–2005 (Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007)

	Year						
Area	1975	1980	1985	1990	1995	2000	2005
Mathematics (including actuarial science, operations research, & joint degrees)	18,833	11,687	16,123	14,852	13,792	13,664	14,610
Mathematics Education	4,778	1,752	2,567	3,116	4,829	4,991	3,369
Statistics	570	467	538	618	1,031	644	855
Total Math Sciences	24,181	13,906	27,928	24,455	22,895	22,614	21,437
Computer Science	na	na	8,691	5,075	2,741	3,315	2,603

The 2005 total marks the first increase since 1985 in the number of undergraduate degrees in mathematics (7%) over a five-year span. This gain is quickly offset, however, by adding in the students receiving undergraduate degrees in mathematics education. The five-year change statistics then shows a 2% decrease since 2000. The drop in degree production in mathematics education alone reported in the 2005 survey suggests a 32% decrease since 2000. This decrease does not bode well for maintaining quality, well-prepared teachers in the nation's secondary school mathematics classrooms. Even drawing on the individuals getting degrees in mathematics without an education degree will not address the demand.

The number of degrees in statistics was 33% higher than it was in 2000, but down from the highpoint production in 1995. Combining mathematics, mathematics education, and statistics, however, still showed a 2% decline in degrees in the total mathematical sciences from 2000 to 2005. Going back ten years to 1995, the decline is 4%.

Over the last half century, the number of women in the mathematical sciences has increased significantly. In 1950, less than 25% of the bachelor's degrees in the mathematical sciences went to women. By 1970, the numbers had increased to 37% (Madison and Hart 1990). The percents of degrees granted to women in 1995 were 44% in mathematics (including actuarial mathematics and operations research), 49% in mathematics education, and 49% in statistics. In 2005, these figures were 40% in mathematics, 60% in mathematics education, and 44% in statistics. This gave an overall 43% of degrees granted to women in 2005 for the mathematical sciences combined. This total was equivalent to the same figure observed in 2000 (Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007).

Data from *The American Freshman* study indicate the changes in the percent of freshman entering college intending to major in mathematics or statistics over the years: 4.5% (1966), 1.0% (1976), 0.7% (1986), 0.5% (1996), 0.7% (2001), and 0.7% (2004) (Sax et al. 2001; Astin et al. 1997; Higher Education Research Institute 2005; National Science Board 2006). The percentage in 2004 is significantly lower than in the 1960s, even when computer science majors are included. The mathematical requirements of majors outside the physical sciences, however, have increased significantly in the same time period. Although some of the mathematics needed to fulfill these requirements is taught outside departments of mathematics and statistics, the increases in these requirements are a major factor in the overall increase in number of courses taken in departments of mathematics and statistics.

Pedagogical Patterns at the Postsecondary Level

The teaching of calculus and elementary statistics courses has been the target of both textual and pedagogical reform in the past decade and a half. Table 9 reports the use of various instructional methods as gleaned from CBMS studies. In the 1995 CBMS assessment, 29% of mainstream and 10% of nonmainstream calculus sections in four-year colleges used a "reform" text. In the 2000 CBMS report, the greater use of computer assignments and graphing calculators and the more frequent appearance of a writing component suggested that these aspects of the reform calculus approach have blended into sections of both traditional and reform courses. The authors of the 2005 CBMS survey suggested that it was no longer possible to discern what constituted a reform-based calculus course, as both texts and teaching methods had incorporated some aspects of the various reform features.

An examination of the data for Calculus 1 shows that the use of hand-held graphing calculators has increased over the past decade at the two-year level and in the teaching of nonmainstream calculus in four-year institutions. The use of calculators in mainstream Calculus 1 at four-year institutions appears to have stayed stable or grown with the exception of their use in large-lecture situations in Ph.D.-granting mathematics departments. About one-fifth of sections at both levels in 2005 were making use of some form of computer assignments in mainstream Calculus 1. Although group projects and writing assignments were used in 19% of the sections of mainstream Calculus 1 in 2005 at the two-year level, they had only penetrated into 9% and 11%, respectively, of the four-year sections. The level of use of computer, group, and writing approaches has decreased from previous survey levels. The picture of pedagogical change in these three areas in nonmainstream calculus mirrors the trend seen in mainstream calculus.

Table 9

Percent of First-Semester Calculus and Statistics Sections Using Various Instructional Methods (Loftsgaarden, Rung, and Watkins 1997; Lutzer, Maxwell, and Rodi 2002; Lutzer et al. 2007)

	Two-Year Colleges								
	Main Calculus 1			**Non-Main Calculus 1**			**Statistics 1**		
Method	1995	2000	2005	1995	2000	2005	1995	2000	2005
Graphing Calculators	65	78	79	44	42	77	na	59	73
Computer Assignments	23	35	20	8	15	9	46	46	45
Group Projects	22	27	19	20	20	14	na	35	24
Writing component	20	31	19	17	20	14	na	50	44

	Four-Year Colleges and Universities*					
	Main Calculus 1			**Non-Main Calculus 1**		
Method	1995	2000	2005*	1995	2000	2005*
Graphing Calculators	37	51	47	26	45	50
Computer Assignments	18	31	20	6	13	4
Group Projects	23	19	9	7	9	4
Writing component	22	27	11	7	14	8

	Stat 1 in Math Dept			**Stat 1 in Stat Dept**		
Method	1995	2000	2005*	1995	2000	2005*
Graphing Calculators	na	47	36	na	4	6
Computer Assignments	51	48	53	59	54	57
Group Projects	na	22	14	na	13	28
Writing component	na	39	25	na	24	50

*Data account for shift in 2005 to report by percent of total sections rather than by percent of students.

The vestiges of reform are more evident in the elementary, noncalculus-based statistics course. Data for instructional approaches in this course are available for two-year college sections and, as taught in mathematics departments and in statistics departments, for four-year college sections. An examination of this data shows more adoption, or adaptation, of recommended pedagogical changes than what was seen in either version of the calculus. Whereas graphing calculators are used heavily at the two-year college sections, their usage is halved when the course is taught in the mathematics department and almost nonexistent in sections taught in statistics departments. The picture is the opposite in the use of computers in statistics sections. Here a clear difference emerges in the use of technology. As one moves from the two-year college to the statistics course in a statistics department, the percent of sections using computer assignments for sections moves from 45% to 57%. Writing and group project use has decreased in the mathematics departments at two- and four-year colleges but has increased in the statistics sections taught in the statistics department. Pedagogical approaches that focus on communication, both interpersonal and in writing, appear to have been viewed as more important in statistics courses than in calculus. Further, statisticians appear to believe more firmly in a focus on communication than those teaching in mathematics departments do.

A facet of university instruction that has shown a marked increase in the past decade has been the use of distance education in providing instruction for those who cannot travel to campus or who need more flexible learning environments. In 1997–98 about one-third of the nation's postsecondary institutions were offering distance-education courses, the majority of them through two-way interactive video. In mathematics, these courses were generally remedial or first-year, entry-level survey courses, accounting for approximately 3% of the enrollments in distance education nationally. About 8% of the institutions offered college-level degree or certificate programs to be completed totally through distance education. By the 2000-2001 school year, 56% of two- and four-year institutions had distance-education programs and degree or certificate programs to be completed by distance education alone were found on 19% of campuses (Lewis et al. 1999; Waits and Lewis 2003). The 2000 CBMS study included a special look at distance education in the undergraduate years. The findings indicated a small increase in the number of sections, but that in many cases the enrollments were quite small. The courses most often taught by distance education at two-year campuses were at the precalculus level or below, elementary statistics, or mathematics for liberal arts. At the four-year level, the only courses with more than 3% of the sections taught by distance education were trigonometry by mathematics departments, and a variety of computer science courses and statistical literacy courses by statistics departments (Lutzer, Maxwell, and Rodi 2002). The 2005 CBMS study shifted its focus in distance education from sections to number of students given the nature of distance education. The two-year responses in 2005 suggest that about 5% of their course enrollments are completed through distance-education arrangements. The courses with the highest numbers of students enrolled in distance-education programs are Elementary Algebra, Intermediate Algebra, College Algebra, and Elementary Statistics. Each of these courses had more than 9,500 students completing them by distance education. No other course had more than five thousand students enrolled for distance credit. At the four-year level, only some of the courses in the precalculus course group (college algebra, trigonometry, and precalculus) had more than five thousand students taking a mathematics course by distance education. The only course offered on a regular basis by statistics departments was the noncalculus-based Elementary Statistics. Enrollment from these offerings accounted for only 990 students in 2005 (Lutzer et al. 2007).

PART IV: Attained Curriculum

The National Assessment of Educational Progress (NAEP)

The U.S. government, with guidance from the National Assessment Governing Board (NAGB) and the expertise of the Department of Education's National Center for Education Statistics (NCES), conducts a large-scale assessment known as the National Assessment of Educational Progress. This program, which periodically assesses the knowledge of, and opportunity to learn, mathematics and other subjects with random samples of American youth, is one of the best barometers of where American mathematics education is headed from assessment period to assessment period. This survey of the mathematical abilities of American youth has taken on even more importance with the use of NAEP as a barometer for measuring states' performances relative to the strictures of the NCLB legislation (NCES 2007).

The most recent NAEP assessment in mathematics was conducted in 2007 and collected data from a national sample of fourth and eighth graders. Data was last collected from the nation's fourth, eighth, and twelfth graders in 2005. With the 2003 assessment, the NAEP program shifted its administration policies to provide expanded accommodations for students as provided by law. Figures 3 and 4 show NAEP data for grades 4 and 8 from studies conducted between 1990 and 2007.

Trend in 4th-Grade NAEP Scores

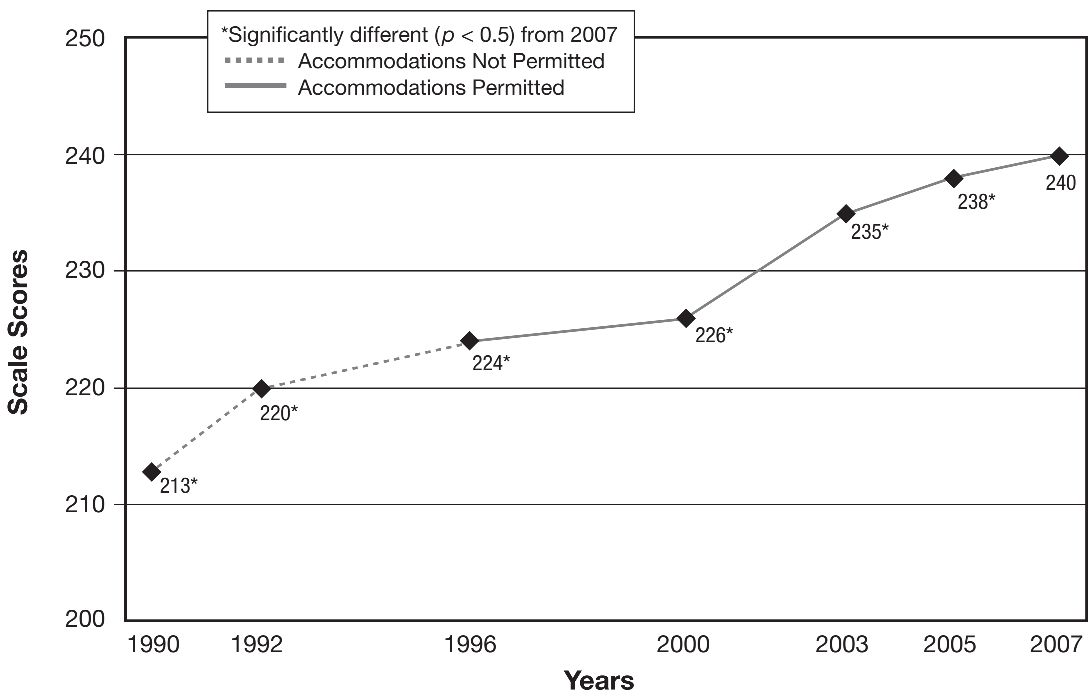

Fig. 3. National NAEP overall mathematics scale results for students with and without accommodations (Lee, Grigg, and Dion 2007)

Students' performance on the NAEP mathematics assessment is reported on a 0–500 scale. Prior to 2005, comparing across grades was possible because the tests were reported on the same scale. With the 2005 examination, however, the cross-grade blocks of items were dropped to test more of each grade's individual curricula in depth. Statistically, the 2007 performances at both grade levels are significantly higher than those observed in either 1990 or 2005. This pattern of continued growth has also been observed in other assessments carried out by states and local districts.

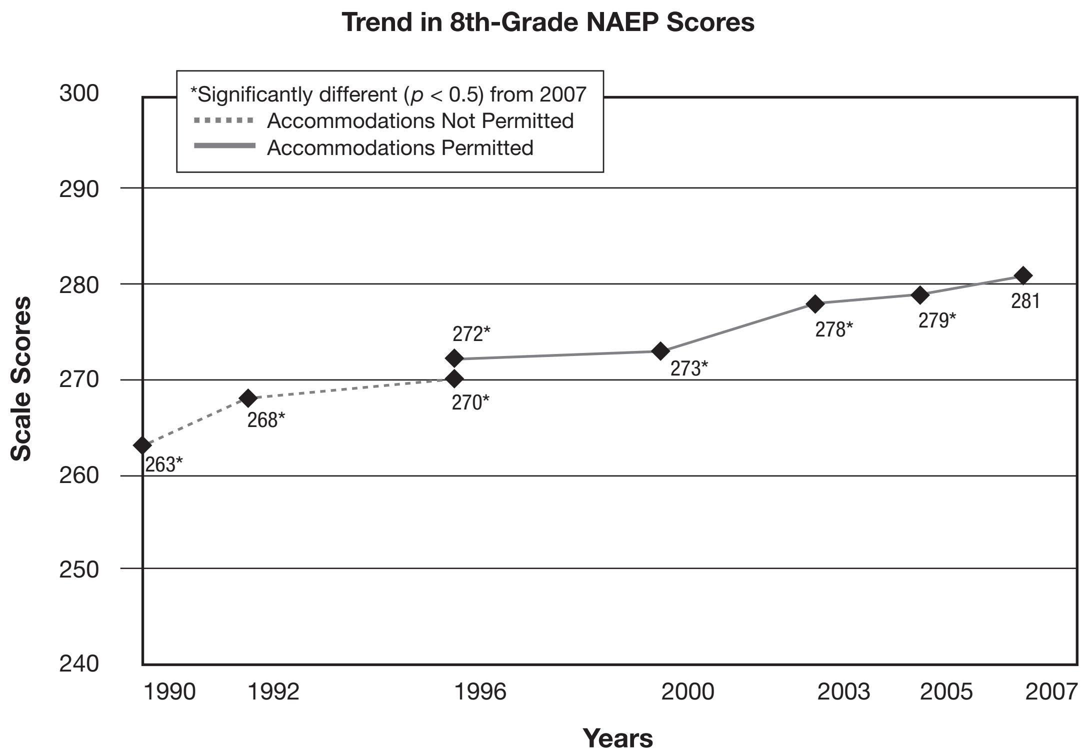

Fig. 4. National NAEP overall mathematics scale results for students with and without accommodations (Lee, Grigg, and Dion 2007)

The content frameworks for the NAEP assessments changed in 2005 and again in 2007. These are small, but significant, changes from previous assessment frameworks. Most notably are changes in the emphasis given to the role of algebra and functions at all grade levels. In addition, more attention was focused on students' proficiency in context-based problem solving, in constructing their own responses, and in knowing when and how to use technology in solving problems on assessments. In addition, the previous use of conceptual, procedural, and problem-solving labels to assign a level of cognitive demands of an item were discontinued and replaced by a factor that focuses on the demand, in terms of complexity, that an item places on students: low, medium, and high (NAGB 2002). The 2007 NAEP mathematics framework continues the changes reflecting the evolution of the school mathematics curriculum as well as adding questions at grade 12 involving content of high school geometry and second-year algebra courses, reflecting the increases in enrollment in those courses (NAGB 2006).

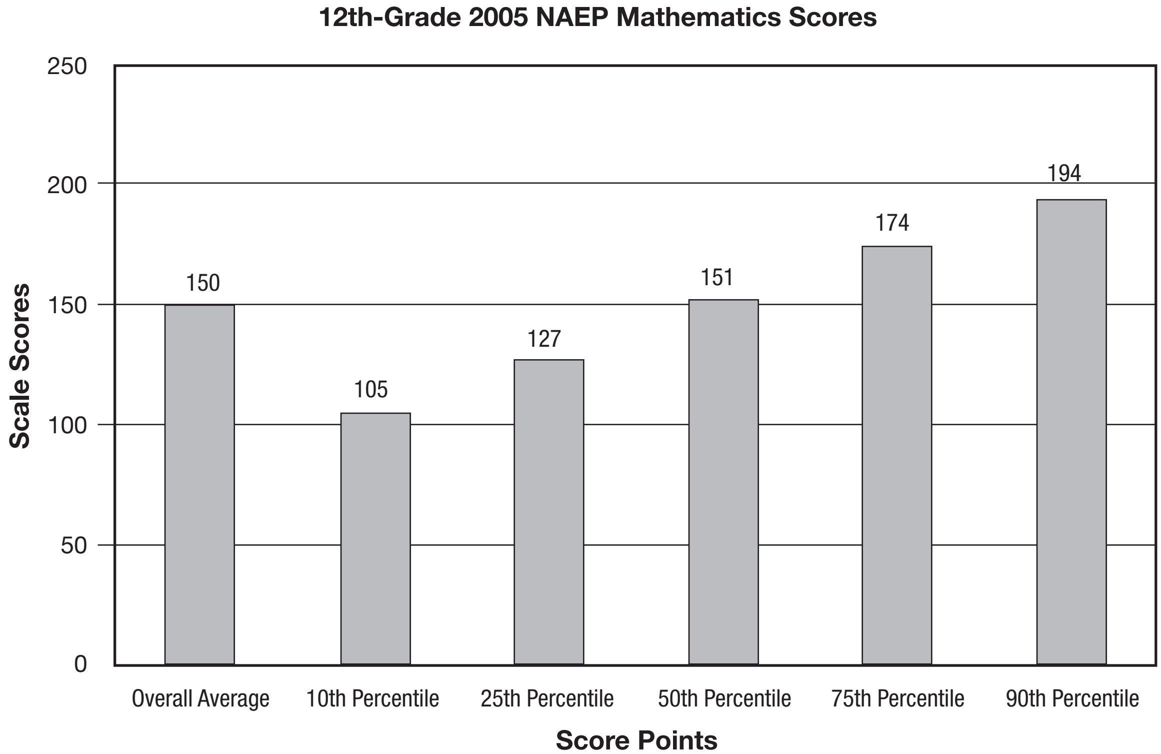

Fig. 5. National NAEP overall mathematics scale results for grade 12 for NAEP 2005 (Grigg, Donahue, and Dion 2007)

NAEP started a new grade-12 mathematics assessment with the 2005 assessment based on the 2005 framework. This framework increased the level of expectations for the grade-12 portion of the assessment to the point that it was necessary to establish a starting point for grade-12 trend comparisons. The 2005 grade-12 percentile point data is shown in figure 5.

The NAEP Longitudinal Study

Although the national NAEP assessments and their frameworks are designed to change as the curriculum and school programs change, the NAEP also administers a long-term trend assessment program with a nationally representative sample of students using a framework that has not changed since the inception of the long-term trend set of studies. On this assessment, students are not allowed to use calculators. As such, the NAEP trend assessment provides valuable information on whether students' performance on items considered important in 1973 (such as paper-and-pencil computation skills, direct application of measurement formulas in geometric settings, and the use of mathematics in daily living skills involving time and money) has changed over time (Perie, Moran, and Lutkas 2005). Data from the NAEP longitudinal study are shown in figure 6.

The long-term trend samples students' performance by age ranges that were used in the initial NAEP assessments in the early 1970s. The 2004 level of performance for both nine- and thirteen-year-old groups is statistically higher than that of the same age groups at every testing period from 1999 or earlier. The 2005 trend line for seventeen-year-olds shows a pattern of insignificant variation from 1990 to the present (Perie, Moran, and Lutkus 2005).

These findings indicate that, on average, elementary and middle school students in 2004 had a better command of the fundamental concepts and skills deemed important in 1973 than their age-related peers across the history of the assessment. The seventeen-year-old group, with the slight exception observed in the 1978–1986 period, showed no appreciable growth or decline in their command of these basic concepts and skills.

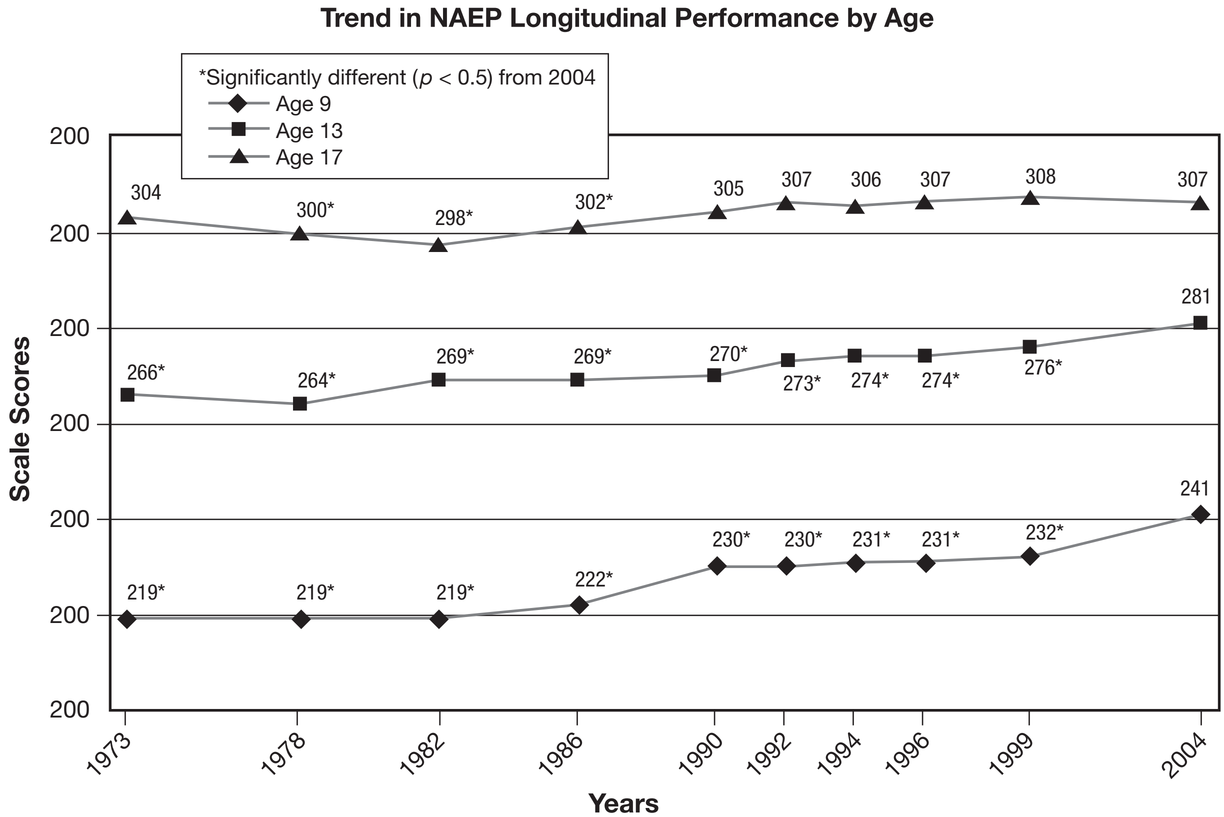

Fig. 6. Average mathematics scale scores for the nation, 1973 to 2004

Longitudinal Trends in Performance at NAEP Benchmark Levels

To help in understanding trends in students' knowledge and skills as measured by NAEP, levels of performance were established by anchoring five points on the mathematics scale: 150, 200, 250, 300, and 350. These five levels are accompanied by descriptions that outline the concepts, procedures, and processes associated with performance at each level. These levels are briefly described in the left column of table 10, which gives the results from the 1978 to 2004 assessments with respect to these levels (Perie, Moran, and Lutkus 2005; NCES 2008).

Analyses of the data in table 10 show a significant increase in the percent of students reaching benchmark levels of 200 and 250 for nine-year-olds and of 250 and 300 for thirteen-year-olds across the period from 1978 through 2004. These analyses indicate that much of the increase in students' long-term trend results comes from growth in mathematical topics, such as basic number facts and operations, and in reading and interpreting graphs, tables, and charts. The data for seventeen-year-olds shows no significant growth at any level since 1986. Few students at any age achieved the 350 benchmark, a level that indicates substantial ability in elementary algebra and geometry and in multistep problem solving.

Table 10
Trends in Percent of Students At or Above Five Mathematics Performance Levels, 1978–2004

Performance Levels	Age	1978	1986	1990	1996	1999	2004
Level 350	9	0	0	0	0	0	0
Multistep Problem Solving and	13	1	0	0	1	1	0
Algebra	17	7	7	7	7	7	7
Level 300	9	1	1	1	2	2	0
Moderately Complex	13	18	16	17	21	23	29
Procedures and Reasoning	17	52	52	56	60	61	59
Level 250	9	20	21	28	30	31	42
Numerical Operations and	13	65	73	75	79	79	83
Beginning Problem Solving	17	92	96	96	97	97	97
Level 200	9	70	74	82	82	83	89
Beginning Skills and	13	95	99	99	99	99	99
Understandings	17	100	100	100	100	100	100
Level 150	9	97	98	99	99	99	99
Simple Arithmetic	13	100	100	100	100	100	100
Facts	17	100	100	100	100	100	100

An analysis was made of the performance of students in the bottom quartile, middle half, and upper quartile of each of the age groups over the same periods of time. The resulting growth patterns for each of the three groups paralleled the increases shown in table 10. This indicates that the increases were not an artifact of the performance of the most able students, but rather an increase indicative of change in the students in each of the three groups (Perie, Moran, and Lutkus 2005).

NAEP Results for Various Subgroups

Considerable research has been done in recent years on the differences in the performance of students of different ethnic and gender groups, a matter of great concern to policy makers (Willingham and Cole 1997; Oakes 1990; Oakes and Wells 1998; Lubienski and Lubienski 2006; Venezia and Maxwell-Jolly 2007). Differences between white and black students' performance on the long-term trend NAEP have varied over the assessment's history, but the differences were significantly smaller in 2004 than in 1973 at all three age levels. Using a conservative ten points on the long-term score scale as a one-year equivalent growth expectation, however, the gap between white and black students' performance widens from 2.3 to 2.6 to 2.8 years in 2004 at ages nine, thirteen, and seventeen, respectively (Perie, Moran, and Lutkas 2005). Similar comparisons between white and Hispanic students' performances show some improvement in closing the gap between 1999 and 2004 for nine-year-olds. When the ten-point score scale rule-of-thumb is applied, differences of 1.7, 2.3, and 2.4 years exist in 2004 between white and Hispanic students' performances at ages nine, thirteen, and seventeen, respectively.

Some studies have suggested that these differences are the result of opportunities afforded the students in school and in their homes and communities; other studies point to the atmosphere of encouragement about education and its role in students' lives (Mullis, Jenkins, and Johnson 1994; Oakes 1990; Eakin and Backler 1993; Venezia and Maxwell-Jolly 2007). Narrowing these gaps remains a central challenge for mathematics education in the United States.

On the 2007 main, or national, NAEP mathematics assessment, male students outscored female students at grades 4 and 8 (Lee, Grigg, and Dion 2007). The same relationship held between the genders for the 2005 twelfth-grade assessment (Grigg, Donahue, and Dion 2007). On the long-term trend NAEP, the gap between male and female performance was not statistically significant in 2004 at any age level (Perie, Moran, and Lutkas 2005).

NAEP Results for Various States

In the late 1980s, the United States Congress passed legislation directing NAEP to allow the possibility of state-by-state analysis of performance. The first state assessment took place in 1990 in grade 8 in thirty-two jurisdictions. The advent of the NCLB legislation has resulted in all states participating in the NAEP assessments at grades 4 and 8. In 2007, all fifty states, plus the District of Columbia and the Department of Defense's system of schools on domestic and foreign bases (DoDEA), participated.

Results, shown in table 11, demonstrate major differences among the states. Taking a difference of ten points as a rough indication of a year (the difference between the national means for grades 4 and 8 being forty-two points using the state-by-state NAEP samples), some jurisdictions differ by two or more years at grade 4 and three or more years at grade 8. The NAEP 2007 state means are reported in table 11 (Lee, Grigg, and Dion 2007).

Table 11

Selected Statistics for Public Schools on the 2007 NAEP

	Grade 4		Grade 8	
Jurisdiction	**Mean Scale Score**	**% Proficient or Above**	**Mean Scale Score**	**% Proficient or Above**
NATION	**239**	**39**	**280**	**31**
Alabama	229	26	266	18
Alaska	237	38	283	32
Arizona	232	31	276	26
Arkansas	238	37	274	24
California	230	30	270	24
Colorado	240	41	286	37
Connecticut	243	45	282	35
Delaware	242	40	283	31
Florida	242	40	277	27
Georgia	235	32	275	25
Hawaii	234	33	269	21
Idaho	241	40	284	34
Illinois	237	36	280	31
Indiana	245	46	285	35
Iowa	243	43	285	35

(Continued on p. 36)

Table 11—*Continued*

Jurisdiction	Grade 4		Grade 8	
	Mean Scale Score	% Proficient or Above	Mean Scale Score	% Proficient or Above
Kansas	248	51	290	40
Kentucky	235	31	279	27
Louisiana	230	24	272	19
Maine	242	42	286	34
Maryland	240	40	286	37
Massachusetts	252	58	298	51
Michigan	238	37	277	29
Minnesota	247	51	292	43
Mississippi	228	21	265	14
Missouri	239	38	281	30
Montana	244	44	287	38
Nebraska	238	38	284	35
Nevada	232	30	271	23
New Hampshire	249	52	288	38
New Jersey	249	52	289	40
New Mexico	228	24	268	17
New York	243	43	280	30
North Carolina	242	41	284	34
North Dakota	245	46	292	41
Ohio	245	46	285	35
Oklahoma	237	33	275	21
Oregon	236	35	284	35
Pennsylvania	244	47	286	38
Rhode Island	236	34	275	28
South Carolina	237	36	282	32
South Dakota	241	41	288	39
Tennessee	233	29	274	23
Texas	242	40	286	35
Utah	239	39	281	32
Vermont	246	49	291	41
Virginia	244	42	288	37
Washington	243	44	285	36
West Virginia	236	33	270	19
Wisconsin	244	47	286	37
Wyoming	244	44	287	36
Other Jurisdictions				
District of Columbia	214	14	248	8
DoDEA	240	37	285	33

College Entrance Examinations

Typically, a student in the United States applies for college in the twelfth grade, the last year of high school. The selectivity of colleges in the United States varies from community colleges and postsecondary institutions that require no more than a high school diploma or its equivalent to selective colleges at which 10% or fewer applicants are accepted. Occasionally, the selectivity of an institution varies with the academic major for which a student applies.

Because college entrance examination scores provide the only easily comparable measure for students coming from different high schools and different areas of the country, they are often given great importance by colleges. As a result, most college-intending students in the United States take a college entrance examination during their junior or senior year.

Two major independent test batteries exist. The SAT tests, administered by the College Board, are more common in the east, south, and west. The ACT tests, administered by the American College Testing Service, are most commonly taken in the middle portion of the country. The percentage of students taking the SAT remained almost constant through the 1990s but increased from 43% to 48% of high school graduates between 1999 and 2003. Since 2003, this statistic has been fairly stable with 48% of the nation's seniors taking the examination in 2007. The percent of high school graduates taking the ACT increased from 38% in 1999 to 40% in 2003. The percent of students taking the ACT has remained at this level, with 40% of the nation's seniors taking the ACT in 2007. Some students take both tests (ACT 2007b; College Board 2007e).

The SAT employs multiple-choice items and items in which students grid-in their answers on an op-scan answer sheet to assess students' knowledge of mathematics, critical reading, and writing. In addition, starting with the 2006 scores, the SAT includes a student-produced essay that is scored on a six-point scale using a standard rubric. The mathematics portion of the test covers number and operations; algebra and functions; geometry; and statistics, probability, and data analysis. The coverage of the mathematics test was increased to include more items testing the content of second-year algebra and more advanced topics from geometry. The focus in these items is on students' critical reasoning skills. A student's results for each of the three sections of the SAT are reported on a 200–800 scale. A student's overall reasoning ability is reported by the sum of the individual scale scores on the 0–2,400 scale (College Board 2007f).

The ACT test assesses high school students' general subject matter knowledge and college/workforce readiness in four skill areas: English, mathematics, reading, and science. The test is composed entirely of multiple-choice items and each of the four skill areas is reported on a 1–36 scale. A general summary score, also on the same 1–36 scale, is used to report a student's overall skill level. In addition, the test has an optional writing section that focuses on a student's skill in planning and writing a short rubric scored essay (ACT 2007a). Both tests have shown a substantial improvement in the mean mathematics score in the past decade (see Table 12).

The general public has come to view these mean scores as a barometer of how well the education system is performing as a whole, despite the examinations not being designed for that purpose and despite their obvious shortcomings when used for this purpose.

Table 12
Mean Scores of High School Seniors on SAT and ACT Mathematics and English Tests Since 1995 (ACT 2007b; College Board 2007e)

	Test			
Year	**SAT—I Math**	**SAT—Reading**	**ACT—Math**	**ACT—English**
1995	506	504	20.2	20.2
1996	508	505	20.2	20.3
1997	511	505	20.6	20.3
1998	512	505	20.8	20.4
1999	511	505	20.7	20.5
2000	514	505	20.7	20.5
2001	514	506	20.7	20.5
2002	516	504	20.6	20.2
2003	519	507	20.6	20.3
2004	518	508	20.7	20.4
2005	520	508	20.7	20.4
2006	518*	503*	20.8	20.6
2007	515*	502*	21.0	20.7

*New SAT test framework

PART V: Developments in Statistical Education

Statistics education is an emerging presence in the pre-kindergarten–grade 12 curriculum and has become an integral part of undergraduate education in U.S. colleges and universities. Several factors contribute to this rapid growth in demand for statistics. More statistics courses have been added to the undergraduate curriculum as more and more fields, such as agriculture, biology, business, economics, engineering, environmental science, neuroscience, psychology, and sociology require, or strongly recommend, their majors take at least one statistics course.

The introduction of the Advanced Placement (AP) Statistics examination in 1997 spurred the growth of statistics teaching in high schools and increased the demand for courses at the undergraduate level as those students asked for second-level statistics courses when they entered college. In 1997, 7,500 students took the first AP Statistics exam; by 2007, the number had increased to more than 98,000. Statistics enrollments at the college level have grown from 27% of calculus enrollments in 1970 (ASA 2007) to nearly equal enrollments in 2005—356,000 in Calculus 1 and 2 and 309,000 in elementary statistics and probability and statistics courses (Lutzer et al. 2007). Perhaps the greatest influence on the demand for statistics has been the federal government's requirement, through the NCLB Act, that each state provide state assessment frameworks for their mathematics standards. Because many states had modeled their standards after NCTM's *Principles and Standards for School Mathematics* (2000), which includes data analysis and probability as an important component of the grades K–12 curriculum, almost all state standards and assessment frameworks include a focus on data analysis, statistics, and probability at the grades K–12 level.

The increased focus on statistics in schools and colleges has led to several attempts to standardize the statistics curriculum. The ASA published the GAISE Report, which contains guidelines for statistics teaching in 2004. Achieve, an organization of business leaders and state governors, published guidelines for mathematics and statistics teaching in 2004 as part of their American Diploma Project Benchmarks. The College Board published the *College Board Standards for College Success: Mathematics and Statistics* (2006b). These three organizations agree strongly regarding the statistics that students need to learn at the high school level to prepare them for the workplace, for introductory college courses, and for acting as statistically literate citizens. The College Board explains their increased emphasis on statistics in the high school curriculum saying,

> This decision was based on the ever-growing influence of statistics and probability in everyday life and in scientific and technological applications across our society. It is fundamental that students understand the basic concepts and applications of statistics and probability and appreciate the modes of reasoning that are used in these fields…. The present and emerging uses of statistics and probability in our society have made these fields a part of the 'new basics' for all students. Such knowledge is critical for students' success in quantitatively based college courses, as well as effective participation in civic life (2006b, p. xv).

The ASA created a taskforce in 2003 to develop guidelines for teaching statistics in prekindergarten through college. Their stated goal was to promote statistical literacy among citizens. The taskforce's recommendations were finalized and endorsed by the ASA Board of Directors in 2004. Both the prekindergarten through grade 12 and the college reports are available on the web at www.amstat.org/education/gaise. In August 2005, the ASA published, in paperback, *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: Pre–K–12 Curriculum Framework* (Franklin et al. 2005), which is available from its Web site, www.amstat.org. This document, intended for teachers, curriculum developers, school boards, and parents, explains why statistical literacy skills are crucial in our technological society and provides a framework for teaching these skills in the preschool through high school curriculum. The related *GAISE College Report* makes recommendations regarding the content and pedagogical approaches for introductory statistics courses at the collegiate level (ASA 2007).

The GAISE Pre-K–12 Recommendations

The GAISE Report has as its foundation the NCTM's *Principles and Standards* (2000) and "is intended to complement the recommendations of the NCTM *Principles and Standards*, not to supplant them" (Franklin el al. 2007, p. 5). Although mathematics and statistics are separate disciplines, many statistical tools rely on and reinforce the mathematical skills students learn in K–12. As such, the authors of the GAISE Report conclude that statistics belongs in the grades K–12 mathematics curriculum as part of an overall school curriculum (Franklin et al. 2007).

The GAISE Report gives guidelines for three developmental levels of statistics education. These levels, labeled A, B, and C, can be thought of as statistical concepts and tools appropriate for introduction in the prekindergarten and elementary school grades (A), middle school (B), and high school (C). The levels reflect developmental stages in learning statistics and suggest that high school or even college students who were not exposed to levels A and B need to start with level A before progressing to level B and then to level C.

At each developmental level students encounter the four process components of a statistical investigation: formulating a question, collecting data, analyzing the data, and interpreting the results. Particular emphasis is placed at each level on interpreting results in the context of the original question. Students learn to distinguish questions that can be answered by collecting and analyzing data (e.g., What proportion of students ride bicycles to school?) and questions that cannot be addressed through data analysis (e.g., Did Sarah ride a bicycle to school this morning?).

The ever presence of variability in data is one of the unifying themes in and across the process components and at each developmental level. Students should look for sources of variation in their data while they investigate real-world questions. Statistical education should emphasize the different types of variation and assist students in learning to explain and quantify the variability observed.

The GAISE Report provides an outline summarizing the prekindergarten through grade 12 curriculum framework for statistics. The table shows the development of each of the four process components at each developmental level, and it describes the nature of variability examined at each level. Some of the wording has been changed from the original report to provide further interpretation.

Statistics Study in Elementary Schools (Grades Pre-K–5 or Pre-K–6)—Level A

I. *Formulate Question:* Students form an awareness of the distinction between statistical questions and nonstatistical questions.

- Teachers pose questions of interest to be investigated by the class.
- Questions are restricted to the classroom population.

II. *Collect Data:* Students do not yet design studies to detect differences.

- Conduct a census of students in the classroom.
- Conduct simple experiments.

III. *Analyze Data:* Students use particular properties of distributions in the context of a specific example (e.g., biggest and smallest values, mean, median, mode, and range).

- Display variability within a group (e.g., using a dot plot or stem plot).
- Compare individual to individual.
- Compare individual to group.
- Begin to develop awareness of group-to-group comparisons (e.g., girls' to boys' means).
- Observe association between two variables (e.g., scatterplots).

IV. *Interpret Results:* Students do not look beyond the data.

- Make no generalization beyond the classroom.
- Observe differences between two individuals with different conditions.
- Observe association in graphical displays.

Variability Considered: Level A focuses on within-group variability, but students learn to distinguish natural variability (e.g., height) from measurement variability (e.g., height measured with a ruler versus a steel tape) and from induced variability (e.g., the differences in plant heights between plants given a cup of water each day and plants given half a cup of water each day).

Statistics Study in Middle School (Grades 5–8 or 6–9)—Level B

I. *Formulate Question:* Students develop increased awareness of the distinction between statistical questions and nonstatistical questions.

- Questions focus on students' own questions of interest.
- Questions are not restricted to the classroom population.

II. *Collect Data:* Students begin to be aware of designing studies to detect differences.

- Conduct sample surveys and begin to use random selection.
- Conduct comparative experiments and begin to use random allocation.

III. *Analyze Data:* Students learn to use particular properties of distributions as tools of analysis.

- Quantify variability within a group.
- Compare group to group in displays (e.g., parallel box plots or dot plots).
- Acknowledge sampling error.
- Begin to quantify the strength of association between two variables; use simple models for association.

IV. *Interpret Results:* Students acknowledge that looking beyond the data is feasible.

- Acknowledge that a sample may or may not be representative of the larger population.
- Note the differences between two groups with different conditions.
- Become aware of distinction between observational study and experiment.
- Note differences in strength of association.
- Learn basic interpretation of models for association.
- Become aware of the distinction between association and cause and effect.

Variability Considered: Level B focuses on sampling variability, but students learn to distinguish variability within a group from variability between groups and become aware of covariability through their study of associations between variables.

Statistics Study in High School (Grades 9–12 or 10–12)—Level C

I. *Formulate Question:* Students can make the distinction between statistical questions and nonstatistical questions.

- Students pose their own questions of interest.
- Questions seek generalization.

II. *Collect Data:* Students design studies to detect differences.

- Create sampling designs that include random selection.
- Create experimental designs that include random allocation.

III. *Analyze Data:* Students understand and use distributions in analysis as a global concept (e.g., sampling distributions).

- Measure variability within a group and between groups.
- Compare group to group in displays and measures of variability.
- Describe and quantify sampling error.
- Quantify the association between two variables; fitting models for association where appropriate.

IV. *Interpret Results:* Students are able to look beyond the data in some contexts.

- Generalize from sample to population.
- Become aware of the effect of randomization on the results of experiments.
- Understand the difference between observational studies and experiments.
- Interpret measures of strength of association.
- Interpret models for association.
- Distinguish between conclusions from association studies and experiments.

Variability Considered: Level C focuses on chance variability and on variability in model fitting.

The GAISE College Report

The *GAISE College Report* (ASA 2007) sets the goal of producing statistically educated students as the primary goal for any introductory college course. The report acknowledges that some introductory statistics courses focus primarily on teaching statistical literacy and others aim to produce students capable of carrying out full statistical analyses on their own. In either case, the course must provide students with a basic understanding of the language of statistics, some knowledge of the ways data are collected and described, a conceptual understanding of statistical inference, and skill in interpreting statistical results in the context of the original problem. The report does not recommend more specific statistical content because course goals differ. It builds on Cobb's (1992) six recommendations for the teaching of introductory statistics courses at the college level.

These recommendations have much wider application, however, and need not be restricted to college courses. The six recommendations are given below.

1. *Emphasize statistical literacy and develop statistical thinking.* Statistical literacy includes understanding statistical terms and concepts. Statistical thinking focuses more on the process of doing statistics and includes understanding that data provide more reliable evidence than anecdotes, that data collection requires careful planning and execution to produce reliable evidence, that it is natural for data collected from individuals in the same population and measured in the same way to vary, and that part of a statistician's job is to measure and explain the observed variability.
2. *Use real data.* Real data engages students' interest more easily than made-up data. It brings with it the opportunity to discuss why and how the data were collected and to show how these issues relate to the choice of which variables to collect, how to measure the variables, and what kinds of data analyses would be appropriate for these data.
3. *Stress conceptual understanding rather than mere knowledge of procedures.* Students should focus on learning core concepts in greater depth and not on being exposed to a wide variety of statistical techniques and procedures. For example, some of the core concepts students need to master include the concepts of a distribution of data, measures of center, and measures of spread. Greater depth of knowledge of these concepts includes understanding the influence of outliers on each.
4. *Foster active learning in the classroom.* Active learning involves using pedagogical techniques in the classroom that encourage students to become actively engaged with the concepts they are learning. In statistics classes, these techniques include using group discussions or problem-solving activities, demonstrations, and laboratory activities, which could include working on the computer or with physical data collection and analysis, in class. They also include providing longer-term group projects that are primarily done outside of class to answer questions students generate.
5. *Use technology for developing concepts and analyzing data.* Technology is an integral part of the work statisticians do, and students should be taught to use technology, such as graphing calculators or computer software, to analyze data. Students should focus on interpreting results and checking conditions for the validity of their data analyses. Technology also facilitates visualizing concepts, such as sampling distributions and conducting simulations that help students see the effects of changing underlying assumptions.

6. *Use assessments to improve and evaluate students' learning.* Assessment is used broadly to include both formative and summative evaluations and both types need to be aligned with learning goals. Prompt feedback is essential for effective formative assessment (ASA 2007, pp. 7-14).

The ASA guidelines present a challenge to educators because most primary and many secondary school teachers have not studied statistics during their professional education, and statistics is not required for teacher certification in most states. Both the College Board and ASA support programs of professional development. The College Board supports short-term programs for AP Statistics teachers, and the ASA has begun to offer daylong workshops for teachers annually at national statistical meetings. Clearly these efforts cannot reach a majority of teachers, especially those teaching at the prekindergarten, primary, and middle school levels. Teacher education, certification, and professional development programs need to initiate or enhance current programs in statistics education to match the degree to which curricula have included data analysis, statistics, and probability concepts and procedures.

PART VI: Programs for Special Populations of Students

Advanced Placement Programs

A high school student who completes the standard mathematics of grades 9–12 before entering twelfth grade and desires to continue the study of mathematics in twelfth grade has three potential paths to follow. One, if the school is very small without a college nearby, then the student may be able to take an individualized course. Two, if the school is near a college or university, then the student may be able to take a college course earning credit toward high school graduation and possibly also college credit if the student passes the course. And three, if enough students in the school are like this student, then the school may wish to offer Advanced Placement (AP) courses.

In 1955, under the auspices of the College Board, the Educational Testing Service (ETS) created the Advanced Placement Program to enable students to take college-level work before they graduate high school (ETS www.ets.org). High schools participating in this program offer courses whose syllabi are designed to be in agreement with introductory college courses. Thirty-seven AP courses now exist in twenty-two different subject areas, and more than 16,000 high schools worldwide participate. In 2006, a total of 666,067 students took one or more AP examinations (College Board 2007b).

Most AP courses are a year in length. Many high schools, however, offer "block schedules" with longer class periods each day, and in these schedules, AP courses are compressed into one-semester configurations. In May of each year, ETS administers nationwide exams for each of the courses. Colleges have the options of offering college credit, placing students into more advanced classes (with or without credit), or ignoring the scores students receive. Many colleges take scores on AP tests into account when placing students into courses.

When AP courses are taken in the eleventh grade or earlier, they can be considered along with a student's application to a college and may factor into admissions decisions. Although scores on AP tests in twelfth grade are not available to colleges before admissions decisions are made, enrollment in AP courses itself tends to signify more serious students and a more scholastically oriented high school and can increase a student's chances of admission to some colleges.

Scores on AP tests range from one to five. The American Council on Education recommends that colleges give credit to students who score three or higher, but some colleges have higher cutoffs, and some give credit for part of the yearlong course, depending on the score (ETS www.wts.org).

Advanced Placement Programs in Calculus

Two AP exams are offered in calculus: Calculus AB (since 1956) and Calculus BC (since 1969). Calculus BC is an extension of Calculus AB, not an enhancement; common topics require a similar depth of understanding. Each test consists of two multiple-choice parts for 105 minutes and a free-response section for ninety minutes. One of the multiple-choice parts does not allow the use of a calculator; the other part and the free-response section require a graphing calculator.

The 211,693 students who took the AB and 64,311 students who took the BC tests in 2007 together equaled more than 9% of graduating high school seniors, the highest numbers on record for these exams. In 2007, 59% of students taking the Calculus AB test scored three or higher, and 80% of students taking the Calculus BC test scored three or higher (College Board 2007d).

The syllabi for Calculus AB and BC are developed, and modified periodically, by a national committee of the College Board and might be said to represent a consensus regarding what a good calculus course should include. Both syllabi are primarily concerned with developing students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed geometrically, numerically, analytically, and verbally. The connections among these representations also are important (College Board 2007c). All Calculus AB topics are covered in Calculus BC; those only in BC are identified here with an asterisk (*).

I. Functions, Graphs, and Limits

- Analysis of graphs
- Limits of functions (including one-sided limits)
- Asymptotic and unbounded behavior
- Continuity as a property of functions
- *Parametric, polar, and vector functions

II. Derivatives

- Concept of the derivative
- Derivative at a point
- Derivative as a function
- Second derivatives
- Applications of derivatives
- Computation of derivatives

III. Integrals

- Interpretations and properties of definite integrals
- *Applications of integrals
- Fundamental theorem of calculus
- Techniques of antidifferentiation
- Applications of antidifferentiation
- Numerical approximations to definite integrals

*IV. Polynomial Approximations and Series

- *Concept of series
- *Series of constants
- *Taylor series

Advanced Placement Program in Statistics

A single AP exam is offered in statistics. AP Statistics is meant to be equivalent to a one-semester, introductory, noncalculus-based college course in statistics. Graphing calculators with statistical capabilities are required on the exam, but the College Board emphasizes that they are not equivalent to computers in the teaching of statistics (College Board 2006c). In 2007, 98,033 students took this exam and 58% of these scored three or higher (College Board 2007d).

Following is an outline of the areas covered by the AP Statistics examination (College Board 2006d):

I. Exploring Data: Describing Patterns and Departures from Patterns
   A. Constructing and interpreting graphical displays of distributions of univariate data (dot plot, stem plot, histogram, cumulative frequency plot)
   B. Summarizing distributions of univariate data
   C. Comparing distributions of univariate data (dot plots, back-to-back stem plots,parallel box plots)
   D. Exploring bivariate data
   E. Exploring categorical data

II. Sampling and Experimentation: Planning and Conducting a Study
   A. Overview of methods of data collection
   B. Planning and conducting surveys
   C. Planning and conducting experiments
   D. Generalizability of results and types of conclusions that can be drawn from observational studies, experiments, and surveys

III. Anticipating Patterns: Exploring Random Phenomena Using Probability and Simulation
   A. Probability
   B. Combining independent random variables
   C. Normal distribution
   D. Sampling distributions

IV. Statistical Inference: Estimating Population Parameters and Testing Hypotheses
   A. Estimation (point estimators and confidence intervals)
   B. Tests of significance

Special Schools and Programs

The National Consortium for Specialized Secondary Schools for Mathematics, Science, and Technology includes more than one hundred institutional members with thirty-seven thousand students. The goal of the Consortium is to foster, support, and advance the efforts of those specialized schools to attract and academically prepare students for leadership in these subject areas. Some members are boarding schools requiring state residence and highly competitive examinations for entrance; a few are local, specialized high schools; others are regional centers that students may attend for a half or full day for a single year (NCSSSMST 2008).

Two types of summer programs in mathematics are available for very capable students. The first type follows a model initiated by the late Julian Stanley at Johns Hopkins University in the 1970s, identifying talent in the upper elementary or middle school grades and offering accelerated courses (usually in the summer but sometimes through the school year) to enable those students to study more advanced mathematics at a younger age. The other type follows a model initiated by Arnold Ross at Notre Dame University at around the same time, in which students are taught mathematics differently than they would normally be in school and in which they are expected to solve problems and deduce propositions in somewhat the same manner as a professional mathematician. These programs recruit either regionally or nationally, and opportunities are available for students across the entire nation (Ohio State University 2008).

The largest organization of mathematics clubs in the United States is Mu Alpha Theta, founded in 1957. Mu Alpha Theta has more than fifteen hundred high school and community college chapters and more than seventy-five thousand student members across the United States. Its purpose is to stimulate interest in mathematics by providing recognition of superior mathematical scholarship in students. In addition to holding regional meetings and an annual national meeting, Mu Alpha Theta also publishes a newsletter and provides several other resources for its student members (Mu Alpha Theta 2008).

In the United States, 803 schools are authorized to offer the program of the International Baccalaureate Organization (IBO); 2,146 are authorized worldwide in 125 countries. Of these 803 schools, 556 offer the Diploma Program, a demanding two-year, precollege program that leads to examinations and is designed for students aged sixteen to nineteen. The remaining 247 schools offer either the Middle Years Program or the Primary Years program, both of which are designed for younger students (IBO 2008).

Programs for College and Graduate Students

The NSF funds a large number of research opportunities for undergraduate students through its Research Experiences for Undergraduates (REU) program. An REU site consists of a group of ten or so undergraduates who work in the research programs of the host college or university. Each student is associated with a specific research project, where the student works closely with the faculty and other researchers. Students are granted stipends and, in many instances, assistance with housing and travel. Undergraduate students supported with NSF funds must be citizens or permanent residents of the United States or its territories. In March 2007, forty REU sites with research opportunities in mathematics were available. A list of the REU sites can be found at the NSF's Web site (NSF 2008).

To increase the number of U.S. citizens, nationals, and permanent residents who pursue careers in the mathematical sciences, the NSF Vertical Integration of Research and Education in the Mathematical Sciences (VIGRE) program supports postdoctoral positions, graduate research traineeships, and research experiences for undergraduates in departments of mathematics and statistics. From 1999 through 2003, VIGRE awards were granted to thirty-nine departments at thirty-seven different universities. A list of the VIGRE sites can also be found at the NSF's Web site (NSF 2008).

Mathematics Competitions

Mathematics competitions in the United States are voluntary both for individuals and schools. Some middle schools and high schools have mathematics teams, often competing in events operated by local professional organizations. Described here are the larger competitions of national scope.

The National Society of Professional Engineers, CNA (an insurance company), and NCTM founded MATHCOUNTS in 1982 to increase interest and involvement in mathematics and to assist in developing a technologically literate population. The competition is now operated by the MATHCOUNTS Foundation; sponsors include the Dow Chemical Company Foundation, the General Motors Foundation, Lockheed Martin, NEC Foundation of America, Texas Instruments Inc., the 3M Foundation, and the National Aeronautics and Space Administration. Participation is restricted to students in grades 7 and 8. In 2003, more than seven hundred thousand students were exposed to MATHCOUNTS materials, and more than forty thousand participated in the national competition at some level (Bauer and Fagan 1999: MATHCOUNTS 2008).

The American Mathematics Competitions (AMC), centered at the University of Nebraska–Lincoln, in 2007 involved more than 413,000 participants. These participants account for 20% of the high schools in the country each year. These competitions began in 1950 as the American High School Mathematics Examination (AHSME) for students in grades 9–12, cosponsored by the MAA and the Society of Actuaries. This exam is now called the AMC 12. Gradually, other organizations became involved. In 1985, an exam for students below grade 9, the American Junior High School Mathematics Examination (AJHSME), now called the AMC 8, was initiated. In 2000, the AMC 10, an exam for students below grade 11, was begun. The AMC 12 is a qualifying exam for the American Invitational Mathematics Examination (AIME), whose highest scorers become eligible to participate in the U.S. Mathematical Olympiad, a six-question, six-hour exam that is used to determine the U.S. Olympiad team. The AMC also operates a summer program for qualifying students (AMC 2008).

The Math League, founded in 1977, specializes in math contests, books, and computer software designed to stimulate interest and confidence in mathematics for students from the fourth grade through high school. In recent years, more than one million students have participated in Math League contests each year. Contest problems are designed to cover a range of mathematical knowledge for each grade level and require no additional knowledge of mathematics beyond the grade level they test (Math League 2008).

The American Regions Mathematics League (ARML), begun in 1976 as the Atlantic Region Mathematics League, is a competition of teams of high school students who represent their school, local area, state, or country (outside the United States). A power contest runs through the school year. A national competition, which takes place toward the end of the school year, occurs at three sites. In May 2007, more than 120 schools participated in the national competition (ARML 2008).

The Consortium for Mathematics and Its Applications (COMAP) operates a Mathematical Talent Search (USAMTS) for very high-performing high school students in its newsletter *Consortium*. It also sponsors—in conjunction with the MAA, NCTM, the Information Science and Operations Research Society, and the Society for Industrial and Applied Mathematics—the High School Mathematical Contest in Modeling (HiMCM). Results from this contest are published in *Consortium*. COMAP also organizes the Mathematical Contest in Modeling for teams of college students and publishes the winning entries in the *UMAP Journal* (COMAP 2008).

The Student Mathematics League (SML) offered by the AMATYC offers two examinations each year to students enrolled in two-year colleges. Nassau Community College in New York founded the SML in 1970; in 1981, the AMATYC assumed sponsorship.

The SML has grown to more than 165 colleges in more than thirty-five states as well as Bermuda, involving more than eight thousand community college mathematics students. The level of the tests is precalculus mathematics. Questions are from a standard syllabus in college algebra and trigonometry and may involve precalculus algebra, trigonometry, synthetic and analytic geometry, and probability; questions that are completely self-contained may be included as well. All questions are short-answer or multiple choice (AMATYC 2008).

The Putnam Exam (officially the William Lowell Putnam Mathematical Competition) for undergraduates, administered by the MAA, is perhaps the most rigorous and prestigious mathematics examination held annually. This examination for undergraduates, which can be entered as individuals and as members of teams representing their college or university, celebrated its sixty-seventh anniversary in December 2007. In 2006, 3,640 individuals and 402 three-person teams competed from 508 colleges and universities in the United States and Canada. Problems and solutions for the 2006 edition are available at www.unl.edu/amc.

PART VII: Teacher Education and Professional Development

Overview

A bachelor's degree and a teaching certificate are needed to teach in most public schools in the United States at any level, kindergarten–grade 12. The teaching certificate is generally obtained through a combination of courses taken at the college level and in-school experience (observations and work in schools, including supervised practice teaching) at or around the grade levels at which the teaching is to take place. Some states also require that the teacher pass a test, which usually consists of specific subject-matter knowledge and general knowledge about teaching and the education system. All fifty states also provide some alternative route to teacher certification based on an individual's prior experiences, education, and, potentially, a bundled set of courses and internship experiences (National Center for Education Information 2008). Also, in cases of teacher shortage or the movement of a teacher from one state to another, provisional certification is possible until all the requirements for full certification have been met.

Most teachers earn certification before having had a full-time teaching position and gain tenure after two to four years of full-time teaching. With tenure comes job security; a tenured teacher cannot be removed from a teaching position without evidence of incompetence, breach of contract, or other wrongdoing.

A teaching certificate, although not required to teach in private or parochial schools, is often desired because the agencies that accredit schools want schools to have certified teachers. (Accreditation is necessary for other schools to recognize automatically a school's graduates and students who transfer from that school.)

University mathematics departments typically offer the mathematics courses taken by preservice and in-service teachers as part of their training, although in some institutions education departments may offer those courses. In 2003, in response to a report of a commission headed by former senator John Glenn (National Commission on Mathematics and Science Teaching for the Twenty-first Century 2000), the MAA initiated a program called Preparing Mathematicians to Educate Teachers (PMET) to help college and university mathematicians to take a larger role in the training and support of classroom teachers (Katz and Tucker 2003). The PMET project had three major components: summer workshops and minicourses for faculty training; articles, Web sites, and other materials, and panels at meetings to support faculty instruction; and minigrants and regional networks to nurture and support grassroots innovation in teacher education on individual campuses. Although the PMET funding has now expired, the project created an energized spirit through the mathematics community in the United States that the responsibility for quality mathematics education resides with mathematicians, mathematics educators, faculty in education departments, and classroom teachers of mathematics.

Data on the quantity and quality of the teacher workforce comes from education agencies in individual states, a schools and staffing survey conducted by NCES, a national survey of science and mathematics education conducted by Horizon Research Inc. (Weiss et al. 2001), projects focusing on mathematics teacher education, and NAEP. The state data are not always complete, and some of the data raise questions about accuracy and completeness (CCSSO 2003). Data from the nation indicate that the number of high school mathematics teachers significantly increased from 1999–2000 to 2003–2004. Many of these teachers teach other subjects because in this time period high school teachers with a main assignment in mathematics increased from 191,000 to 213,000. The percent of these teachers who are female (55%) and male (45%) remained constant (Snyder, Dillow, and Hoffman 2007).

The opposite trend held in the national data for teachers in prekindergarten through grade 8. Staffing data reported from schools showed that the numbers of teachers with full-time mathematics positions and a mathematics degree decreased from twenty-six thousand in 1999–2000 to nineteen thousand in 2003–2004 (Snyder, Dillow, and Hoffman 2007). This finding is alarming given the push to increase the number of highly qualified teachers of mathematics in the middle school.

The Council of Chief State School Officers (CCSSO) conducted a 2003–2004 survey focusing on mathematics teachers in middle and high schools. They obtained data from thirty-five states, DoDEA, Guam, and the Virgin Islands. The resulting data indicate that 79% of the junior high school teachers and 85% of high school teachers who were teaching mathematics were teaching it as half or more of their teaching assignment. Of all teachers teaching mathematics at the middle school level (grades 7–8), 71% were certified as mathematics teachers for this level in their state. At the high school level, 89% of all teachers who were teaching mathematics were certified as mathematics teachers in their state (Blank and Langesen 2005)

Elementary School Mathematics Teachers and Teacher Education

The last fifty years of the twentieth century saw vast changes in the preparation of teachers for the nation's elementary schools. In 1952, nearly half of the nation's six hundred thousand public elementary school teachers did not hold college degrees (Lucas 1997). By the early 1990s, all states required an undergraduate degree for an individual to receive a teaching certificate. Even at present, however, the amount of mathematics included in the collegiate program for someone preparing for teaching grades K–6 is minimal. Individual university programs vary, as do state requirements for certification. Most programs for students preparing to teach in grades K–6, however, consist of a major in education with only a modicum of coursework beyond the institution's general education requirements (Hawkins, Stancavage, and Dossey 1998; Smith, Arbaugh, and Fi 2007).

The 2000 national NAEP assessment asked teachers to indicate their college majors, but they could provide multiple responses that are difficult to interpret. From these surveys, of fourth-grade teachers, about 3% had an undergraduate or graduate degree in mathematics, 4% had a degree in mathematics education, 87% had a degree in education, and the remaining 6% had a degree in some other major. Weiss et al. (2001), however, report that virtually no teachers in grades K–4 in their national sample had either a mathematics or mathematics education major. The data from the 2003 NAEP assessment corroborates the findings of the CCSSO and earlier NAEP studies in finding that whereas 88% of the nation's students in grade 4 were learning in classrooms taught by a certified teacher, only 2% of the nation's K–4 students had teachers with a degree in mathematics and another 2% of students at this level had teachers with a degree in mathematics education. As expected, the majority of the degrees held by teachers at this level were in elementary education (Smith, Arbaugh, and Fi 2007).

Elementary school teachers have been found not to have great depth of understanding of mathematics. Liping Ma (1999) compared the mathematical knowledge of elementary school mathematics teachers in the United States with their counterparts in Shanghai, China, most of whom teach only mathematics. She found the U.S. teachers had far less depth of knowledge than their Chinese counterparts. The Ma book argues for deep conceptual knowledge for teachers and for its role in the teachers' planning and guidance of lessons in their classrooms. This work had great influence on the CBMS (2001) document *The Mathematical Education of Teachers* discussed below.

Of the teachers grades K–4 surveyed in the Weiss et al. (2001) national studies, 96% reported having completed a course in mathematics for elementary school teachers; 42%, a course in college algebra, trigonometry, or elementary functions; 33%, a course in probability or statistics; 21%, a course in applications of mathematics or problem solving; 21%, a course in geometry for teachers; and 12%, a course in calculus. These findings are not always consistent with the 2000 (1996) NAEP, in which 83% (84%) of students in grade 4 had teachers who had taken a course in the teaching of mathematics, 39% (43%), in number systems and numeration; 31% (37%), in measurement; 30% (34%), in geometry; 46% (45%), in college algebra; and 39% (36%), in probability and statistics. In general, these data suggest that the overall mathematics preparation of elementary school teachers is short of the goals outlined in the CBMS recommendations for the mathematical education of elementary school teachers (CBMS 2001). This document calls for preservice elementary school teachers to have at least nine semester hours (equivalent to three courses) of coursework in mathematics that should provide experiences in number and operations, in geometry and measurement, in algebra and functions, and in data analysis, statistics, and probability. Further, this training should be taught with the goal of developing teachers' in-depth understanding of the mathematics they teach.

Such goals will take significant effort to accomplish, as many prospective elementary school teachers take much of their general content coursework at community colleges and then transfer to a four-year college or university to complete their undergraduate degree program. In some states, these students must complete a fifth-year program before gaining certification. These points of discontinuity in their preparation make the development of a carefully articulated sequence of courses and experiences in mathematics and mathematics education a formidable task.

Middle School Mathematics Teachers and Teacher Education

In 2007, forty-five states had either a middle school or junior high school certification or endorsement requirement (National Middle School Association 2007). Many of these states also have special mathematics requirements for that certification or endorsement by the teachers' selected area of content expertise. In mathematics, these special requirements range from passing a test to completing the equivalent of an undergraduate minor in mathematics. The CBMS (2001) recommendations call for the teaching of mathematics in grades 5–8 to be conducted by mathematics specialists, teachers specifically educated to teach mathematics to the students of the grade levels they instruct. These teachers should have at least twenty-one semester hours in mathematics, including at least twelve semester hours on fundamental ideas of mathematics appropriate for middle school teachers.

The 2003 national NAEP results (Smith, Arbaugh, and Fi 2007) lead to estimates that 85% of the nation's eighth graders are taught by teachers who are certified by their state. When examined by teachers' degrees, 30% of the nation's eighth graders had teachers with an undergraduate degree in mathematics; 26% had teachers with an undergraduate degree in mathematics education; and the remaining students were taught by a teacher with a degree in some other discipline. These percents show a decrease in majors and an increase in minors from the prior NAEP data. Under any interpretation, however, the fact that at least one-third of the nation's eighth-grade students are still being taught mathematics by teachers without substantial mathematics training is a matter of major concern.

Secondary School Mathematics Teachers and Teacher Education

For secondary school mathematics teacher certification, states require from eighteen (in South Dakota) to forty-five (in California) semester hours of mathematics, equivalent to six to fifteen semester courses, or they require a major in the subject. When a number of credit hours of mathematics are specified for the certificate, almost half require thirty credit hours. When specific courses are mentioned, they include three courses in calculus (two single-variable and one multivariable), linear algebra, geometry, and abstract algebra, plus a host of various electives.

The 2000 study conducted by Iris Weiss and her colleagues at Horizon Research (2001) reports that 58% of mathematics teachers in grades 9–12 in their sample had an undergraduate major in mathematics, 21% had a degree in mathematics education, 10% had a degree in some other education field, and 10% had a degree in a field other than education or mathematics. In this sample, 96% of teachers had completed a course in calculus, 86% in probability and statistics, 82% in geometry, 81% linear algebra, 70% in advanced calculus, 68% in computer programming or other computer science, 65% in differential equations, and 64% in abstract algebra. NAEP does not collect data on grade 12 teachers.

The CBMS report (2001) recommends that high school teachers of mathematics have a major in mathematics that includes a six-hour capstone course connecting their college mathematics courses with high school mathematics. This recommendation stems from the view that teachers need to know the subjects they will teach, they need to understand the broad range of the mathematical sciences their students will encounter in their careers (i.e., core subjects plus dynamical systems, graph theory, combinatorics, operations research, computer science, and so on), and they need to develop the habits of mind and dispositions toward doing mathematics that characterize effective workers in the field.

In addition to specific courses, the CBMS (2001) report notes that teachers of the high school grades (9–12) need to develop understanding and skills associated with the use of technology in representing and exploring mathematical concepts and relationships in teaching. This includes experience writing computer programs in a high-level language, such as C++, and experience with a computer algebra system, dynamic geometry software, and a statistical software package. These experiences should also be designed to enable teachers to become thoughtful and effective in using educational technology and to keep abreast of changes in the field.

Professional Development Programs

In 1998, forty-four states reported policies defining requirements for continuing professional development of teachers to maintain certification by the state. The policy in half the states is six semester credits (equivalent to two college courses) every five years; three require more. Seven states require from 120 to 180 clock hours of professional development every five years. Four states require continuing education units, which are given after various kinds of professional development experience. Eight other states allow a combination of credits, units, and contact hours (CCSSO 1998).

In-service opportunities are widely offered within school districts, by professional organizations, by local colleges and universities, by regional education centers, and by commercial enterprises. Within state policies, local school districts and often individual teachers have the freedom to choose the kinds of in-service activities they desire. Unless given within the teacher's school district, an individual rarely is required to participate in a particular in-service program.

NCTM and its more than 250 affiliated national, state, and local organizations in mathematics education provide a number of professional development opportunities for teachers of mathematics. In addition to journals and publications, these organizations hold a number of regional and state-affiliated conferences with special sessions for teachers of all grade levels from kindergarten through undergraduate teacher preparation.

The annual meeting of NCTM, held in the spring of each year, has been attended by eighteen to twenty-three thousand mathematics teachers and other mathematics educators in recent years. In addition, NCTM sponsors three regional meetings geographically scattered across the United States and Canada throughout the fall and spring of the year to serve teachers on a regional basis, serving about twelve thousand teachers. All these meetings feature nationally known speakers, workshops, grade-level curriculum and teaching sessions, and displays of the most recent text materials, manipulatives and other supplementary materials, and technology for teaching mathematics.

In recent years, new methods of assessment have been a popular subject for professional development. Sometimes these sessions revolve around new tests that school systems and states have developed, often as a result of the NCLB legislation. Other professional development sessions emphasize assessment using open-ended questions, contextualized real-world tasks, and portfolios, as recommended in reports of the Mathematical Sciences Education Board (MSEB) and NCTM (MSEB 1991; MSEB 1993a; MSEB 1993b; Stenmark 1991; NCTM 2000).

Another popular subject for professional development programs is technology. The largest program in this area is Teachers Teaching with Technology (T^3). The T^3 group has established chapters in more than twenty-five countries and focuses on bringing teachers together to work with and learn from one another with the goal of increasing the appropriate use of educational technology in the teaching and learning of mathematics (Teachers Teaching with Technology 2008).

PART VIII: Doctoral Programs and Research Centers

Doctoral Programs in Mathematics Education

Mathematics education is a small area within education. Of 38,723 doctorates awarded in the United States in education in the six-year period from 2000 to 2005, only 520 were in mathematics education (Hoffer et al. 2001; Hoffer et al. 2002; Hoffer et al. 2003; Hoffer et al. 2004; Hoffer et al. 2005; Hoffer et al. 2006). Still, more doctorates in mathematics education were earned in this period than in any other content specialty within education. Of these 520 doctorates, 60% were awarded to women. An analysis of the recipients' citizenship showed that 85% were U.S. citizens.

The number of doctorates per year in mathematics education from 2000 to 2005 was eighty-six per year, equaling the 1970 to 1999 national average of eighty-six doctorates per year. The ten U.S. institutions awarding the greatest number of doctoral degrees in mathematics education during the period from 2000 to 2005, in descending order, are Teachers College–Columbia University (41), University of Georgia (33), Illinois State University (28), North Carolina State University (20), University of Texas at Austin (17), Rutgers University (14), Florida State University (13), Ohio State University (13), University of Oklahoma (13), and University of Maryland (11) (Reys et al. 2007). All except Teachers College are public institutions.

The most recent survey of features of doctoral programs in mathematics education (Reys et al. 2007), showed that most doctoral programs (87%) reported requiring the Graduate Record Examination (GRE) for admittance. Other information required by a number of programs included letters of recommendation, scores from the Test of English as a Foreign Language, and interviews. Faculty associated with doctoral programs also indicated that they take into account the applicant's knowledge of mathematics, school and research experience, and goals.

For doctoral students planning to specialize in elementary mathematics education, programs expected applicants to enter with a bachelor's degree in mathematics or mathematics education and about 75% suggest that students have experience in teaching at the elementary level. In about a quarter of these programs, experience in teaching at the elementary school level is an entrance requirement. For students planning to specialize in middle or secondary school mathematics education, more than half the doctoral programs either require or strongly suggest that entering students have at a minimum the equivalent of a bachelor's degree in mathematics or mathematics education. In addition, 60% of the programs indicate that students exiting programs with this level of specialty will have achieved the master's level of mathematical knowledge by the time they complete their degree.

In contrast with admittance requirements for applicants seeking an elementary school emphasis, nearly 75% of institutions require or strongly encourage entering students seeking a secondary school emphasis to have a teaching certificate at the middle or secondary school level. Twenty-seven percent of the institutions require middle or secondary school teaching experience prior to entering their doctoral program, and more than half (54%) report that they strongly encourage students to have this experience.

To move to the level of candidacy for the doctorate, approximately 60% of the programs require passing a qualifying examination. Schools reported requiring from forty-five to

125 semester hours of coursework or research for the degree. The allocation of this coursework to content areas varied widely. Most programs, however, had identifiable blocks of work dealing with research in mathematics education (98%) and research methods (quantitative and qualitative) (97%), mathematics content (90%), learning theories (83%), and mathematics curriculum (80%). The wide variance in amount of coursework reflects whether schools expected a completed master of sciences degree on entrance or not. The majority of schools required a residency (76%), and almost all (89%) required that the student pass a set of comprehensive exams. All institutions required a doctoral dissertation, and all allowed these dissertations to involve quantitative or qualitative research (Reys et al. 2007).

Government-Supported Research and Training Centers

In 2006, the U.S. Education Department awarded new five-year contracts to ten Regional Educational Laboratories charged with carrying out research, development, dissemination, training, and technical assistance activities. Among their many activities, the laboratories collaborate with states in developing plans for the provision of professional development, identifying and disseminating exemplary practices, developing strategies for addressing needs of underserved and underrepresented groups, and providing information and facilitating communication among their constituents. Administered by the Department's National Center for Education Evaluation and Regional Assistance in the Institute of Education Sciences (IES), the laboratories provide a crucial link between research and practice. These regional laboratories, by the regions they serve, are as follows:

- **Appalachia:** CNA Corporation, Alexandria, Virginia
- **Central:** Mid-Continent Research for Education and Learning, Denver, Colorado
- **Mid-Atlantic:** Pennsylvania State University, State College, Pennsylvania
- **Midwest:** Learning Point Associates, Naperville, Illinois
- **Northeast:** Education Development Center, Newton, Massachusetts
- **Northwest:** Northwest Regional Educational Laboratory, Portland, Oregon
- **Pacific:** Pacific Resources for Education and Learning, Honolulu, Hawaii
- **Southeast:** University of North Carolina at Greensboro, North Carolina
- **Southwest:** Edvance Research Inc., Austin, Texas
- **West:** WestEd, San Francisco, California

NSF-Funded Centers

Since 2000, the NSF has funded more than twenty-five Centers for Learning and Teaching (CLTs). Each CLT program addresses the need to enrich and diversify the national infrastructure for standards-based science, technology, engineering, and mathematics education. The goal is to increase the number of educators of grades K–12 prepared in content, pedagogy, and assessment methodologies. Each CLT has a specific concentration, but all offer an environment that merges education research, high-quality professional development, and the teaching of innovative instructional practices. Each CLT consists of at least one doctoral degree-awarding university and one or more school districts, plus partnering organizations.

Of the funded centers, several are devoted to the learning and teaching of mathematics. The following is a list of the largest of these centers, each with a URL and list of partners.

Mid-Atlantic Center for Mathematics Teaching and Learning (initiated in 2000)
www.education.umd.edu/mac-mtl/
University of Maryland
Pennsylvania State University
University of Delaware
Delaware State Department of Education
Pittsburgh Public Schools (Pennsylvania)
Prince George's County Public Schools (Maryland)

Appalachian Collaborative Center for Learning, Assessment and Instruction in Mathematics (ACCLAIM) (initiated in 2001)
www.acclaim-math.org
University of Tennessee
Ohio University
University of Kentucky
University of Louisville
West Virginia University
Marshall University (West Virginia)

Center for Learning and Teaching in the West (CLT-West) (initiated in 2001)
www.cltw.org/ universities
Montana State University
Colorado State University
Portland State University
University of Montana
University of Northern Colorado

Diversity in Mathematics Education (DIME) (initiated in 2001)
www.wcer.wisc.edu/dime
University of Wisconsin—Madison
University of California, Berkeley
University of California, Los Angeles (UCLA)
Berkeley Unified School District (California)
Madison Metropolitan School District (Wisconsin)
California Subject Matter Project (at UCLA)

Center for Proficiency in Teaching Mathematics (CPTM) (initiated in 2002)
www.cptm.us/
University of Georgia

University of Michigan (Ann Arbor)
Aquinas College (Michigan)
Grand Valley State University (Michigan)
Henry Ford Community—College (Michigan)
University Of Michigan—Dearborn
University Of Michigan—Flint
Western Michigan University
Gwinnett County Public Schools (Georgia)
Oakland Intermediate School District (Michigan)
Morgan County Schools (Georgia)
Social Circle Schools (Georgia)
South Redford School District (Michigan)

Center for the Study of Mathematics Curriculum (CSMC) (initiated in 2004)
mathcurriculumcenter.org/
University of Missouri
Michigan State University
Western Michigan University
University of Chicago School Mathematics Project
Horizon Research, Inc.
Columbia Public Schools (Missouri)
Grand Ledge Public Schools (Michigan)
Kalamazoo Public Schools (Michigan)

Center for the Mathematics Education of Latinos/as (CEMELA) (initiated in 2004)
math.arizona.edu/~cemela
University of Arizona
University of California, Santa Cruz
University of Illinois At Chicago
University of New Mexico
Albuquerque Public Schools (New Mexico)
Bernalillo Public Schools (New Mexico)
Chicago Public Schools (Illinois)
North Monterey County Unified School District (California)
Pajaro Valley Unified School District (California)
Socorro Consolidated Schools (New Mexico)
Sunnyside Unified School District (Arizona)
Tucson Unified School District (Arizona)

Mathematics in America's Cities (Metro-Math) (initiated in 2004)
www.metromath.org
Rutgers University
Graduate Center of the City University of New York
University of Pennsylvania
New York City Department of Education (New York)
Newark Public Schools (New Jersey)
Plainfield Public Schools (New Jersey)
School District of Philadelphia (Pennsylvania)

PART IX: Resources

Professional Organizations in Mathematics Education

Closed-membership organizations

Conference Board of the Mathematical Sciences (CBMS) (founded 1960)
e-mail: rosier@georgetown.edu; Web site: www.cbmsweb.org

CBMS is an umbrella organization consisting of the major professional societies in the mathematical sciences in the United States and composed of the CBMS Executive Committee and the presidents and executive directors of the member societies. Its purpose is to promote understanding and cooperation among the national professional organizations in mathematics so they can work together, supporting one another in research, the improvement of education, and the expansion of the mathematical sciences. The following societies belong: American Mathematical Association of Two-Year Colleges (AMATYC), American Mathematical Society (AMS), Association of Mathematics Teacher Educators (AMTE), American Statistical Association (ASA), Association for Symbolic Logic (ASL), Association for Women in Mathematics (AWM), Association of State Supervisors of Mathematics (ASSM), Benjamin Banneker Association (BBA), Institute for Operations Research and the Management Sciences (INFORMS), Institute of Mathematical Statistics (IMS), Mathematical Association of America (MAA), National Association of Mathematicians (NAM), National Council of Supervisors of Mathematics (NCSM), National Council of Teachers of Mathematics (NCTM), Society for Industrial and Applied Mathematics (SIAM), and the Society of Actuaries (SOA).

Mathematical Sciences Education Board (MSEB) (founded 1986)
e-mail: mseb@nas.edu; Web site: www7.nationalacademies.org/mseb

MSEB is a standing board of the National Research Council (NRC) Center for Education with appointed members. The current mission of MSEB is to provide a continuing national leadership and guidance for policies, programs, and practices supporting the improvement of mathematics education of all students at all levels. MSEB is currently pursuing initiatives that focus on the learning, instruction, and assessment of mathematics; equity in mathematics; attracting and retaining students in mathematics majors and in mathematically intensive careers; capacity building and professionalization of mathematics education; evidence of effectiveness in mathematics education; and the public perception of mathematics education enterprise.

United States National Commission on Mathematics Instruction (USNC/MI) (founded 1978)
Web site: www7.nationalacademies.org/usnc-mi/index.html

The national adhering body to the International Commission on Mathematical Instruction (ICMI) is the U.S. National Academy of Sciences (NAS). The NAS, through the NRC, appoints the USNC/MI to conduct the work of the ICMI and foster other international collaborations in mathematics education. The NRC Board of Mathematical Sciences, MSEB, CBMS, and NCTM provide nominees for selection to the USNC/MI.

Open-membership organizations—grades K–12

National Council of Supervisors of Mathematics (founded 1969)
e-mail: ncsm@mathforum.org; Web site: ncsmonline.org/
Journal: *Journal of Mathematics Education Leadership*

National Council of Teachers of Mathematics (NCTM) (founded 1920)
e-mail: infocentral@nctm.org; Web site: www.nctm.org
Journals: *Teaching Children Mathematics, Mathematics Teaching in the Middle School, Mathematics Teacher, Journal for Research in Mathematics Education*

School Science and Mathematics Association (SSMA) (founded 1902)
e-mail: office@ssma.org; Web site: www.ssma.org
Journal: *School Science and Mathematics*

Women and Mathematics Education (WME) (founded 1978)
Web site: www.wme-usa.org

Open-membership organizations—college level

American Mathematical Association of Two-Year Colleges (AMATYC) (founded 1974)
e-mail: amatyc@amatyc.org; Web site: www.amatyc.org
Journal: *The AMATYC Review*

American Mathematical Society (AMS) (founded 1888)
e-mail: ams@ams.org; Web site: www.ams.org
Journal: *Bulletin of the American Mathematical Society; Notices of the American Mathematical Society*

American Statistical Association (ASA) (founded 1839)
e-mail: asainfo@amstat.org; Web site: www.amstat.org
Journals: *The American Statistician, Chance, Stats* (and others devoted to research in statistics)

Mathematical Association of America (MAA) (founded 1915)
e-mail: maahq@maa.org; Web site: www.maa.org
Journals: *The American Mathematical Monthly, College Mathematics Journal, Mathematics Magazine*

The National Association of Mathematicians (founded 1969)
e-mail: dawnalott@aol.com; Web site: www.nam-math.org/
Journals: *NAM Newsletter*

Open-membership organizations—special focus

Association of Mathematics Teacher Educators (AMTE) (founded 1993)
e-mail: nbezuk@mail.sdsu.edu; Web site: www.amte.net
Journals: *AMTE Connections*

Benjamin Banneker Association (founded 1986)
e-mail: director@bannekermath.org; Web site: www.bannekermath.org

Research Council on Mathematics Learning (founded 1974)
e-mail: r.pourdavood@csuohio.edu; Web site: www.unlv.edu/RCML/
Journals: *FOCUS on Learning Problems in Mathematics*

TODOS: Mathematics for ALL (founded 2003)
e-mail: maleiva@email.uncc.edu; Web site: www.todos-math.org/index.html
Journals: *Noticias de TODOS*

Grades K–12 Textbook Publishers

Textbooks for grades K–12 are not listed in *Books in Print*, and currently available textbooks are likely not to be listed even in online bookstore catalogs. For this reason, this list of publishers, with their locations and a URL, is provided to assist those who might be interested in obtaining more information about grades K–12 textbooks and other curriculum materials used in the United States. These publishers and their college publishing counterparts often have auxiliary materials available online. Additional information can be found at the Association of American Publishers' School Division Web site, www.aapschool.org/. This organization represents the nation's leading developers of instructional materials, technology-based curricula, and assessments. The National Association of School Textbook Administrators (www.nasta.org) has another interesting Web site. NASTA is composed of the individuals responsible for the selection and administration of school textbook policies for the different states that have state adoption processes. This group of states controls such a large percent of the U.S. textbook purchases that their decisions shape, to a great extent, the actual contents and coverage sequences found in contemporary, U.S. grades K–12 textbooks.

Amsco School Publications, New York, NY 10013; www.amscopub.com

AnsMar Publishers, Inc., Poway, CA 92064; www.excelmath.com

Bates Publishing Company, Sandwich, MA 02563; batespub.com

Carnegie Learning, Pittsburgh, PA 15219; www.carnegielearning.com

CORD Communications, Waco, TX 76702; www.cord.org

CPM Educational Program, Sacramento, CA 95822; www.cpm.org

Curriculum Research and Development Group, Honolulu, HI 96822; www.hawaii.edu/crdg

Education Development Center, Newton, MA 02458; www2.edc.org

Glencoe/McGraw-Hill, Blacklick, OH 43004; www.glencoe.com

Globe Fearon, Lebanon, IN 46052; www.agsglobe.com

Harcourt School Publishers, Lewisville, TX 75067; www.harcourtschool.com

Holt, Rinehart & Winston, Orlando, FL 32887; www.hrw.com

Houghton Mifflin Company, Boston, MA 02116; www.hmco.com

It’s About Time, Inc., Armonk, NY 10504; www.its-about-time.com

Kendall/Hunt Publishing Company, Dubuque, IA 52004; www.kendallhunt.com

Key Curriculum Press, Emeryville, CA 94608; www.keypress.com

Macmillan/McGraw-Hill, Desoto, TX 75115; www.mhschool.com

McDougal Littel, Geneva, IL 60134; www.mcdougallittell.com

Pearson Learning Company, Lebanon, IN 46052; www.pearsonlearning.com

Prentice Hall School Division, Lebanon, IN 46052; www.phschool.com

Sadlier-Oxford, New York, NY 10005; www.sadlier-oxford.com

Saxon Publishers, Orlando, FL 32887; saxonpublishers.harcourtachieve.com

SRA/McGraw-Hill, Desoto, TX 75115; www.sra4kids.com

W. H. Freeman & Co., New York, NY 10003; www.whfreeman.com

Wright Group/McGraw-Hill, Desoto, TX 751 15; www.wrightgroup.com

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National Council of Teachers of Mathematics:

Founded in 1920, the National Council of Teachers of Mathematics (NCTM) is a nonprofit education association with more than one hundred thousand members and nearly 250 affiliates located throughout the United States and Canada. NCTM is dedicated to improving mathematics teaching and learning, kindergarten through high school, and facilitates ongoing dialogue and constructive discussion with all stakeholders about what is best for our nation's students. For more information on NCTM, visit www.nctm.org on the Web or contact NCTM at infocentral@nctm.org or call (703) 620-9840, ext. 2113.

United States National Commission on Mathematics Instruction:

The U.S. National Commission on Mathematics Instruction (USNC/MI) is a committee of the U.S. National Academy of Sciences. The roles of the USNC/MI are to facilitate U.S. participation in the activities of the International Commission on Mathematical Instruction, and to engage the U.S. mathematics education community through the National Council of Teachers of Mathematics, the Conference Board of the Mathematical Sciences, and the National Research Council to advance mathematics education in the United States and throughout the world. Support to the USNC/MI is provided by the National Science Foundation. Further information is available by contacting usncmi@nas.edu.