Overcoming Obstacles to Make Mathematics Work for Each and Every Student

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Goals

- **Provide an overview of *Principles to Actions.***
- **Examine some of these issues through the window of communication with parents and and needed advocacy.**
- **Questions and Responses**

High Quality Standards are Necessary for Effective Teaching and Learning, But Insufficient

Standards do not describe or prescribe the essential conditions required to make sure mathematics works for all students.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Guiding Principles for School Mathematics

1. *Teaching and Learning*
2. *Access and Equity*
3. *Curriculum*
4. *Tools and Technology*
5. *Assessment*
6. *Professionalism*

→ Essential Elements of Effective Math Programs

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Principles to Actions: Ensuring Mathematical Success for All

The overarching message is that effective teaching is the non-negotiable core necessary to ensure that all students learn mathematics.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

We Must Focus on Instruction

Teaching has 6 to 10 times as much impact on achievement as all other factors combined ... Just three years of effective teaching accounts on average for an improvement of 35 to 50 percentile points.

Schmoker, M. (2006). *Results now: How we can achieve unprecedented improvements in teaching and learning*. Alexandria, VA: Association for Supervision and Curriculum Development.

Teaching and Learning Principle

Teaching and Learning. An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Eight Research-Informed Instructional Practices

- Establish mathematics **goals** to focus learning.
- Implement **tasks** that promote reasoning and problem solving.
- Use and connect mathematical **representations**.
- Facilitate meaningful mathematical **discourse**.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Eight Research-Informed Instructional Practices

- Pose purposeful **questions**.
- Build **procedural fluency** from conceptual understanding.
- Support **productive struggle** in learning mathematics.
- **Elicit and use evidence** of student thinking.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Obstacles to Implementing Research-Informed Instructional Practices

Dominant cultural beliefs about the teaching and learning of mathematics continue to be obstacles to consistent implementation of effective teaching and learning in mathematics classrooms.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Discussion Question

With a shoulder partner:

What cultural beliefs about the teaching and learning of mathematics do you believe stand as obstacles to implementation of effective teaching and learning in mathematics classrooms?

Unproductive Belief

Students need only learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Eight Research-Informed Instructional Practices

Build procedural fluency from conceptual understanding.

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding …

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

What are Teachers Doing?

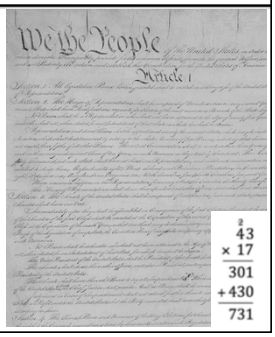
Connecting student-generated strategies and methods to more efficient procedures.

Using visual models to support students' understanding of general methods.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Our Founding Fathers Did NOT Establish the "Standard Algorithms"

Standard algorithms were developed in India in the first centuries of the modern era, and further honed by traders and engineers in the Iraq-Persia region.

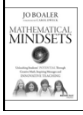


The Pedagogical Value of Standard Algorithms

Standard algorithms sacrifice ease of understanding in favor of computational efficiency, and that made sense once. In today's world we have readily accessible machines to do calculations, so we can turn the educational focus on understanding the place-value system that lies beneath those algorithms.

What Employers Need

What employers need … is people who can ask good questions, set up models, analyze results, and interpret mathematical answers. It used to be that employers needed people to calculate; they no longer need this. What we need is people to think and reason.

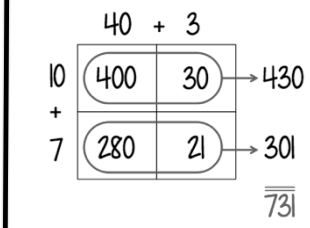


Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovated teaching.* San Francisco, CA: Jossey-Bass. P. 27

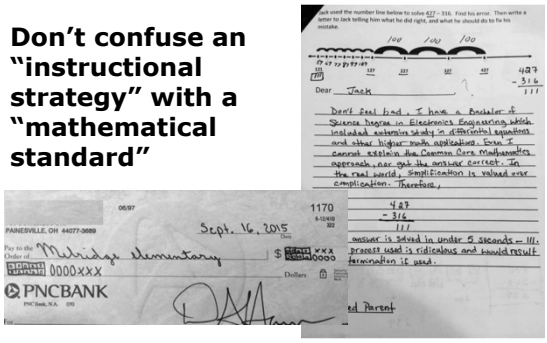
The Area Model Builds Understanding of the Standard Algorithm

43×17

$$\begin{array}{r} {\scriptstyle 2}\,43 \\ \times\ 17 \\ \hline 301 \\ +430 \\ \hline 731 \end{array}$$



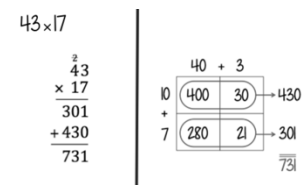
Don't confuse an "instructional strategy" with a "mathematical standard"



Moving Forward: Reflect on How you Communicate with Stakeholders

We should emphasize **visual representations or models to build understanding** -- not "alternate" "different" or "new" algorithms

How, Why and When



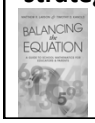
Which Makes More Sense? Which is More Grounded in Place Value?

$$\begin{array}{r} \overset{2}{\cancel{2}}\\ 43 \\ \times\ 17 \\ \hline 301 \\ +430 \\ \hline 731 \end{array}$$

$$\begin{array}{r} 43 \\ \times\ 17 \\ \hline 21 \\ 280 \\ 30 \\ 400 \\ \hline 731 \end{array}$$

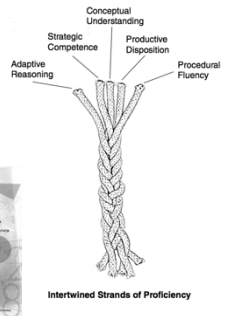
We Need to Be Clear: There is No "New Math" & There is No Such Thing as Common Core Math

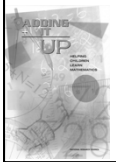
With respect to standards "Math is Math" – fraction expectations shouldn't be different in Maine and California. At the K-8 level there is no "new math," but there are "new" research-informed instructional strategies!



Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents.* Bloomington, IN: Solution Tree.

Mathematical Literacy is Multifaceted





How, Why and **When** while building a **Positive Mathematics Identity**!

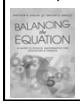
Multiple Methods Confuse Some Parents

The emphasis in learning multiple methods must be on how the methods are related to one another to build conceptual understanding and not on committing still more procedures to memory.

Bay-Williams, J., Duffett, A., & Griffith, D. (2016). *Common core math in the K-8 classroom: Results from a national teacher survey.* Washington, DC: Thomas B. Fordham Institute.

Confront the Homework Issue

- It isn't parents' responsibility to "do" homework. In fact that can do more harm than good.
- Make homework comprehensible.
- Children should practice their preferred and efficient strategy.
- Parents should support perseverance, monitor progress, and ask questions.



Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents.* Bloomington, IN: Solution Tree.

Unproductive Belief

An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.

Eight Research-Informed Instructional Practices

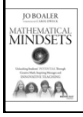
<u>Support productive struggle in learning mathematics</u>.

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

The "Didactic Contract"

A common situation in mathematics classes occurs whereby teachers are called over to students who ask for help; the students expect to be helped, and teachers know it is their role to help them, so the teachers break down the problem and make it easier. In doing so they empty the problem of its cognitive demand.



Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovated teaching.* San Francisco, CA: Jossey-Bass. P. 179

Support Productive Struggle in Learning Mathematics

Teachers sometimes perceive student frustration or lack of immediate success as indicators that they have somehow failed their students.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Struggle vs. Frustration

Struggle does not mean needless frustration or extreme levels of challenge. It means students expend some effort to make sense of mathematics.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* . Charlotte, NC: Information Age Publishing

Productive Perseverance: Learning from Our Mistakes

With parents we should talk about **perseverance and learning from mistakes**, not "productive struggle."

Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents.* Bloomington, IN: Solution Tree.

If your students are going home at the end of the day less tired than you are, the division of labor in your classroom requires some attention.

Wiliam, D. (2011). *Embedded formative assessment.* Bloomington, IN: Solution Tree Press.

Eight Research-Informed Instructional Practices

<u>Facilitate meaningful mathematical discourse</u>.

Effective teaching of mathematics facilitates discourse among students in order to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Facilitate Meaningful Mathematical Discourse

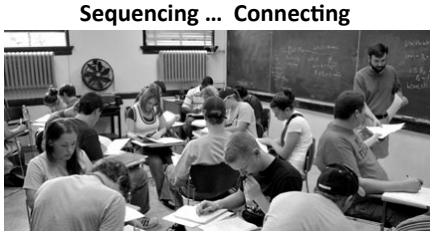
Students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding …

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary schools.* Portsmouth, NH: Heinemann.

Five Practices to Promote Productive Math Discussions

Anticipating … Monitoring … Selecting …

Sequencing … Connecting



Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: NCTM.

Parents Often Ask: Why Does My Child Have to Explain? Isn't the Answer Enough?

We must emphasize that mathematics is more than answer getting. Mathematics is reasoning and sense making. When you explain your work you are engaged in reasoning. Explaining your work (reasoning) is what mathematicians do. And it is what most adults do in their own jobs.

Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents.* Bloomington, IN: Solution Tree.

Eight Research-Informed Instructional Practices

Pose purposeful questions.

Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.

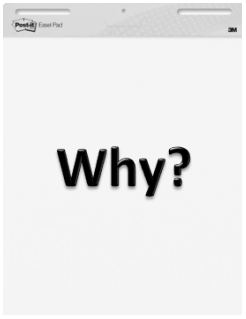
NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

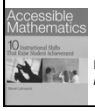
Effective Teachers are Effective Questioners

"Effective mathematics teachers … pose more questions with higher cognitive demand and ask more follow-up questions"

McRel. (2010). *What we know about mathematics teaching and learning*, third edition. Bloomington, IN: Solution Tree Press.

Make "Why?" "How do you know?" "Can you explain?" "Do you agree/ disagree?" Classroom Mantras





Leinwand, S. (2009). *Accessible mathematics: 10 instructional shifts that raise student achievement.* Portsmouth, NH: Heinemann.

Five Essential Elements of Effective Mathematics Programs

Effective teaching and learning, while the non-negotiable core of successful mathematics programs, are part of a system of essential elements of excellent mathematics programs.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Five Essential Elements of Effective Mathematics Programs

Access and Equity

Curriculum

Tools and Technology

Assessment

Professionalism

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Guiding Principles for School Mathematics: Access and Equity

Access and Equity. An excellent mathematics program requires that all students have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Access and Equity Obstacles

A range of obstacles exists … one of these involves the quality of instruction available to different groups of students … another involves differential opportunities to learn high-quality grade level mathematics content and to be held to high expectations for mathematics achievement.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Mathematics Education is a Powerful Force

Mathematics education often reinforces, rather than moderates, inequalities in education.

OECD. (2016). *Equations and inequalities: Making mathematics accessible to all.* Paris: PISA OECD Publishing. Downloaded at http://dx.doi.org/10.1787/9789264258495-en.

Tracking and Identity

The power and status of school mathematics often manifest themselves in decisions about what content gets taught, to which students, and by which teachers … what gets taught in the mathematics classroom shapes the mathematics identities of both students and teachers.

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: Rethinking equity-based practices.* Reston, VA: NCTM.

Different Opportunities for Different Students

The learning opportunities provided for low-ability, average-ability, and high ability-grouped classrooms are hierarchically different.

Boaler, J., Wiliam, D., & Brown, M. (2000). Students' experiences of ability grouping – disaffection, polarisation and the construction of failure. *British Educational Research Journa*l, 26(5), 631-648.

Opportunity to Learn is a Serious Issue

Across OECD countries, more than 70% of students attend schools whose principal reported that students are grouped by ability for math … reducing ability-grouping can reduce the influence of socio-economic status on students' opportunities to learn.

OECD. (2016). *Equations and inequalities: Making mathematics accessible to all.* Paris: PISA OECD Publishing. Downloaded at http://dx.doi.org/10.1787/9789264258495-en.

Educide by the Low-Slow Group

Low expectations often result in self-fulfilling prophecies. Once placed in the low tracks, it is very difficult for students to move to a higher track.

Flores, A. (2008). The opportunity gap. *TODOS Research Monograph: Promoting High Participation and Success in Mathematics by Hispanic Students: Examining Opportunities and Probing Promising Practices*, 1(1), 1-18.

Educide by the Low-Slow Group

Too often, schools serving large populations of minority students emphasize "slowing down" or providing less mathematics content, rather than providing more challenging content.

Walker, E. N. (2007). Why aren't more minorities taking advanced math? Educational Leadership, 65(3), 48-53.

On Paper We've De-Tracked High School

Even with universal algebra policies, there is the real possibility that 'tracking' will create different 'algebras' for different populations of students.

Stein, M. K., Kaufman, J. H., Sherman, M., & Hillen, A. F. (2011). Algebra: A challenge at the crossroads of policy and practice. *Review of Educational Research*, 81, 453-492.

Is it Even Really Algebra?

Nearly all of the class of 2005 graduated having taken "Algebra 1."

However, based on the course materials, fewer than one in four studied the kind of challenging topics needed to prepare for college-level mathematics.

Nord, C., Roey, S., Perkins, R., Lyons, M., Lemanski, N., Brown, J., and Schuknect, J. (2011). *The Nation's Report Card: America's High School Graduates* (NCES 2011-462). U.S. Department of Education, National Center for Education Statistics. Washington, DC: U.S. Government Printing Office.

Are the Teachers Tracked*?*

Teachers themselves are tracked, with those judged to be the most competent, experienced, or high status assigned to the top tracks and those with the least experience and training assigned to the lower tracks.

Darling-Hammond, L. (2007). The flat earth and education: How America's commitment to equity will determine our future. *Educational Researcher*, 36(6), 318-334.

Who is Teaching Whom?

In a study of 29 districts in 16 states, marginalized students in grades 4 though 8 had access to less effective instruction than non-marginalized students, and that lack of access persisted over time.

Isenberg, E., Max, J., Gleason, P., Potamites, L., Santillano, R., Hock, H., & Hansen, M. (2013). *Access to effective teaching for disadvantaged students* (NCEE 2014-4001). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

We Need to Reflect on Course and Teacher Assignment Practices

Schools should carefully study whether their teacher assignments and tracking practices are helping or hindering equity …

Lubienski, S. T. (2007). What can we do about achievement disparities? Educational Leadership, 65(3), 54-59.

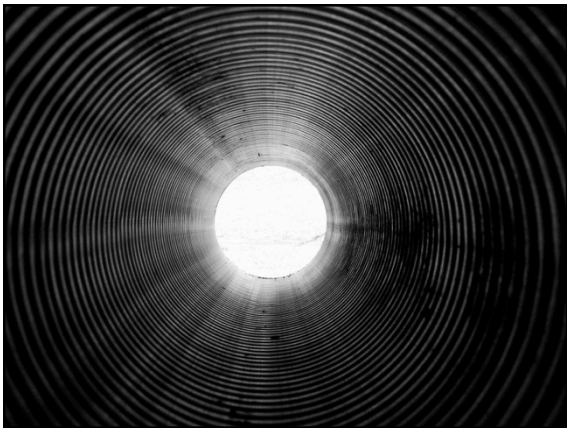
We expect that the very best doctors will treat the most grievously ill patients.

It should be no different in education. Great teachers have the skills to help the students who struggle the most.

Education Trust. (2005). *Gaining traction, gaining ground: How some high schools accelerate learning for struggling students*. Washington, DC: Education Trust.

Access, Equity, and Empowerment

NCTM is re-framing its work to focus on Access, Equity, and Empowerment, to capture the critical constructs of *identity, agency, and social justice*.



High Quality Mathematics Education Can Be Part of the Solution to Social Inequality By:

- Eliminate tracking.
- Supporting students in acquiring higher-order mathematics knowledge.
- Supporting positive attitudes (identity and agency) by providing students with multiple opportunities to learn key concepts and offering tailored support to students.

OECD. (2016). *Equations and inequalities: Making mathematics accessible to all.* Paris: PISA OECD Publishing. Downloaded at http://dx.doi.org/10.1787/9789264258495-en.

Our Future Depends on Educating Every Student to a High Level

The economy has become increasingly bifurcated into a low-skill, low-wage sector and a high-skill, high-wage information sector. Largely gone are the manufacturing jobs that provided a middle class wage without a college degree. As a result, education success has become increasingly essential to economic success.

Reardon, S. F. (2013). The widening income achievement gap. *Educational Leadership*, 70, 10-16.

But We Need to Also Focus on Mathematics For Participation in Our Democratic Society

Traditionally, mathematics education has been connected to issues of national economic survival, rather than to the development of democratic citizenship through critical thinking in mathematics.

Tate, W. F. (2013). Race, retrenchment, and the reform of school mathematics. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: Teaching social justice by the numbers, second edition* (pp. 42-51). Milwaukee, WI: Rethinking Schools.

But We Need to Also Focus on Mathematics For Participation in Our Democratic Society

Mathematics literacy is essential to informed and active participation in our democratic society. We live in a world where mathematics is increasingly used to characterize societal problems and formulate solutions.

But We Need to Also Focus on Mathematics For Participation in Our Democratic Society

Without mathematics literacy and a positive mathematics identity and agency, ordinary citizens are unlikely to comprehend, let alone challenge, many of the decisions and actions of those in power in political, social, scientific, and economic institutions.

An Important Goal of Mathematics Education

We want to live in a society where citizens not only have the agency to improve their own lives, but the lives of others and society at large.

Each and everyone of us is engaged in an enterprise much bigger than the specific tasks of our daily job.

It Can All Seem Overwhelming and Change Often Feels Sisyphean!

Change is Hard!

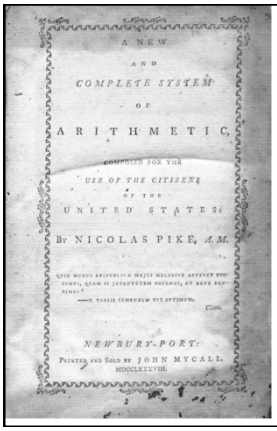
The most likely reason for the stability of teaching practices over time is that teaching is a cultural activity and cultural activities, by their very nature, are highly resistant to change.

Stigler, J. W., & Thompson, B. J. (2009). Thoughts on creating, accumulating, and utilizing shareable knowledge to improve teaching. *The Elementary School Journal*, 109(5), 442-457.

Some Practices are a Cultural Trap

Cultural routines evolve over time to enable adaptation to the environment. However, sometimes the environment changes, and yet, the cultural routine persists, even if it is now highly maladaptive.

Stigler, J. W., & Thompson, B. J. (2009). Thoughts on creating, accumulating, and utilizing shareable knowledge to improve teaching. *The Elementary School Journal*, 109(5), 442-457.


A NEW AND COMPLETE SYSTEM OF ARITHMETIC, COMPOSED FOR THE USE OF THE CITIZENS OF THE UNITED STATES. By NICOLAS PIKE, A.M. NEWBURY-PORT: Printed and Sold by JOHN MYCALL. MDCCLXXXVIII.

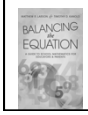
We live in the educational shadow of the 18th century.

Nicholas Pike's 1788 *Arithmetic*

Cultural Teaching Script: State a Rule, Provide an Example, Practice the Rule

Moving Forward: Support Research-Informed Instructional Practices

We expect physicians to use research-informed treatments. We must do the same.



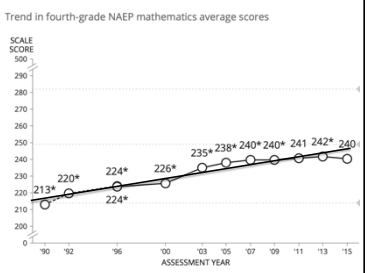
Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents.* Bloomington, IN: Solution Tree.

Moving Forward: Support and Implement Research-Informed Instructional Practices

The six guiding principles constitute the foundation of high-quality mathematics education.

Standards-Based Reform Has Improved Mathematics Learning

Math achievement in this country is up over the long-term … Since we've been doing Standards-based reform!



Standards-Based Reform Has Improved Mathematics Learning

Based on the NAEP long-term trend assessment, initiated in 1973, today's fourth and eighth graders are performing at a significantly higher level than their parents and grandparents did in mathematics.

NCTM. (2016). *Mathematics education in the United States 2016: A capsule summary fact book.* Reston, VA: NCTM.

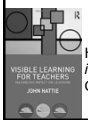
We Know What to Do – We Just Need to Do It!

It is critical that schools learn the lesson that "best practice" in effective organizations is rarely new practice … the most effective actions are well-known practices, with the extra dimension that they are reinforced and carried out reliably.

Schmoker, M. (2011). *Focus: Elevating the essentials to radically improve student learning.* Alexandria, VA: ASCD.

YOU Can Make it Happen!

[Effective] Teachers and leaders believe that success and failure in student learning is about what they, as teachers or leaders, did or did not do … We are change agents!



Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning.* New York: Routledge, Taylor & Francis Group.