

Atlanta | July 14–16, 2016

Engaging Students *in* Learning: Mathematical Practices

AN NCTM INTERACTIVE INSTITUTE FOR GRADES 9–12



PROGRAM WORKBOOK

NCTM® | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

nctm.org

Welcome!

On behalf of the NCTM High School Institute Advisory Group, we welcome you to this interactive institute focusing on mathematical practices and teaching practices. This institute is one component of NCTM's ongoing initiative to help achieve the vision of reasoning and sense making as a part of the mathematics classroom every day. This professional learning experience has been designed to help you engage students in learning mathematics by examining the Standards for Mathematical Practice, the NCTM Process Standards, and the eight effective teaching practices explained in NCTM's *Principles to Actions*, with guidance from recognized mathematics leaders as well as thoughtful reflection and discussion with your peers.

One goal of the institute is to work on using and creating tasks that address the mathematical practices and processes. The two and a half days of the institute and the networks that you develop here are part of achieving that goal. We encourage you to take full advantage of this Institute: participate in all plenary and breakout sessions, actively engage in task group work, and network with colleagues from throughout the United States and beyond. Then, at the end of the day, meet up with friends or family and enjoy the Atlanta area.

We wish to thank the staff at NCTM for helping us with the planning and logistics for the Institute, marketing, registration, and on-site work. We also thank all the presenters for agreeing to participate and share their expertise, views, and insights. Finally, we thank everyone in attendance, and we hope that you will find the institute helpful as you work to make reasoning and sense making through mathematical practices and processes a daily classroom experience for all students..

High School Institute Advisory Group

Ed Dickey
University of South Carolina

Fred Dillon
Baldwin Wallace University, Berea, Ohio
Ideastream/PBS, Cleveland, Ohio

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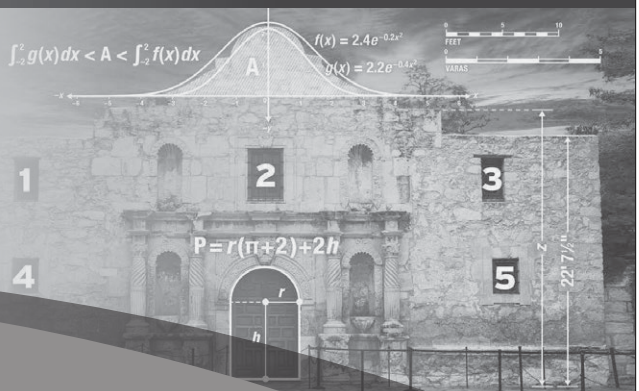
Save the Date!

PREMIER MATH EDUCATION EVENT

NCTM ANNUAL MEETING & EXPOSITION 2017

April 5–8 | San Antonio



Creating Communities and Cultivating Change

It's never too early to plan ahead for the leading math education event of the year. Network with thousands of your peers and fellow math education professionals to exchange ideas, engage with innovation in the field and discover new learning practices that will drive student success.

The latest teaching trends and topics will include:

- **Access and Equity:** Teaching Mathematics with an Equity Stance
- **Assessment:** A Tool for Purposeful Planning and Instruction
- **Building Conceptual and Procedural Understanding**
- **Professionalism:** Learning Together as Teachers
- **Teaching, Learning, and Curriculum:** Best Practices for Engaging Students in Productive Struggle
- **The "M" in STEM/STEAM**
- **Tools and Technology:** Using Technology to Effectively Teach and Learn Mathematics

Registration will open Nov. 2016

The NCTM Annual Meeting & Exposition is ideal for:

PRE-K–12 TEACHERS
MATH TEACHER EDUCATORS
NEW AND PROSPECTIVE TEACHERS
MATH COACHES AND SPECIALISTS
MATH RESEARCHERS
SCHOOL AND DISTRICT ADMINISTRATORS

Learn more at **nctm.org/annual** and follow us on #NCTMannual

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Task Discussion Groups

Resources

National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-1502;
Telephone (703) 620-9840; Fax (703) 476-2970; E-mail nctm@nctm.org; Web www.nctm.org

INSTITUTE INFORMATION

Range of Activities

All presentations are open to all Institute participants. Admission is on a first-come, first-served basis. Reserving spaces in line or saving seats is not permitted. The following activities are available:

Keynote Sessions—(60–75 minutes)—Well-known leaders in math education will address crucial topics related to and supporting the mathematical practices and process standards.

Breakout Workshops—(90 minutes)—Math education practitioners will engage participants in hands-on activities and strategies for implementing these activities in the classroom. Rooms are set up with round tables for hands-on work. Choose from seven different workshops during each breakout workshop time slot.

Task Discussion Groups—(90–120 minutes)—Participants will engage in guided activities and facilitated discussions that address the CCSSM mathematical practices. Discussions on the mathematical practices will be rooted in a content strand. Attend task groups according to the content strands that you selected when registering.

Program Updates

Program updates, including speaker updates, will be available at the Information Desk.

Materials Pickup / Information Desk

Located in the Atrium, the NCTM Information Desk is available to correct or replace badges and to offer general assistance. On-site registration for new attendees will not be available, and no payments will be collected at the meeting.

Materials Pickup & Information Desk Hours

Wednesday, July 13	4:00 p.m.–7:00 p.m.
Thursday, July 14	8:00 a.m.–4:30 p.m.
Friday, July 15	8:00 a.m.–4:00 p.m.

You must wear your badge to enter all presentations.

By registering for the NCTM Interactive Institute, participants grant NCTM the right to use, in promotional materials, their likeness or voice as recorded on, or transferred to, videotape, film, slides, audiotapes, or other media.

For Your Child's Safety

Because of the size and nature of the 2016 High School Institute, this event is not an appropriate setting for children under 16 years of age. Your hotel concierge can recommend activities for children while you are attending the Institute. We appreciate your understanding and cooperation.

NCTM Book Display

The NCTM Bookstore, located in the Atrium, will feature NCTM publications with a focus on grades 9–12 mathematics education. NCTM publications are for display purposes only; attendees can purchase books on-site and save 25 percent off the list price in addition to receiving free shipping. Books can be purchased after the meeting by using the special conference discount code HSPRAC16; however, shipping fees will be an additional charge. Free shipping and the 25 percent discount apply only to on-site purchases from July 13 to July 16. This discount applies to all NCTM publications and is not limited to those on display. Offer expires August 31, 2016.

Book Display Hours

Wednesday, July 13	4:00 p.m.–7:00 p.m.
Thursday, July 14	8:00 a.m.–4:30 p.m.
Friday, July 15	8:00 a.m.–4:30 p.m.
Saturday, July 16	8:00 a.m.–11:30 a.m.

Lost-and-Found

Attendees who have lost or found items may retrieve or turn them in at the Information Desk located in the Atrium. After the institute, all lost-and-found items will be turned over to hotel security.

Boxed Lunches

The networking lunch will take place in the Grand Ballroom. You will need to show your badge to pick up a lunch.

Wireless Internet Access

You will be able to access the Internet through a wireless connection in all meeting rooms by using the log-in information below:

Network name: **Westin-MeetingRoom**
Password: **nctm2016**

Social Media

Twitter: Find us on Twitter at #NCTMINST and, of course, at #NCTM.

Facebook: http://www.facebook.com/TeachersofMathematics

Handouts

Handouts and PDFs of the presentations will be available online within one to two weeks after the close of the Institute at **http://nctm.org/hs16**.

Welcome Reception

Enjoy light hors d'oeuvres and a cash bar provided by the Westin Atlanta Airport in Candler on Thursday, July 14, from 5:00 to 6:00 p.m.

SCHEDULE AT A GLANCE

Wednesday, July 13

4:00 p.m.–7:00 p.m.	Materials Pickup

Thursday, July 14

8:00 a.m.–4:30 p.m.	Materials Pickup
9:00 a.m.–10:15 a.m.	Opening Session: Margaret (Peg) Smith
10:30 a.m.–12:30 p.m.	Task Discussion Groups
12:30 p.m.-1:30 p.m.	Boxed Lunches / Networking
1:30 p.m.–3:00 p.m.	Breakout Workshops
3:15 p.m.–4:15 p.m.	Keynote Session: Dylan Wiliam
5:00 p.m.–6:00 p.m.	Welcome Reception

Friday, July 15

8:30 a.m.–9:30 a.m.	Keynote Session: David Pugalee
9:45 a.m.–11:45 a.m.	Task Discussion Groups
11:45 a.m.–12:45 p.m.	Boxed Lunches / Networking
12:45 p.m.–2:15 p.m.	Breakout Workshops
2:30 p.m.–3:30 p.m.	Keynote Session: Barbara Dougherty

Saturday, July 16

8:00 a.m.–9:30 a.m.	Breakout Workshops
9:45 a.m.–11:15 a.m.	Task Discussion Groups
11:30 a.m.–12:30 p.m.	Closing Session: Cathy Seeley

THURSDAY, JULY 14

8:00 a.m.–4:30 p.m.

Materials Pickup—Atrium

9:00 a.m.–10:15 a.m.

Opening Session

Effective Teaching Practices: The Key to Supporting Students' Learning of Ambitious Standards

The recent publication *Principles to Actions* from NCTM (2014) describes a core set of effective teaching practices that are essential for promoting deep learning of mathematics. This session will focus on describing these eight practices, discussing how they impact student learning, and engaging participants in a discussion of how best to help teachers learn and refine them.

Margaret (Peg) Smith
pegs@pitt.edu
University of Pittsburgh, Pennsylvania
Grand Ballroom III and IV

10:30 a.m.–12:30 p.m.

Task Discussion Groups

Algebra 1 (Blue Circle)—***Grand Ballroom II***

Algebra 1 (Red Circle)—***College Park***

Geometry (Yellow Circle)—***Jasmine***

Geometry (Green Circle)—***Dogwood***

Algebra 2 (Orange Circle)—***Gardenia***

Algebra 2 (Purple Circle)—***Magnolia***

12:30 p.m.–1:30 p.m.

Networking Lunch

Grand Ballroom III and IV

1:30 p.m.–3:00 p.m.

Breakout Workshops

Fostering Flexibility with Functions

(Beginning Algebra)

Understanding functions allows students to describe and make sense of relationships between quantities. Students should move flexibly among tabular, graphical, symbolic, and contextual representations of functions. Participants will engage in activities that enhance student understanding of representations of functions and their parameters.

Kim Knighton
kknig@profile.k12.nh.us
Profile School, Franconia, New Hampshire
Grand Ballroom II

When Traditional Won't Do: Algebra Tasks That Promote Understanding

(Beginning Algebra)

Designing tasks that challenge learners to think deeply and critically about mathematics is a crucial part of what teachers do. Learners in beginning algebra are often denied access to relevant and rigorous mathematics and the opportunity to engage and reason with mathematics. This session will provide participants with an opportunity to experience tasks designed to engage learners in developing mathematical understanding by connecting to students' lived experiences.

Crystal Morton
cranhill@iupui.edu
IUPUI, Indianapolis, Indiana
Grand Ballroom I

Connecting Algebra and Geometry through Coordinates: Transforming Lines to Make Polygons

(Geometry)

The effects of transformations on lines and the resulting polygons will be analyzed. Slopes of lines, translations, reflections, rotations, systems of equations, and relationships in special right triangles will be discussed. Participants will engage in a rich task that makes connections between algebra 1 and geometry. The Mathematics Teaching Practices from *Principles to Actions* will be discussed with an emphasis on facilitating meaningful mathematical discourse.

Benjamin Sinwell
bsinwell@anderson4.org
Pendleton High School, Pendleton, South Carolina
Jasmine

Piecewise Functions and the Mathematics Teaching Practices

(Intermediate Algebra)

Piecewise functions are a natural step in solving many real-world problems. We will work with tasks that build piecewise functions from linear and nonlinear functions, that have multiple entry points, and that use multiple representations. The session will focus on setting goals, selecting tasks, connecting multiple representations, and using discourse and questions to enhance student learning.

Fred Dillon
fdillon.nctm@gmail.com
Baldwin Wallace University, Berea, Ohio
Ideastream/PBS, Cleveland, Ohio
College Park

A Multiple Representation Mash-Up Problem for Intermediate Algebra

(Intermediate Algebra)

We will work on a challenging intermediate algebra task that was created by mashing two classic multiple representation problems. You will leave with ideas for transforming your favorite activities from early algebra into cognitively demanding tasks appropriate for the intermediate algebra classroom.

Nicole Bannister
nbannis@clemson.edu
Clemson University, Clemson, South Carolina
Gardenia

Mathematical Practices through a Statistical Lens, Part 1: Statistics versus Mathematics

(Statistics)

In this first of two 90-minute workshops, we will use a variety of frameworks to illustrate the similarities and differences between good mathematical work and good statistical work in the 9–12 classroom. We will work through classroom-ready tasks in exploratory data analysis and probability that illustrate how these differences look and sound. Bring a laptop or handheld device with Internet access if possible.

William Thill
wthill@hw.com
Harvard Westlake, Studio City, California
Dogwood

Developing Equitable Teaching Practices

(Equity)

This workshop features strategies for planning equitable mathematics lessons, which includes posing purposeful questions to encourage sense making and using exit tickets to increase access and participation. Participants will watch video to identify equitable teaching in action, and examine problems of practice related to designing and enacting equitable mathematics instruction. Participants will learn about using a specially designed equity sticky note to plan lessons to facilitate equitable math teaching.

Imani Masters
imanigoffney@gmail.com
University of Maryland, College Park
Magnolia

3:15 p.m.–4:15 p.m.

Keynote Session

Classroom Formative Assessment: Engaging Learners and Responding to Their Needs

As every math teacher knows, students do not learn what we teach. That is why assessment is a key process in effective education—it is only through assessment that we can establish what has been learned. This keynote will present the five key strategies of classroom formative assessment, coupled with a number of techniques for classroom use.

Dylan Wiliam
dylanwiliam@mac.com
Institute of Education, University of London, London, U.K.
Grand Ballroom III and IV

5:00 p.m.–6:00 p.m.

Welcome Reception

Candler

8:30 a.m.–9:30 a.m.

Keynote Session

Writing and Mathematical Thinking: Grade-Level Strategies to Support Development of the Mathematical Practices

Students are increasingly asked to write as part of their mathematics experience. Writing in mathematics sustains the development of reasoning, communication, and connections while developing metacognitive thinking. Writing in mathematics supports both deep understanding of content and students' effective development of the mathematical practices. This session will highlight a plan for creating and implementing an effective mathematical writing program. Information will bridge the divide between language and mathematics instruction with user-friendly connections to theory and the standards, and ready-to-implement strategies for the classroom. Writing strategies tested in the mathematics classroom will be used to demonstrate support of the eight mathematical practices, thereby highlighting writing as a tool to develop these essential mathematical habits of mind.

David Pugalee
David.Pugalee@uncc.edu
University of North Carolina at Charlotte
Grand Ballroom III and IV

9:45 a.m.–11:45 a.m.

Task Discussion Groups

Algebra 1 (Blue Circle)—***Grand Ballroom II***

Algebra 1 (Red Circle)—***College Park***

Geometry (Yellow Circle)—***Jasmine***

Geometry (Green Circle)—***Dogwood***

Algebra 2 (Orange Circle)—***Gardenia***

Algebra 2 (Purple Circle)—***Magnolia***

11:45 a.m.–12:45 p.m.

Networking Lunch

Grand Ballroom III and IV

12:45 p.m.–2:15 p.m.

Breakout Workshops

Opening Up to Algebraic Thinking
(Beginning Algebra)

Many students view algebra as routine equation solving and graphing, consisting primarily of right and wrong answers. This view can be changed as teachers pose more "open" questions that allow for different approaches and rich mathematical discourse among students. Participants will explore techniques to adapt existing questions and practical strategies to optimize the effectiveness of discussions.

Kim Knighton
kknig@profile.k12.nh.us
Profile School, Franconia, New Hampshire
Grand Ballroom II

Building Mathematical Discourse with Beginning Algebra Tasks
(Beginning Algebra)

Struggling algebra students often disengage, making classroom discourse a challenge. Providing small groups of students with related tasks allows for discussion within the group. Comparing and contrasting each related task with other groups' results promotes productive struggle and conceptual understanding of linear functions. Participants will focus on questioning strategies promoting classroom discourse. A laptop or tablet will be helpful.

Jerel Welker
jwelker@lps.org
Lincoln Public Schools, Lincoln, Nebraska
Grand Ballroom I

Just Keep Spinning: Bridging Geometry to Calculus
(Geometry)

Engage in rich tasks to promote deep understanding of geometric solids. We will use symmetry and transformational geometry to develop volume formulas for cones, cylinders, and frustrums. Experience classroom-tested activities and projects for which students model, create, and construct solids of revolution.

Brian Shay
brian.shay@sduhsd.net
Canyon Crest Academy, San Diego, California
Jasmine

Using Transformations to Make Connections between Polygons and Circles
(Geometry)

This session will focus on the use of dilations to help students understand similarity. Teachers will explore modeling right triangle similarity by graphing linear equations. A regular hexagon will be constructed (and proved) with the use of compasses or dynamic geometry software, and connections will be made between geometric constructions and polygons graphed in the coordinate plane.

Benjamin Sinwell
bsinwell@anderson4.org
Pendleton High School, Pendleton, South Carolina
Dogwood

Building a Stronger Foundation for Understanding in Advanced Trigonometry
(Intermediate Algebra)

As topics in algebra 2 become more abstract, establishing and building on strong foundations of understanding can support greater retention and the purposeful application of abstract procedures. Through approaches that build procedural fluency from conceptual understanding, this session seeks to lower the barriers of abstraction that lead students to blindly misapply procedures, not knowing what questions are asking them to do and not knowing whether their answers even make sense. In this session, we will engage with strategies and tasks that both establish a stronger foundation of understanding in right triangle trig and that extend nicely to support a more successful move beyond SOHCAHTOA into nonright triangle trigonometry.

Jason Slowbe
jason.slowbe@gmail.com
Great Oak High School, Temecula, California
College Park

Building Functions
(Intermediate Algebra)

Participants will work on two rich tasks that involve creating equations to describe number relationships and building a function to model a relationship between two quantities, and will discuss how the activities promote mathematical discourse.

Thomas Durante
thomasduarte@yahoo.com
Anaheim High School, Anaheim, California
Gardenia

Re-Examining Mathematics Teaching Practices with a Lens on Equity
(Equity)

This workshop will continue to leverage the tasks and activities from the first equitable mathematics instruction workshop. The activities in this workshop will discuss additional case studies and examine problems of practice related to designing and enacting equitable mathematics instruction. We will continue to explore the use of the equity sticky note as a planning tool, and we will also explore teaching strategies like the use of smartness wordless and re-examining discipline practices that inhibit linguistically and culturally diverse students and economically disadvantaged students from fully engaging in the mathematics class.

Imani Masters
imanigoffney@gmail.com
University of Maryland, College Park
Magnolia

2:30 p.m.–3:30 p.m.

Keynote Session

2*b* or Not 2*b*: Misconceptions in Algebraic Reasoning

Students are often able to solve simple linear equations but have significant misconceptions about the roles that a variable can assume. This keynote will share some student misconceptions and describe ways to support stronger understanding for more sophisticated uses of variable. Instructional techniques will be included in the discussion.

Barbara Dougherty
doughertyb@missouri.edu
University of Missouri–Columbia
Grand Ballroom III and IV

8:00 a.m.–9:30 a.m.

Breakout Workshops

Equity and Data: Beginning Algebra Tasks That Promote Problem Solving
(Beginning Algebra)

Carefully crafted tasks can help teachers engage in mathematics teaching practices that empower learners to recognize and use mathematics as an analytical tool to understand and possibly change society. In this session, participants will engage in tasks designed to examine and critique social, political, and economic issues. In doing these tasks, participants will consider the strategies used by learners to solve the task and how learners can use their mathematical knowledge as a tool for social change. Participants will discuss the Mathematics Teaching Practices from *Principles to Actions* (NCTM 2014), with a particular focus on access and equity.

Crystal Morton
cranhill@iupui.edu
IUPUI, Indianapolis, Indiana
Grand Ballroom II

Engaging Beginning Algebra Students Using Technology
(Beginning Algebra)

How can I best use the digital devices in my algebra classroom? Participants will explore applications allowing students to investigate mathematical ideas, promoting learning using visual representations of functions, developing mathematical models for algebraic functions, and providing immediate feedback to the student. A laptop or tablet and a TI-83/84 calculator will be helpful during this session.

Jerel Welker
jwelker@lps.org
Lincoln Public Schools, Lincoln, Nebraska
Grand Ballroom I

Construction Junction, What's Your Function?
(Geometry)

Constructions are the foundation for all geometry and provide rich connections between circles, angles, quadrilaterals, and triangles. Experience rich and thought-provoking constructions that move student thinking beyond the four basic constructions to deep relationships. These constructions are tactile ways for students to experience, see, and feel the depths and connections within geometry.

Brian Shay
brian.shay@sduhsd.net
Canyon Crest Academy, San Diego, California
Jasmine

Sustaining High Cognitive Demand in Intermediate Algebra Problems
(Intermediate Algebra)

Beyond choosing good problems, teachers should be equipped with strategies for implementing and modifying problems to sustain their cognitive demand for a wide range of diverse learners. In this session, we will address questioning strategies for developing purposeful questions that create "higher ceilings" or even "different ceilings" to meaningfully extend the runway of any intermediate algebra problem or task.

Jason Slowbe
jason.slowbe@gmail.com
Great Oak High School, Temecula, California
College Park

Let's Talk about Intermediate Algebra
(Intermediate Algebra)

This goal of this session is to use a shared problem-solving experience to make sense of larger ideas about facilitating meaningful mathematical discourse in the intermediate algebra classroom. Participants will experience a discourse-intensive intermediate algebra task and debrief the collaborative problem-solving experience in the context of the *Principals to Actions* Mathematics Teaching Practices.

Nicole Bannister
nbannis@clemson.edu
Clemson University, Clemson, South Carolina
Gardenia

Access and Equity for All Learners
(Equity)

This session will focus on providing access and equity for ELLs and students with special needs. Participants will engage in a math game to increase mathematical discourse, discuss how to use design activities that allow every student access to high-level mathematics, and use alternative assessments to provide insight into student understanding as well as enhance students' writing skills.

Thomas Durante
thomasduarte@yahoo.com
Anaheim High School, Anaheim, California
Dogwood

Mathematical Practices through a Statistical Lens, Part 2: Studies, Simulation, and Inference
(Statistics)

In this second workshop, attendees will participate in a controlled, randomized experiment. We will do this to better understand how small, thoughtfully designed, nontraditional activities better help students to reason statistically and to apply newly acquired procedural skills appropriately. We will work through an activity that will help participants better understand the essential statistical concepts of random assignment, simulating an experiment based on a randomization model, and making inferences from data to a larger context. Participants do not need to attend part 1 to benefit from part 2.

William Thill
wthill@hw.com
Harvard Westlake, Studio City, California
Magnolia

9:45 a.m.–11:15 a.m.

Task Discussion Groups

Algebra 1 (Blue Circle)—***Grand Ballroom II***
Algebra 1 (Red Circle)—***College Park***
Geometry (Yellow Circle)—***Jasmine***
Geometry (Green Circle)—***Dogwood***
Algebra 2 (Orange Circle)—***Gardenia***
Algebra 2 (Purple Circle)—***Magnolia***

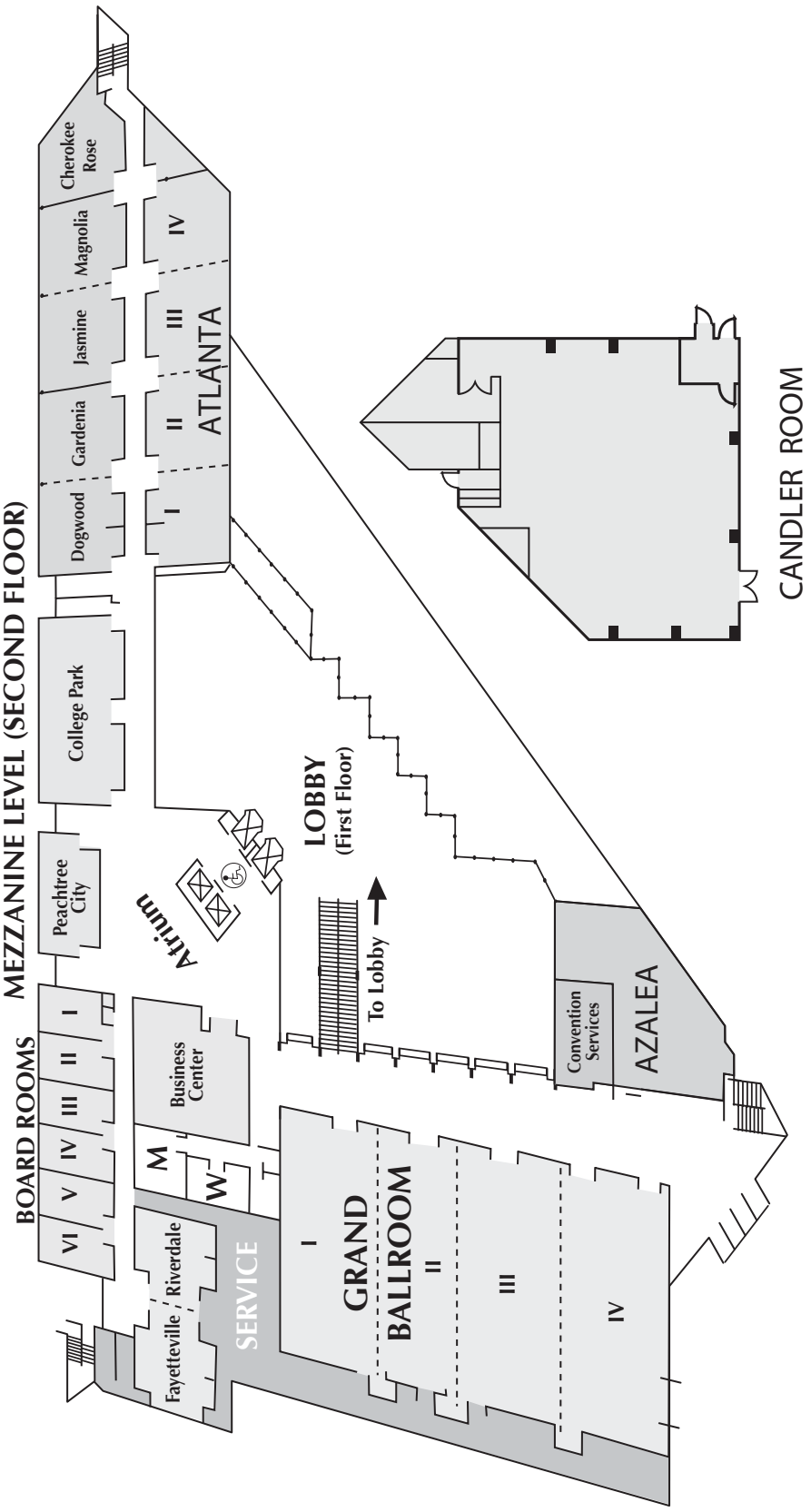
11:30 a.m.–12:30 p.m.

Closing Session

Processing and Practicing What You've Learned

We will look back over the experiences of the institute as you consider next steps for the coming school year. Let's think together about how you can process what you've learned and put it all into practice in rich classrooms focused on every student becoming a mathematical thinker.

Cathy Seeley
cseeley@utexas.edu
Charles A. Dana Center at the University of Texas at Austin (retired)
NCTM Past President
Grand Ballroom III and IV


MEZZANINE LEVEL (SECOND FLOOR)
BOARD ROOMS
I
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Peachtree City
College Park
Dogwood
Gardenia
Jasmine
Magnolia
Cherokee Rose
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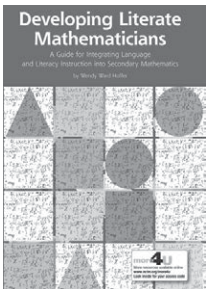

Developing Literate Mathematicians: A Guide for Integrating Language and Literacy Instruction into Secondary Mathematics

BY WENDY WARD HOFFER

How can we integrate literacy instruction authentically into mathematics content to support mathematical understanding? Busy secondary mathematics teachers who seek to respond to the needs of their students and the demands of the Common Core State Standards will welcome this book, which offers lively classroom examples, usable research, and specific ideas and resources. Enrich your students' understanding of mathematics by attending to reading, vocabulary, discourse, and writing through a workshop model.

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More Lessons Learned from Research, Volume 2: Helping All Students Understand Important Mathematics

EDITED BY EDWARD A. SILVER AND PATRICIA ANN KENNEY

Applying research to strengthen teaching practice and ensure students' success in mathematics

More than seventy years of research point to the importance of teaching mathematics for understanding. Successful students actively construct understanding rather than passively receive knowledge. Implications of this fundamental lesson from research are explored in different ways through twenty-four chapters presented in this book. Chapters cover investigations of a wide range of topics, approaches, and settings, and mathematics teachers at all levels will find examples of research that are relevant to the challenges they face.

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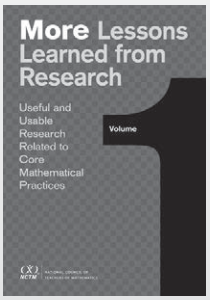

DON'T MISS! More Lessons Learned from Research, Volume 1

EDITED BY EDWARD A. SILVER

Helps to link classroom teachers to all that original research has to offer

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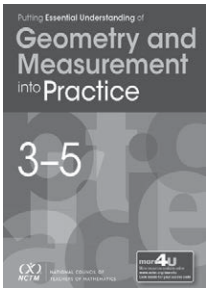
All books available as eBook.

Putting Essential Understanding of Geometry and Measurement into Practice in Grades 3–5



BY KATHRYN CHVAL, JOHN LANNIN, AND DUSTY JONES

KATHRYN CHVAL, VOLUME EDITOR

BARBARA J. DOUGHERTY, SERIES EDITOR

Do your students have "concept images" that limit their ideas of shapes to specific examples, oriented in particular ways? Do they confuse the size of an angle with the length of the rays in a drawing of an angle? This book demonstrates how to use multifaceted knowledge to address the big ideas and essential understandings that students must develop for success with geometry and measurement—not only in their current work, but also in higher-level mathematics and a myriad of real-world contexts.

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Problem Solving in All Seasons, Grades 3–5

BY KIM MARKWORTH, JENNI MCCOOL, AND JENNIFER KOSIAK

Holidays and seasonal activities offer perfect backdrops for mathematical tasks that can be related to other topics and themes in the classroom. This book delivers thirty-six appealing, real-world mathematical tasks, arranged in grade-level order, to engage young learners in problems tied to the Common Core and designed to allow children to participate in the Common Core Standards for Mathematical Practice. Each task includes a complete implementation guide, and handouts and ancillary materials can be accessed online. This is your all-in-one practical handbook for problem solving in the primary years.

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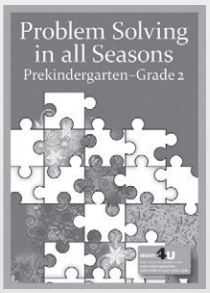

DON'T MISS! Problem Solving in All Seasons, Pre-K–Grade 2

BY KIM MARKWORTH, JENNI MCCOOL, AND JENNIFER KOSIAK

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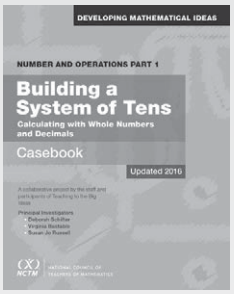
ADDITIONAL NEW TITLES



Developing Mathematical Ideas: Building a System of Tens, Casebook and Facilitators Guide

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On the Money: Math Activities to Build Financial Literacy in High School

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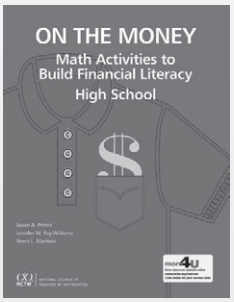

Annual Perspectives in Mathematics Education 2016

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Discovering Lessons for the Common Core Standards in Grades 9–12

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Task 1: Analyzing Change

Promoting Students' Algebraic Thinking by Using a Task from Japan

Participants will explore an open-ended task that provides opportunities for developing linear and quadratic functions from a visual geometric pattern and will then compare these functions. A major focus of the session will be on attending to the mathematical structures of functions.

Task 2: Counting Trains

Promoting Students' Algebraic Thinking by Using a Hands-On Activity

Participants will engage in a classroom-tested activity in which linear, quadratic, and exponential functions are developed by using patterns built from Cuisenaire® rods. A major focus of the session will be on using reasoning and sense making to develop the various functions. Participants will also have opportunities to develop and explore extensions to the activity.

Task 3: Analyzing Data–Correlation and Causation

Promoting Students' Algebraic Thinking by Using Data and Statistics

Participants will use the Statistics and Probability strand of CCSSM and the Standards for Mathematical Practice by collecting and graphing data that suggest a linear relationship. A major focus of the session will be on finding a line of best fit and assessing how well the linear model fits the data by using the correlation coefficient, the coefficient of determination, and residuals. Participants will also interpret the slope and y-intercept in context and interpolate, extrapolate, and distinguish between correlation and causation.

Facilitators:

Matthew (Blue) Taylor
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New Visions for Public Schools, New York City

Barbara Lynch
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Lakewood Schools, Lakewood, Ohio

$$2\left(\frac{n(n+1)}{2}\right) = n(n+1) = n^2 + n$$

Task 1: Rotating Squares

Constructing Valid Mathematical Arguments

Participants will examine the concepts of congruence, transformation, and similarity as they discover a relationship between congruent polygons and their areas. Both abstract and quantitative reasoning will be used as participants make and justify conjectures as well as discuss what it means to provide a valid and formal proof of a mathematical argument.

Task 2: Packaging Soda Cans

Seeing Structure and Making Sense of Problems

What is the most cost-effective, efficient way of packaging soda cans? Participants will apply geometric understandings of area of cross sections and other circle properties to determine how to best fit circles into rectangles of different sizes. The task employs student use of repeated reasoning and looking for mathematical structure.

Task 3: Constructing Geometric Meaning

Using Tools Effectively

Participants will examine how integrating other construction tools into their teaching will help students make connections between geometric constructions and broader geometric ideas, including symmetry, congruence, transformation, and reflection. Ideas for appropriate use of tools and structuring effective discussion in the mathematics classroom will be discussed and demonstrated by using a variety of strategies.

Facilitators:

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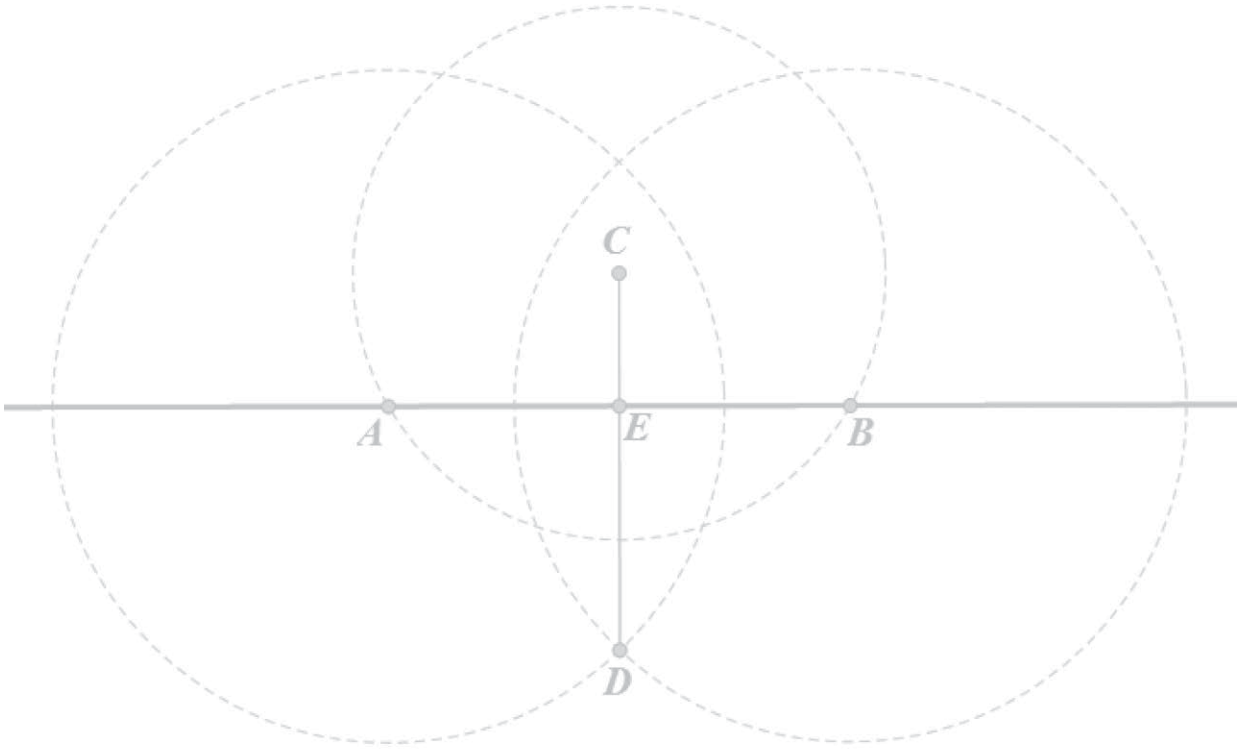

Task 1: Passing through Points

Building Functions through Given Points

How many different functions pass through three points? Attendees will collaborate in writing equations of functions that pass through the given points, and they will discover new ways of writing such functions. Attendees will also discuss strategies for implementing this task through the Common Core's Standards for Mathematical Practice in ways that advance students' content knowledge and support them in reasoning and sense making about functions.

Task 2: Adapting to Standards

Building Polynomial Functions

Creating new resources yourself is difficult, time-consuming, and often unnecessary. Many resources created prior to the Common Core still hold potential as meaningful "Common Core" tasks. Attendees will collaborate in adapting a polynomials task to incorporate the Standards for Mathematical Practice and align the task with the spirit of the Common Core. Interaction between graphical and symbolic representations can promote a better understanding of the mathematics involved. The questioning strategies we will discuss in this session can be leveraged to increase the rigor in any task, empowering teachers to take materials created by others and adapt them for use in their own classrooms.

Task 3: Collecting and Analyzing Data

Making Connections among Algebra II Topics

Statistics is a good topic to use to apply previously learned topics in a real-world context, as well as to show connections among different mathematical concepts. The classic M&Ms® experiment allows participants to compare different curves of fit made with their own data. The appropriateness of a model will be determined using scatterplots, residual plots, and transformed data. The data collected can be used to make connections between exponential and logarithmic functions and can be used as an impetus to exploring inverse functions. An additional activity about the kidney filtration of blood further enhances understanding of the concepts involved. Sample student work will be provided to allow attendees to reflect on student thinking, to make connections among topics, and to formulate questions that create opportunities for productive classroom discourse.

Facilitators:

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Dover High School, Dover, Ohio

$$2\left(\frac{n(n+1)}{2}\right) = n(n+1) = n^2 + n$$

SAMPLE REASONING AND SENSE-MAKING TASK

TITLE—*Should be a catchy title that gives a hint of the context of the problem.*
Source—*Include statement of source if this is a direct adaptation of a task from another source.*
Adapted/Prepared by _____—*Include name and affiliation of author(s).*

<table>
<tr><td>Purpose</td><td colspan="2">This should be a two- or three-sentence broad-brush summary of the task meant to stimulate further interest. Mention where this may fit into the curriculum, including prerequisites, as well as major reasoning habits or mathematical practices (see below). Some indication of the context may also be useful.</td></tr>
<tr><td>Focus on Reasoning and Sense Making</td><td>Focus in High School Mathematics: Reasoning and Sense Making (FHSM) Reasoning Habits

List the major reasoning habits (see pp. 9–10 of FHSM. For each, give the general category, followed by a hyphen and then the specific habit. Use semicolons to separate multiple habits from a category. With “Seeking and using connections,” you might pick a particular item from the list following the stem.

Principles and Standards for School Mathematics (PSSM) Process Standards

List the major process standards from PSSM. For each, give the standard name, followed by a hyphen and the area of emphasis within the standard. Use semicolons to separate multiple areas of emphasis from the same standard.</td><td>Common Core State Standards (CCSS) Mathematical Practices

List the practices from the CCSS Standards for Mathematical Practice (see pp. 7–8 of CCSSM) by standard number.</td></tr>
<tr><td>Focus on Mathematical Content</td><td>FHSM Key Elements

List the key elements from FHSM (see chapters 4–7). For each, give the content strand, followed by a hyphen and the key element. Use semicolons to separate multiple key elements from the same strand.</td><td>CCSS Content Standards

Give the high school standards addressed. For each, give the conceptual area and domain (as abbreviated in CCSS), along with the standard number, all separated by hyphens (e.g., F-BF-3). Then give the full statement of the standard.</td></tr>
<tr><td>Materials and Technology</td><td colspan="2">• Include a bulleted list of any handouts provided, other materials, and technological resources that a teacher might need to implement the task.</td></tr>
</table>

SAMPLE REASONING AND SENSE-MAKING TASK

In the Classroom

Task

State the task. Refer to any handouts distributed.

Use in the Classroom

This section should give teaching suggestions showing how the task might be used to promote student reasoning and sense making. *The focus of this discussion should be on what students will actually be doing and how the teaching will help this to happen.*	*Explain how the teaching suggestions support the reasoning habits and mathematical practices. These comments should line up with the teaching suggestions discussed.* *In general, one of the comments should explicitly address all reasoning habits and mathematical practices listed above. This should be aligned with the focus of this discussion sentence.*

Focus on Student Thinking

The body of this section should reflect student thinking related to the implementation of the ideas in the "In the Classroom" section. The range of student thinking should contain examples of both successful and unsuccessful student thinking (e.g., both student insights and misconceptions).

Assessment

This section should give suggestions for how a teacher will know whether the students are achieving the intended outcomes. This section might include classroom observations, classwork to be turned in, homework assignments, associated projects, or other write-ups.

Source(s)

Give the complete sources of any resources from which the task was drawn.

TASK TEMPLATE

TITLE—

Purpose		
Focus on Reasoning and Sense Making	***Focus in High School Mathematics: Reasoning and Sense Making* (*FHSM*) Reasoning Habits** ***Principles and Standards for School Mathematics* (*PSSM*) Process Standards**	***Common Core State Standards* (*CCSS*) Mathematical Practices**
Focus on Mathematical Content	***FHSM* Key Elements**	***Common Core State Standards* (*CCSS*) Content Standards**

TASK TEMPLATE

In the Classroom
Task
Use in the Classroom
Focus on Student Thinking
Assessment
Source

NOTES

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Take Your Professional Development to a New Level of Excellence

Developing Mathematical Ideas Professional Development Series

Number and Operations, Part 1: Building a System of Tens

Calculating with Whole Numbers and Decimals

BY DEBORAH SCHIFTER, VIRGINIA BASTABLE, AND SUSAN JO RUSSELL

Casebook: NCTM Stock No. 15032
List: $40.95 | **Member: $32.76**

Facilitator's Guide: NCTM Stock No. 15053
List: $62.95 | **Member: $50.36**

Building a System of Tens is the first module in the seven-part **Developing Mathematical Ideas Series**. The complete module consists of a casebook for participants and an online facilitator's package that contains everything necessary to prepare for and lead the seminar, including access to the casebook content and classroom videos.

The DMI program engages participants in collaborative learning where they make sense of the content, recognize where and how their grade content is situated in the trajectory of learning from K–grade 8, build connections among different concepts, and analyze student thinking from a mathematical perspective. Teachers learn how to orient their instruction to specific mathematical goals and to develop a mathematics pedagogy in which student understanding is central.

The ***Building a System of Tens Casebook*** was designed as a key component of the modules. The thirty cases, written by teachers describing real situations and actual student thinking in their classrooms, provide the basis of each session's investigation into specific mathematical concepts and teaching strategies. Guided by the facilitator, participants explore the base-ten number structure, consider how that structure is exploited in multidigit computational procedures, and examine how basic concepts of whole numbers reappear when working with decimals.

The online ***Building a System of Tens Facilitator's Package*** includes an introduction, preseminar preparation for the facilitator, and all the readings, materials, and videos needed to lead the eight sessions of the seminar.

Here is what educators are saying about the Developing Mathematical Ideas Series:

"Our teachers love DMI seminars. The seminars provide them with an opportunity to dig deeply into important mathematics content … teachers here say that it was what they learned in our DMI seminars that helped them transform their teaching."

LINDA RUIZ DAVENPORT
Director of K–12 Mathematics
Boston Public Schools

"The ongoing support from DMI trainers, case studies, and videos has provided us with a foundation for changing the way we engage our teachers and students in math instruction."

BRIAN SHINDORF
Director of Elementary Education
St. Joseph School District
St. Joseph, Missouri

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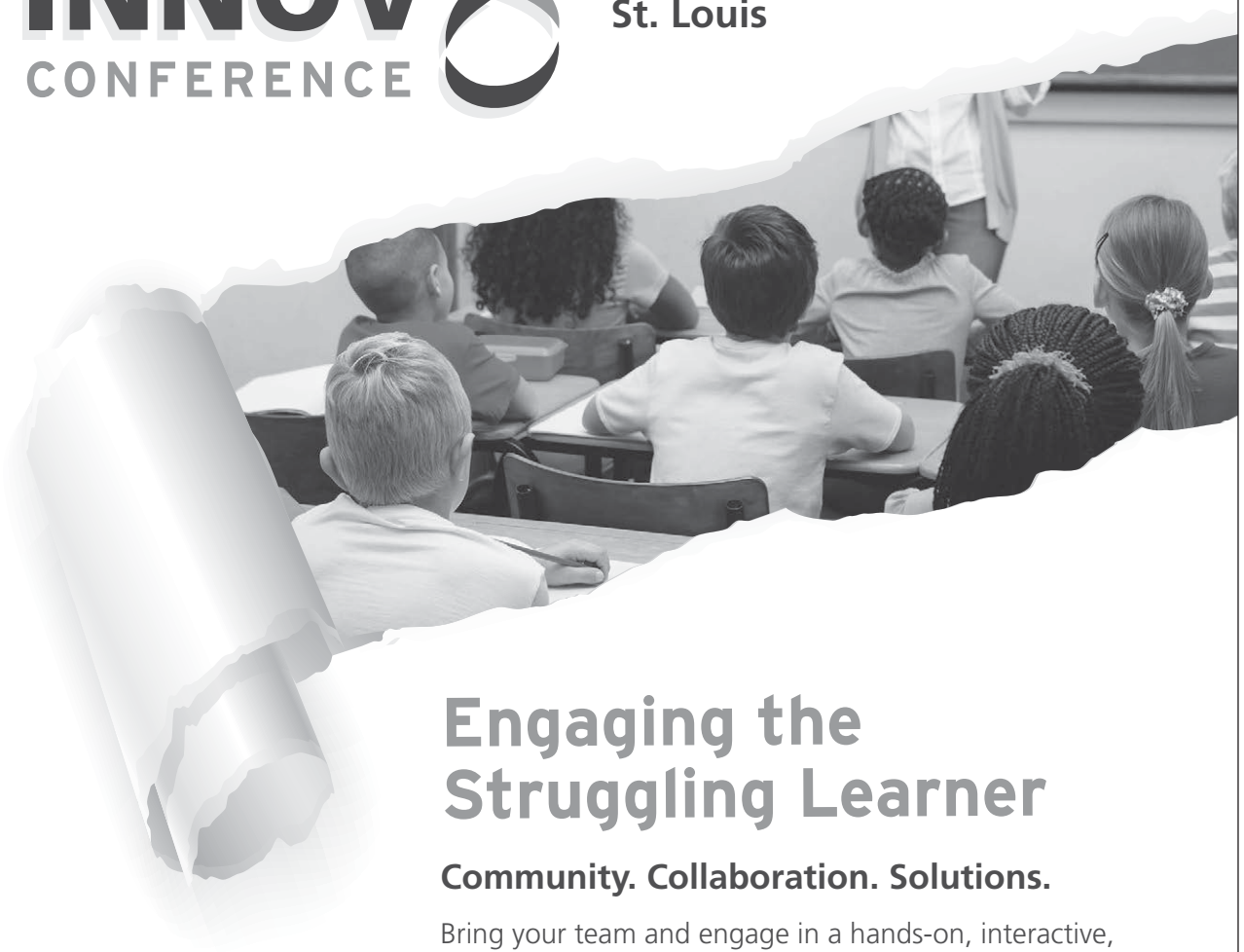

Engaging the Struggling Learner

Community. Collaboration. Solutions.

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With a focus on "Engaging the Struggling Learner," become part of a team environment and navigate your experience through three different pathways:

- Response to Intervention (RtI)
- Supporting productive struggle
- Motivating the struggling learner

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Worthwhile Mathematical Tasks

The teacher of mathematics should pose tasks that are based on—

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences; and
- knowledge of the range of ways that diverse students learn mathematics—

and that

- engage students' intellect;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions; and
- promote the development of all students' dispositions to do mathematics.

National Council of Teachers of Mathematics (NCTM). *Professional Teaching Standards.* Reston, Va.: NCTM, 1991

Principles to Actions: Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Principles to Actions: Ensuring Mathematical Success for All is an official position of the National Council of Teachers of Mathematics as approved by the NCTM Board of Directors, February 2014.

National Council of Teachers of Mathematics (NCTM). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, Va.: NCTM, 2014

NCTM PROCESS STANDARDS

Problem Solving

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems; and
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs; and
- select and use various types of reasoning and methods of proof.

Communication

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

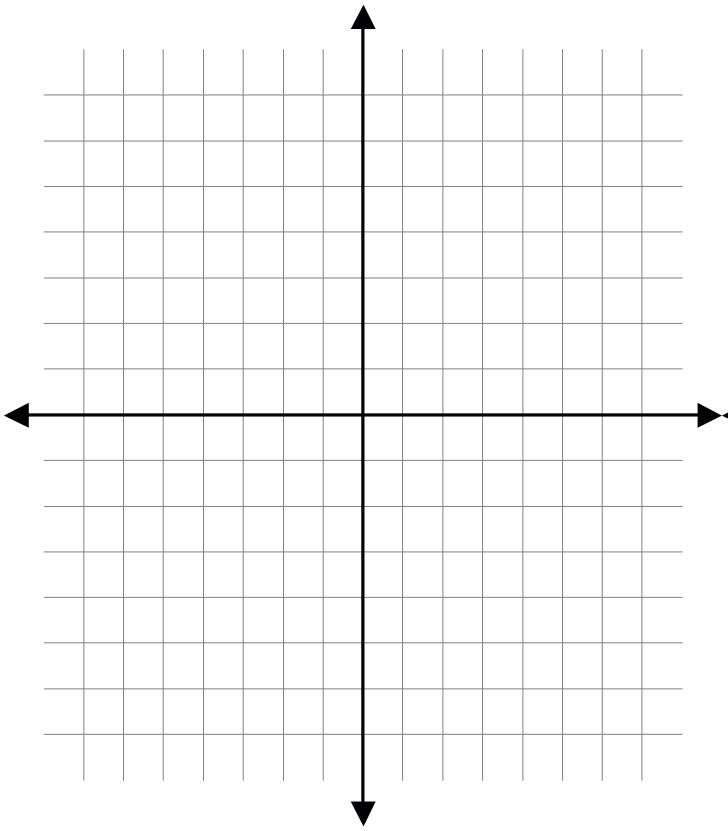
Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

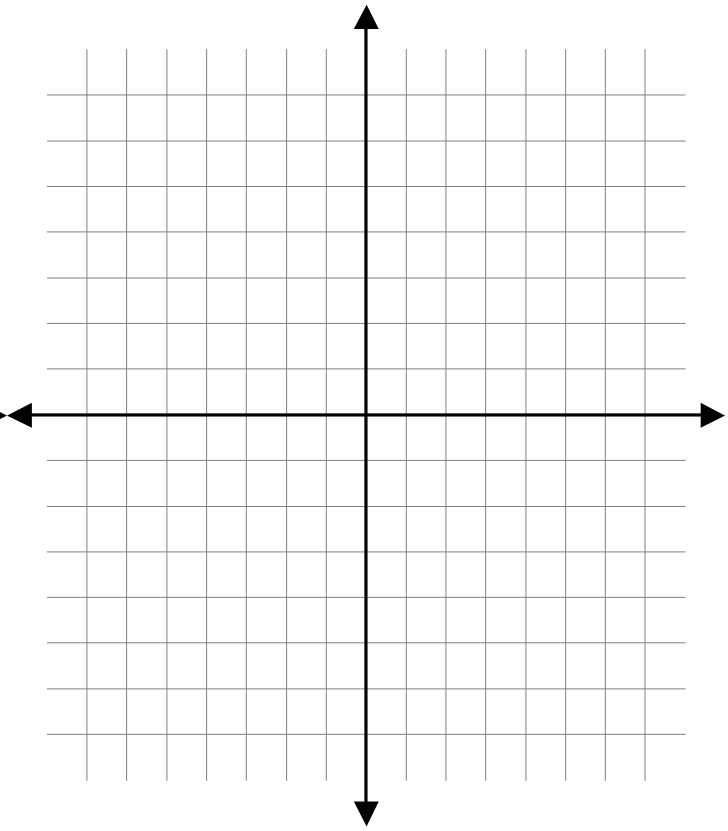
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

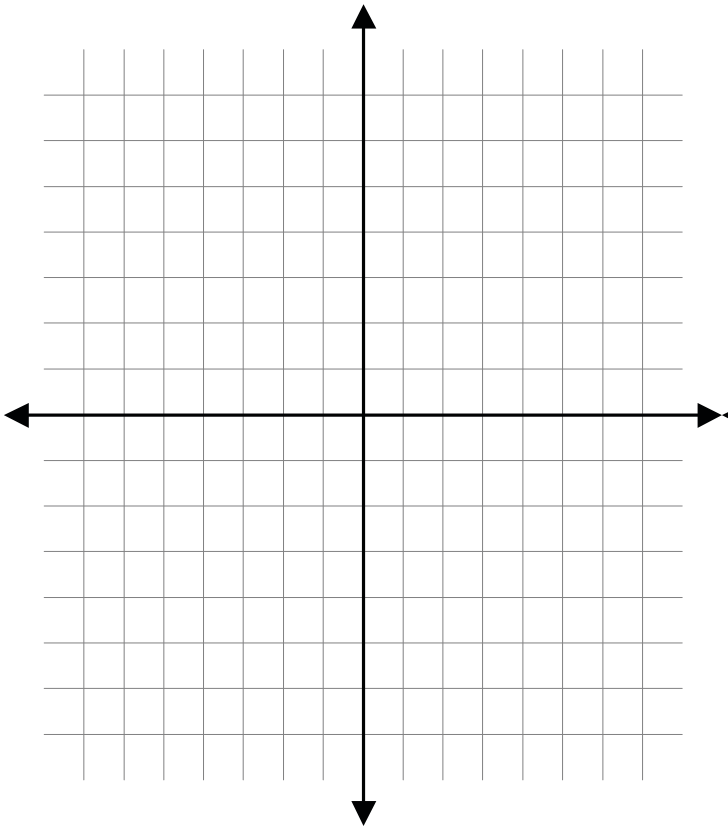
The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

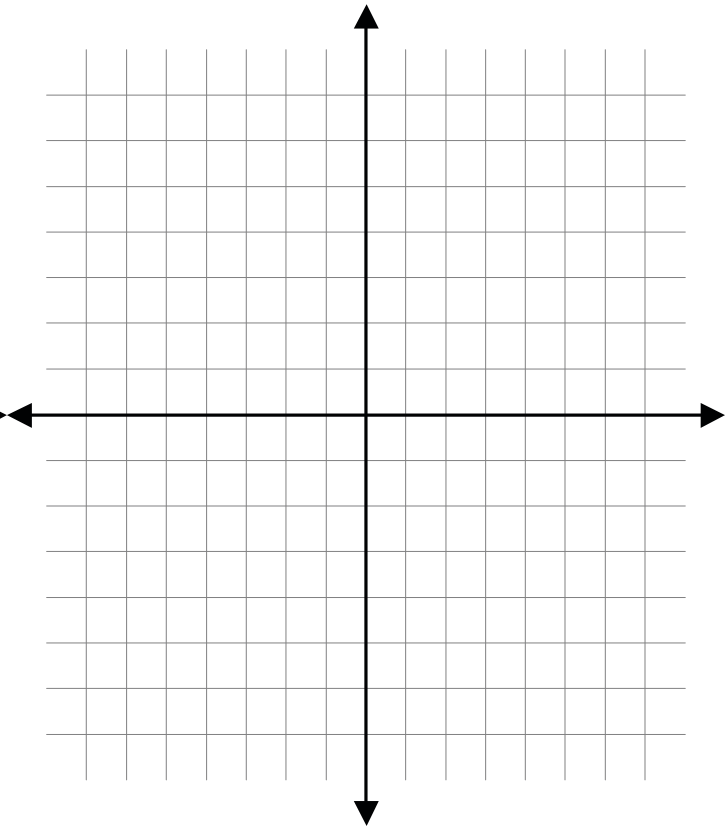
In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

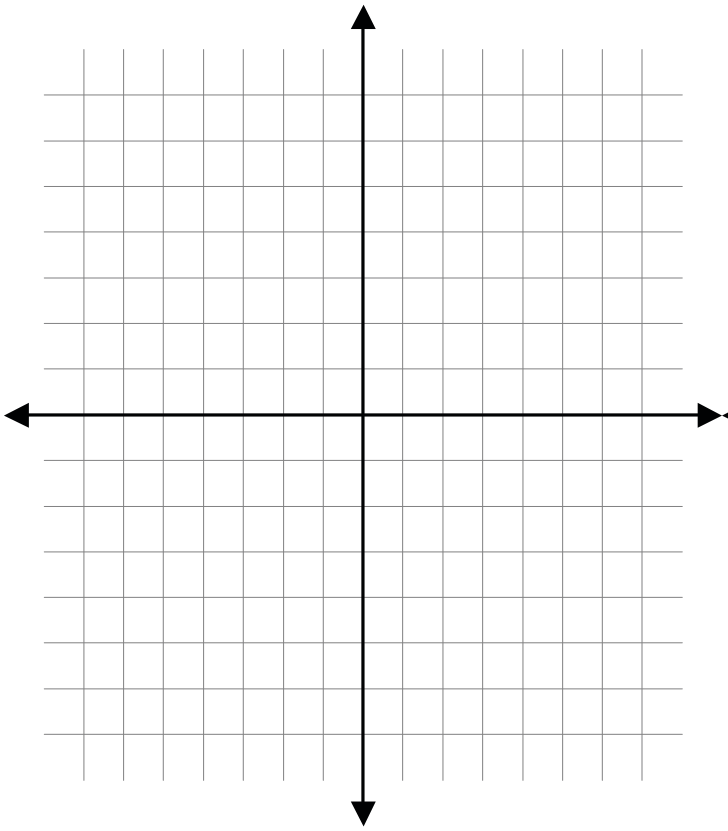
National Governors Association Center for Best Practices and the Council of Chief State School Officers (NGA Center and CCSSO). *Common Core State Standards for Mathematics.* Washington, D.C.: NGA Center and CCSSO, 2010. http://www.corestandards.org

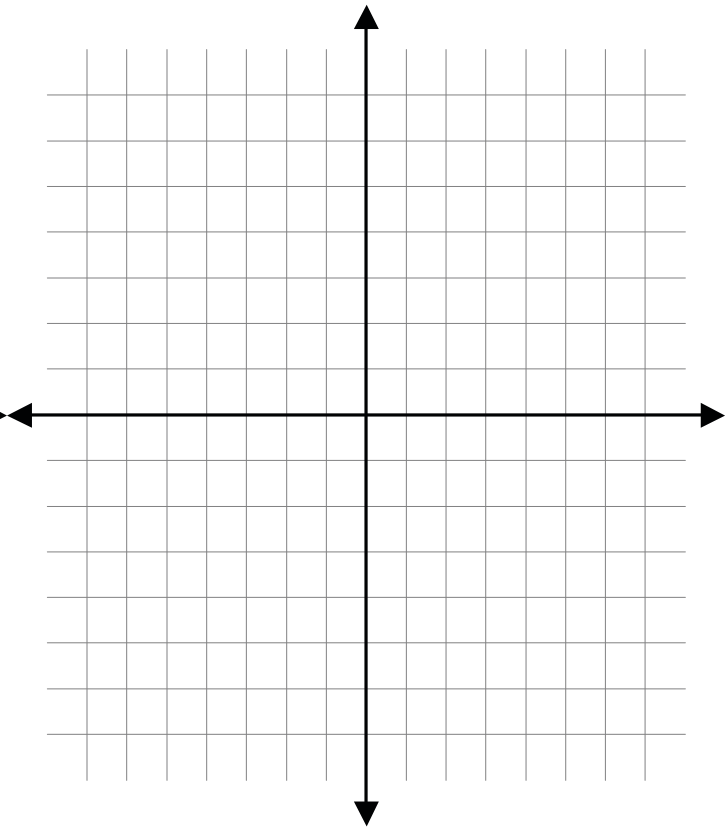


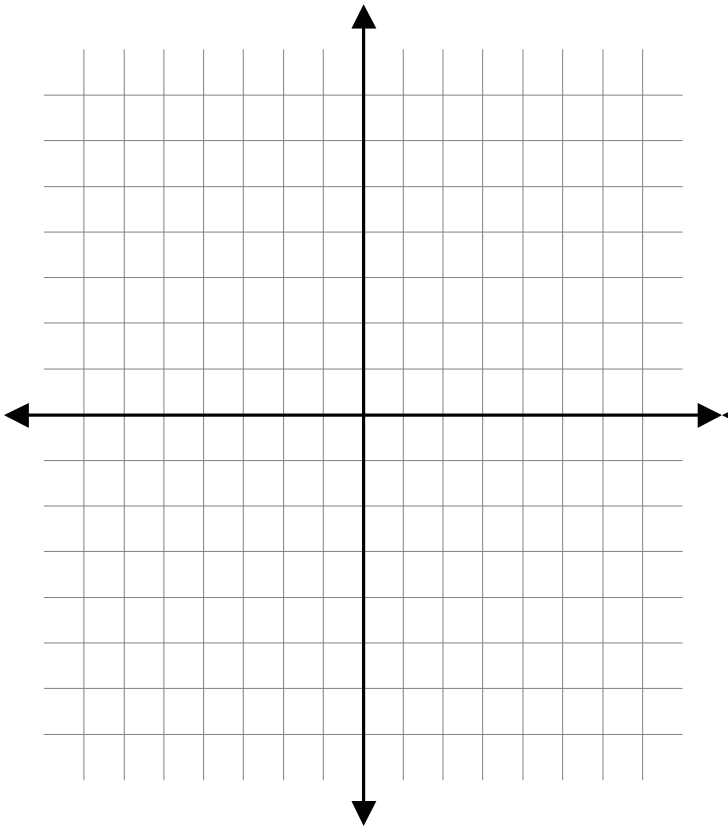


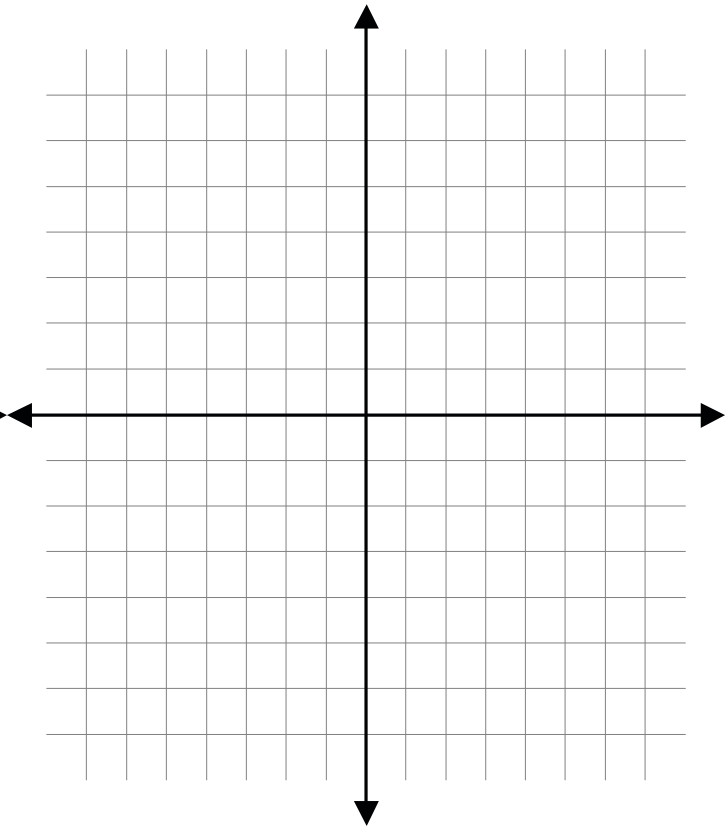










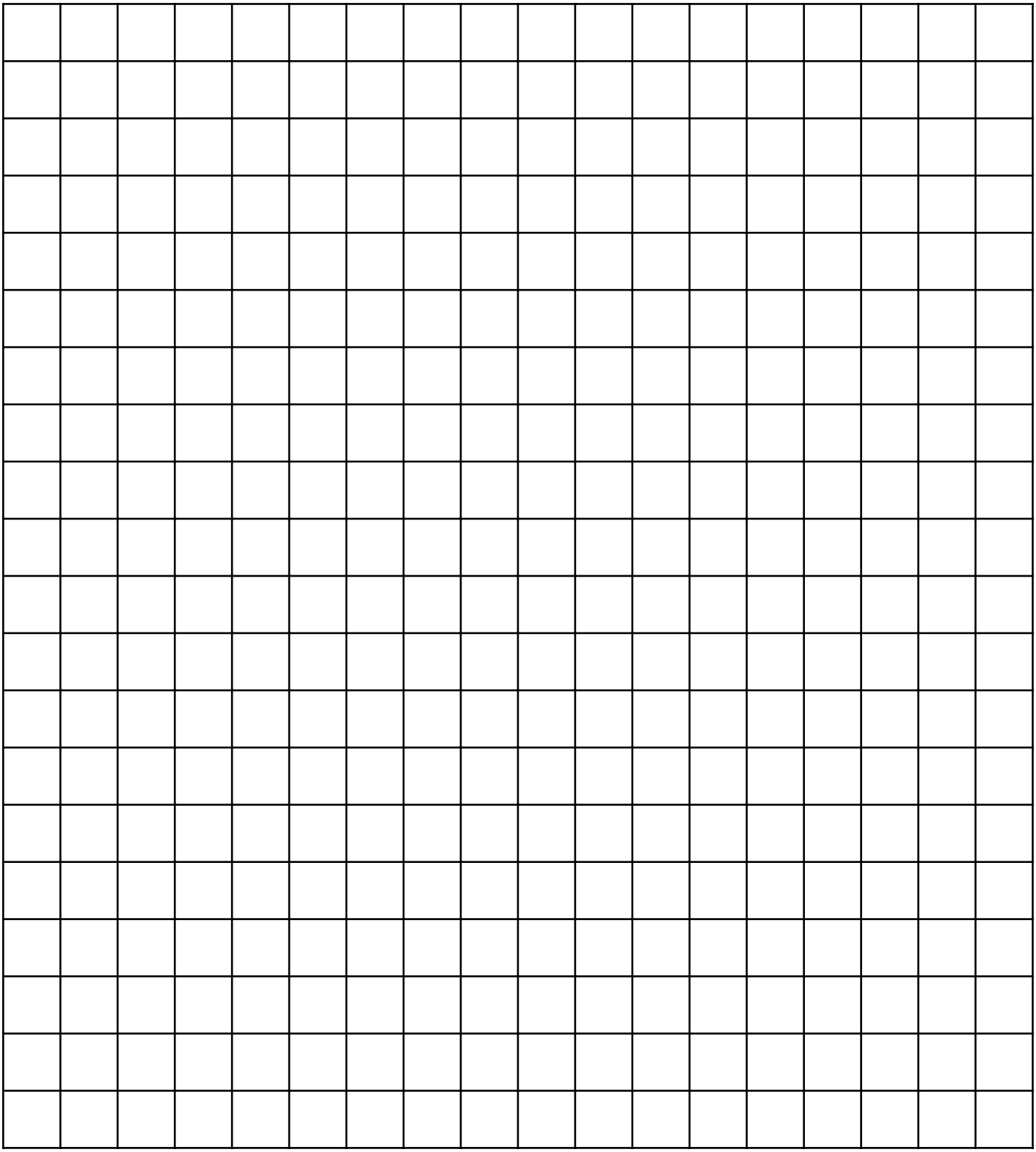


Graph Paper 1 cm

NAME _____

Graph Paper 1 cm

NAME _____



Graph Paper 1 cm

NAME _____

Graph Paper 1 cm

NAME _____

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NCTM® | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

This certificate is presented to

_____

in recognition of attendance and participation at the NCTM 2016 Interactive Institute on High School Mathematics—Engaging Students in Learning: Mathematical Practices

Atlanta, Georgia • July 14–16, 2016

Matthew Larson
President, NCTM

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

NCTM Interactive Institute on
High School Mathematics
July 14–16, 2016
Atlanta, Georgia

Name of Provider: National Council of Teachers of Mathematics

Educator's Name: _____

Description of Professional Development Activity: This is a two-and-a-half-day Institute sponsored by the National Council of Teachers of Mathematics. The focus of the Institute is on the CCSS Mathematical Practices, NCTM's Process Standards, and the eight effective teaching practices outlined in *Principles to Actions*. Participants will attend three task discussion groups based on the strand they selected at registration. Participants will select three out of twenty-one breakout workshops and will attend five keynote sessions.

Note: PD time earned should be the time actually spent in sessions and/or workshops.

Date	Session Type	Session Title	Presenter(s)/ Facilitator's Name(s)	Start/End Time	PD Time earned
TOTAL Professional Development Hours Accrued:					

I certify that the above named educator accrued the indicated number of Professional Development hours.

Matthew Larson
President, National Council of Teachers of Mathematics

Please check with your state education agency and local administration to determine if these Institute hours can be used for professional development credits.