

Denver | July 21-23, 2016
Connecting Number
and Operations in the Classroom

AN NCTM INTERACTIVE INSTITUTE FOR PRE-K-GRADE 5


PROGRAM WORKBOOK

## Welcome!

On behalf of NCTM and the Number and Operations Institute Advisory Group, we welcome you to the Interactive Institute for Pre-K-Grade 5, "Connecting Number and Operations in the Classroom." The learning experiences in this institute are designed to allow you to explore classroom activities and tasks to support students' development of a sense of number and to deepen your understanding of number-a mathematical foundation for all. You will hear from and have multiple opportunities to engage with recognized mathematics leaders during the next three days. Take full advantage of this institute: participate in every keynote session and all workshops designed specifically for your grade band, as well as network with colleagues from across the United States and beyond. Then, at the end of the day, meet up with friends or family and enjoy the Denver area.

We thank everyone in attendance and hope that you will find the institute useful and applicable in your implementation of the Common Core State Standards, as well as to your work in developing foundational understandings necessary for fluency and proficiency with number for each and every student.

## Number and Operations Institute Advisory Group



John SanGiovanni
NCTM Board of Directors Howard County Public School System Ellicott City, Maryland


Dorothy White
University of Georgia, Athens


## Save the Date!

PREMIER MATH EDUCATION EVENT

# NCTM ANNUAL MEETING \& EXPOSITION April 5-8 | San Antonio 

## Creating Communities and Cultivating Change

It's never too early to plan ahead for the leading math education event of the year. Network with thousands of your peers and fellow math education professionals to exchange ideas, engage with innovation in the field and discover new learning practices that will drive student success.

The latest teaching trends and topics will include:
Access and Equity: Teaching Mathematics with an Equity Stance
Registration will open
Building Conceptual and Procedural Understanding
Professionalism: Learning Together as Teachers
Nov. 2016
Teaching, Learning, and Curriculum: Best Practices for Engaging Students
in Productive Struggle
The " $M$ " in STEM/STEAM
Tools and Technology: Using Technology to
Effectively Teach and Learn Mathematics

The NCTM Annual Meeting \&
Exposition is ideal for:
PRE-K-12 TEACHERS
MATH TEACHER EDUCATORS
NEW AND PROSPECTIVE TEACHERS
MATH COACHES AND SPECIALISTS
MATH RESEARCHERS
SCHOOL AND DISTRICT ADMINISTRATORS


## One mentor can make a difference. 70,000 mentors can make a career.

As a member of the world's largest community of mathematics teachers, you are connected to a supportive and diverse network of peers in mathematics education.

Take full advantage of the cumulative knowledge of this extensive and supportive network. Member-exclusive resources include:

- Leading research, curriculum, and assessment tools
- Discounts on in-person professional development



## General Information and Program

Institute Information ..... 2
Schedule at a Glance ..... 3
Program Information ..... 4
Floor Plans ..... 9
2016 Institute Sponsors ..... 11
Standards
Principles to Actions:
Mathematics Teaching Practices ..... 13
NCTM Process Standards ..... 14
CCSSM Standards for Mathematical Practice ..... 15
Resources
Graph Paper ..... 19
Number Lines ..... 23
Certificate of Participation ..... 25
Workshop Handouts

The publications and programs of the National Council of Teachers of Mathematics present a variety of viewpoints. The content, affiliations, and views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of the Council. References to particular commercial products by a speaker should not be construed as an NCTM endorsement of said products. NCTM reserves the right to change speakers, change facilities, or modify program content.

Some speakers on this program have elected to print their e-mail addresses as a means for individual correspondence with conference attendees. Unsolicited commercial e-mail or unsolicited bulk e-mail, whether or not that e-mail is commercial in nature, is expressly prohibited. Any use of e-mail addresses beyond personal correspondence is not authorized by NCTM.

National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-1502; Telephone (703) 620-9840; Fax (703) 476-2970; E-mail nctm@nctm.org; Web www.nctm.org

## INSTITUTE INFORMATION

## Range of Activities

The following activities are available:
Keynote Sessions-Well-known leaders in math education will address crucial topics related to and supporting number. Speakers may use technology, share handouts, or incorporate audience participation.

Breakout Workshops-Math education practitioners will engage participants in hands-on activities focused on developing understanding of number. Participants will attend a series of four workshops within their grade band.

PD Strand-Participants in the PD (professional development) strand will attend their session as a group.

## Program Updates

Program updates, including speaker updates, will be available at the information desk.

## Materials Pickup \& Information Desk

Located in the Ballroom Prefunction area, the NCTM Information Desk is available to correct or replace badges and to offer general assistance. On-site registration for new attendees will not be available, and no payments will be collected at the meeting.

## Materials Pickup \& Information Desk Hours

| Wednesday, July 20 | 4:00 p.m.-7:00 p.m. |
| :--- | :--- |
| Thursday, July 21 | 8:00 a.m.-5:00 p.m. |
| Friday, July 22 | 8:00 a.m.-5:00 p.m. |

You must wear your badge to enter all presentations.
By registering for the NCTM Interactive Institute, participants grant NCTM the right to use, in promotional materials, their likeness or voice as recorded on, or transferred to, videotape, film, slides, audiotapes, or other media.

## For Your Child's Safety

Because of the size and nature of the Number and Operations Institute, this event is not an appropriate setting for children younger than 16 years. The hotel concierge can recommend activities for children while you are attending the institute. We appreciate your understanding and cooperation.

## NCTM Book Display

The NCTM Bookstore, located in the Ballroom Prefunction area, will feature NCTM publications with a focus on pre-Kgrade 5 mathematics education. NCTM publications are for display purposes only; however, attendees can purchase books on-site and save 25 percent off the list price in addition to receiving free shipping. Books can be purchased after the meeting by using the special conference discount code NUM16; shipping fees will be an additional charge. Free shipping and the 25 percent discount apply only to on-site purchases from July 20 to July 23. This discount applies to all NCTM publications and is not limited to those on display. Offer expires August 31, 2016.

## Book Display Hours

$$
\begin{array}{ll}
\text { Wednesday, July 20 } & \text { 8:00 a.m.-7:00 p.m. } \\
\text { Thursday, July 21 } & \text { 8:00 a.m.-5:00 p.m. } \\
\text { Friday, July 22 } & \text { 8:00 a.m.-5:00 p.m. } \\
\text { Saturday, July 23 } & \text { 8:00 a.m.-1:00 p.m. }
\end{array}
$$

## Lost-and-Found

Attendees who have lost or found items may retrieve or turn them in at the information desk located at the registration counter in the Ballroom Prefunction area. After the institute, all lost-and-found items will be turned over to hotel security.

## Boxed Lunches

The networking lunch will take place in the Colorado Ballroom CD. You will need to show your badge to pick up a lunch.

## Wireless Internet Access

You will be able to access the Internet through a wireless connection in all meeting rooms by using the log-in information below:

## Network name: NCTM Institute <br> Password: NCTM16

## Social Media

## Twitter

Find us on Twitter at \#NCTMINST and, of course, at \#NCTM.

## Facebook

http://www.facebook.com/TeachersofMathematics

## Accessing Institute Content

Handouts and PDFs of the presentations will be available online within one to two weeks after the close of the institute at: www.nctm.org/num16.

Wednesday, July 20
4:00 p.m.-7:00 p.m. Materials Pickup

Thursday, July 21
8:00 a.m.-5:00 p.m. Materials Pickup
8:30 a.m.-11:30 a.m. PD Strand Workshop (additional registration required)
12:30 p.m.-1:45 p.m. Opening Session: Jennifer Bay-Williams
2:00 p.m.-4:45 p.m. Breakout Workshop 1

Friday, July 22
8:00 a.m.-9:15 a.m. Keynote Sessions (Intervention/Math for ALL)
9:30 a.m.-12:00 p.m. Breakout Workshop 2
12:00 p.m.-1:00 p.m. Boxed Lunches/Networking
1:00 p.m.-3:30 p.m. Breakout Workshop 3
3:45 p.m.-4:45 p.m. Assessment Keynote

Saturday, July 23
8:00 a.m.-9:15 a.m. Keynote Sessions (Intervention/Math for ALL)
9:30 a.m.- 12:00 p.m. Breakout Workshop 4
12:00 p.m.-1:00 p.m. Boxed Lunches/Networking
1:00 p.m.-2:15 p.m. Closing Keynote: Max Ray-Riek
2:30 p.m.-3:30 p.m. Closing Reflection Session

8:00 a.m.-5:00 p.m.
Materials PickupBallroom Prefunction area

## 8:30 a.m.-11:30 a.m.

## PD Strand Workshop (additional registration required)

This interactive workshop will engage participants in the Standards for Mathematical Practice and develop teacher content and pedagogical knowledge associated with important number-related domains and standards. Activities will encompass the content domains and standards of the Common Core State Standards. We will focus on number and operations in base ten and fractions, with an emphasis on developing number sense. Leave with plans for developing mathematics learning communities at the school and district level.

## Delise Andrews

Lincoln Public Schools, Lincoln, Nebraska

## Colorado Ballroom A

12:30 p.m.-1:45 p.m.

## Opening SessionRedefining Help: How to Support Conceptual Understanding and Procedural Fluency

What does it mean to help a child understand place value? Know basic facts from memory? Understand and select efficient ways to multiply fractions? We will look to the research and see how we can best support conceptual understanding and procedural fluency-in other words, explore what we really need to do to help each student become mathematically proficient.
Jennifer Bay-Williams
Board of Directors, NCTM
University of Louisville, Kentucky

## Colorado Ballroom CD

All Attendees

2:00 p.m.-4:45 p.m.

## Breakout Workshop 1

## Pre-K-Grade 2

Green Star Group-Big Thompson
Red Star Group-Platte River
Black Star Group-Telluride A
Blue Star Group-Telluride B
Grades 3-5
Blue Circle Group-Durango
Red Circle Group-Steamboat
Green Circle Group-Aspen
Yellow Circle Group-Vail

## 8:00 a.m.-9:15 a.m.

## Keynote Sessions

## Supporting Students Who Struggle: Number and Algebraic Reasoning

Mathematics in the early grades forms a significant foundation for students. When students struggle, they develop gaps in their understanding. This session focuses on common misconceptions and errors that young students have. Mathematical tasks that can bridge these gaps and associated instructional strategies will be shared.

## Barbara Dougherty

University of Missouri-Columbia

## Colorado Ballroom CD

Star Groups (Pre-K-Grade 2)

## Facilitating Access and Equity in Pre-K-5 Mathematics Classrooms

Every student and teacher brings a wealth of strengths to the mathematics classroom. Teachers can build on these strengths by embracing and infusing their students' and their own cultures and interests into their mathematics teaching and classroom practices. Learn how to establish equitable learning environments and enhance curriculum, instruction, and assessment to support students' mathematics learning.

## Kathryn Chval

University of Missouri-Columbia

## Colorado Ballroom A

Circle Groups (Grades 3-5)
9:30 a.m.-12:00 p.m.

## Breakout Workshop 2

## Pre-K-Grade 2

Green Star Group-Big Thompson Red Star Group-Platte River Black Star Group-Telluride A Blue Star Group-Telluride B

Grades 3-5
Blue Circle Group-Durango
Red Circle Group-Steamboat
Green Circle Group-Aspen
Yellow Circle Group-Vail

12:00 p.m.-1:00 p.m.
Networking Lunch

## Colorado Ballroom CD

1:00 p.m.-3:30 p.m.

## Breakout Workshop 3

## Pre-K-Grade 2

Green Star Group-Telluride A
Red Star Group-Telluride B
Black Star Group-Big Thompson
Blue Star Group-Platte River
Grades 3-5
Blue Circle Group-Aspen
Red Circle Group-Vail
Green Circle Group-Durango
Yellow Circle Group-Steamboat

3:45 p.m.-4:45 p.m.

## Keynote Session

## Formative Assessment Strategies Support the Mathematical Practices

This session will examine formative assessment strategies to help teachers seamlessly incorporate the mathematical practices in their instruction. We will examine how the algorithms that students use in addition and multiplication align with the algebraic structure of mathematics.

Anne Collins<br>Lesley University, Cambridge, Massachusetts

## Colorado Ballroom CD

All Attendees

## 8:00 a.m.-9:15 a.m.

## Keynote Sessions

## Facilitating Access and Equity in Pre-K-5 Mathematics Classrooms

Every student and teacher brings a wealth of strengths to the mathematics classroom. Teachers can build on these strengths by embracing and infusing their students' and their own cultures and interests into their mathematics teaching and classroom practices. Learn how to establish equitable learning environments and enhance curriculum, instruction, and assessment to support students' mathematics learning.

## Kathryn Chval

University of Missouri-Columbia

## Colorado Ballroom A

Star Groups (Pre-K-Grade 2)

## Supporting Students Who Struggle: Number and Algebraic Reasoning

Mathematics in the early grades forms a significant foundation for students. When students struggle, they develop gaps in their understanding. This session focuses on common misconceptions and errors that young students have. Mathematical tasks that can bridge these gaps and associated instructional strategies will be shared.

## Barbara Dougherty

University of Missouri-Columbia
Colorado Ballroom CD
Circle Groups (Grades 3-5)

> 9:30 a.m.-12:00 p.m.

## Breakout Workshop 4

## Pre-K-Grade 2

Green Star Group-Telluride A Red Star Group-Telluride B Black Star Group-Big Thompson
Blue Star Group-Platte River
Grades 3-5
Blue Circle Group-Aspen
Red Circle Group-Vail
Green Circle Group-Durango
Yellow Circle Group-Steamboat

12:00 p.m.-1:00 p.m.
Networking Lunch
Colorado Ballroom CD

1:00 p.m.-2:15 p.m.

## Keynote Session

## The Thinking Teachers Do So That Students Do All the Thinking

Throughout the Institute, teachers have been in the role of learners, encountering new and powerful strategies, representations, and conceptual understandings. When we go back to our classrooms, we are in the role of teachers again, and our learners need to be the ones doing the hard work of building their understanding and trying out new strategies and representations. How do we plan for instruction, structure math discussions, and take the role of facilitator so that our students are the ones making sense and building connections? In this session, we will practice specific teaching practices (selecting student work to share, sequencing share-outs, and planning prompts and questions to support sense-making and connections). At the end of the session, participants will work with colleagues to produce a proposal for building time into each week to plan instruction around student thinking. Participants will also reflect on the Institute as a whole and make commitments to implement what they are taking away.

## Max Ray-Riek

The Math Forum at NCTM, Reston, Virginia
Colorado Ballroom CD
All Attendees

2:30 p.m.-3:30 p.m.
Closing Reflection Session
Advisory Group
Colorado Ballroom CD
All Attendees

## BREAKOUT WORKSHOP DESCRIPTIONS

Participants will attend four workshops within their grade band group. Workshop leaders will engage participants in hands-on activities focused on developing a sense of number:

## Pre-K-Grade 2 Strand

## Counting and Cardinality

The goal of this session is to explore tasks and instructional strategies to support students' development of naming, counting, and comparing numbers. Guided by the mathematical practices identified in CCSSM, the session will describe questioning techniques to promote students' conceptual knowledge and procedural fluency with numbers. During this session, participants will explore counting principles, examine the differences between rote and rational counters, develop a core understanding of the relationships between numbers and quantities, and examine common misconceptions related to counting and cardinality.

## CCSSM: K.CC, K.OA

## Number in Context

In this session, participants will explore numbers in context, emphasizing iterative units for measures such as length, time, and money. The CCSSM Standards for Mathematical Practice will ground this exploration as participants learn strategies to engage students in modeling, reasoning, and using appropriate tools to order, measure, and compare objects of varying lengths; read and write time for analog and digital clocks; and count money. The session will also focus on having students learn to solve and pose mathematical problems in contexts.
CCSSM: 1.MD, 2.MD

## Facilitators: Counting and Cardinality and Number in Context

## Dorothy White

University of Georgia, Athens

## Latrenda Knighten

Former Member, Board of Directors, NCTM
East Baton Rouge Parish, Louisiana

## Addition and Subtraction

The focus of this session will be on addition and subtraction of whole numbers and mental strategies to develop students' computational proficiency in adding and subtracting numbers. Through problem-based tasks, teachers will explore addition and subtraction problem types, the commutative and associative properties of addition, and the use of mathematical symbols to represent addition and subtraction problems.
CCSSM: K.NBT, 1.NBT, K.OA, 1.OA, 2.OA

## Base 10 and Place Value

The goals of this session are to understand the characteristics of the base-ten numeration system and children's place value development from counting by ones to grouping in tens and ones. During the session, participants will examine the importance of the positions of the digits in determining the value of two- and three-digit numbers, and the role of composing and decomposing numbers in laying the foundations for students' mathematical readiness to compute. The session will also include ideas for supporting students' computational knowledge through skip-counting by $5 \mathrm{~s}, 10 \mathrm{~s}, 100 \mathrm{~s}$.
CCSSM: K.NBT, 1.NBT, 2.NBT

Facilitators: Addition and Subtraction and Base 10 and Place Value<br>Jennifer Ward<br>University of South Florida<br>Courtney Baker<br>George Mason University, Fairfax, Virginia

## BREAKOUT WORKSHOP DESCRIPTIONS

## Grades 3-5 Strand

## Multiplication and Division (1)

This session will focus on the conceptual development of students' understanding of multiplication and division, with a particular emphasis on developing understandings related to a variety of representations of these operations. These include representations of equal-sized groups, rectangular arrays, and area models, as well as representations with number lines, including open number lines; expanded forms; and equations. Participants will also discuss strategies for acquiring automaticity with the related multiplication and division facts, and apply the commutative, associative, and distributive properties as strategies for multiplication and division. A variety of problem-based contexts will be used to engage participants, connecting directly to the CCSSM Standards for Mathematical Practice. The mathematical practices and their use in planning, teaching, and formative assessment will be an important consideration for participants as they discuss how they will engage students in grades 3-5 in foundational, number sense-building experiences related to these important operations.

## CCSSM: 3.OA.1, 2, 3, 4, 5, 6, 7; 4.OA.1, 2

## Multiplication and Division (2)

This session will extend the previous session related to multiplication and division and focus on the use of strategies based on place value, the properties of operations, and varied representations that lead to access to and fluency with a standard algorithm for multiplication and division. Number sense topics that will receive emphasis are the importance of place value in mental mathematics and estimation strategies in multiplying and dividing whole numbers. Further, participants will consider multiplication as scaling when comparing a product to one factor on the basis of the size of the other factor. A variety of problem-based contexts will be used to engage participants, with a particular emphasis on the CCSSM Standards for Mathematical Practice. The mathematical practices and their use in planning, teaching, and formative assessment will be important considerations for participants as they discuss how they will engage students in grades 3-5 in these important operations.
CCSSM: 4.NBT.1, 5, 6; 5.NBT.2, 5, 6; 6.NS. 2

## Facilitators: Multiplication and Division

## John SanGiovanni

Board of Directors, NCTM
Howard County Public Schools, Maryland
Jennifer Suh
George Mason University, Fairfax, Virginia

## Fractions as Numbers (1)

The focus of this session will be on fractions as numbers, emphasizing magnitude and equivalence. Participants will engage in problem-based tasks that involve a variety of representations, sharing and proportionality, comparing and ordering equivalent fractions and decimals, and reasoning with ratios and rates. The CCSSM Standards for Mathematical Practice will be an important consideration for participants as they approach the content focus of this session and discuss how they will engage students in using the mathematical practices to develop fraction sense.
CCSSM: 2.G.3; 3.G.2; 4.NF.1, 2, 5,6, 7; $6 . R P .1$

## Fraction Operations (2)

This session will emphasize understandings critical to operations with fractions and decimals, with particular attention to building fractions from unit fractions, joining and separating parts of the same whole, applying and extending prior knowledge of multiplication and division of whole numbers to fractions and decimals. Participants will engage in problem-based tasks involving a variety of representations for fractions, decimals, and common percents. The CCSSM Standards for Mathematical Practice will be an important consideration for participants as they approach the content focus of this session and discuss how they will engage their students in using the mathematical practices to develop conceptual understanding and computational proficiency with fractions and decimals.
CCSSM: 4.NF.3, 4; 5.NF.3, 4, 5, 6, 7; 6.NS.1, 3; 6.RP.3

## Facilitators: Fractions as Numbers and as Operations

## Angela Waltrup

Frederick County Public Schools, Maryland
Julie McNamara
TeachingWorks, University of Michigan


Banquet level


## 2016 INSTITUTE SPONSORS

We thank our sponsor for generously supporting NCTM by offering products and services to enhance your Institute experience.

## In-Kind Sponsor

Hands-on learning materials provided by:

NOTES

## Principles to Actions

## Research-Based Teaching Strategies to Strengthen Student Learning

With more than 30,000 copies in print, the landmark publication Principles to Actions: Ensuring Mathematical Success for All is a vital tool for teachers, mathematics coaches, administrators, parents, and policymakers.

Make Principles to Actions your go-to source and enhance your teaching expertise with-

- research-based descriptions of eight essential Mathematics Teaching Practices;
- a review of the conditions, structures, and policies that must support the Teaching Practices;
- an understanding of obstacles, unproductive and productive beliefs, and key actions that must be acknowledged and addressed by all stakeholders; and
- tools to engage students in mathematical thinking, reasoning, and sense making to significantly strengthen teaching and learning.

NCTM Stock No. 14861
ISBN: 978-0-87353-774-2
List: \$28.95 | Member: \$23.16

Visit nctm.org/PrinciplestoActions to learn more and purchase your copy today.

To Order: Call 800.235.7566 | Online: www.nctm.org/catalog

## Build Your Professional Resource Library with New Books from NCTM

 SAVE 25\%! Use code Summer16 when placing order. Offer expires 10/31/2016.

## Developing Literate Mathematicians:

 A Guide for Integrating Language and Literacy Instruction into Secondary Mathematics BY WENDY WARD HOFFERHow can we integrate literacy instruction authentically into mathematics content to support mathematical understanding? Busy secondary mathematics teachers who seek to respond to the needs of their students and the demands of the Common Core State Standards will welcome this book, which offers lively classroom examples, usable research, and specific ideas and resources. Enrich your students' understanding of mathematics by attending to reading, vocabulary, discourse, and writing through a workshop model.
©2015, Stock \#14522


More Lessons Learned from Research, Volume 2: Helping All Students Understand Important Mathematics
EDITED BY EDWARD A. SILVER AND PATRICIA ANN KENNEY
Applying research to strengthen teaching practice and ensure students' success in mathematics

More than seventy years of research point to the importance of teaching mathematics for understanding. Successful students actively construct understanding rather than passively receive knowledge. Implications of this fundamental lesson from research are explored in different ways through twenty-four chapters presented in this book. Chapters cover investigations of a wide range of topics, approaches, and settings, and mathematics teachers at all levels will find examples of research that are relevant to the challenges they face.
©2016, Stock \#14439


DON'T MISS!
More Lessons Learned from Research, Volume 1
EDITED BY EDWARD A. SILVER
Helps to link classroom teachers to all that original research has to offer
©2015, Stock \#14117

All books available as eroork

## Putting Essential <br> Understanding of Geometry and Measurement into Practice in Grades 3-5

BY KATHRYN CHVAL, JOHN LANNIN, AND DUSTY JONES KATHRYN CHVAL, VOLUME EDITOR

bARBARA J. DOUGHERTY, SERIES EDITOR
Do your students have "concept images" that limit their ideas of shapes to specific examples, oriented in particular ways? Do they confuse the size of an angle with the length of the rays in a drawing of an angle? This book demonstrates how to use multifaceted knowledge to address the big ideas and essential understandings that students must develop for success with geometry and measurement-not only in their current work, but also in higher-level mathematics and a myriad of real-world contexts. © 2016, Stock \#14543

## Problem Solving in All Seasons,

 Grades 3-5by Kim markworth, Jenni mccool, AND JENNIFER KOSIAK
Holidays and seasonal activities offer perfect backdrops for mathematical tasks that can be related to other topics and themes in the classroom. This book delivers thirty-six appealing, real-world mathematical tasks, arranged in grade-level order, to engage young learners in problems tied to the Common Core and designed to allow children to participate in the Common Core Standards for Mathematical Practice. Each task includes a complete implementation guide, and handouts and ancillary materials can be accessed online. This is your all-inone practical handbook for problem solving in the primary years.
©2016, Stock \#14809


DON'T MISS!
Problem Solving in All Seasons, Pre-K-Grade 2
BY KIM MARKWORTH, JENNI MCCOOL, AND JENNIFER KOSIAK ©2015, Stock \#14808

## ADDITIONAL NEW TITLES



## Developing

Mathematical Ideas: Building a System of Tens, Casebook and Facilitators Guide
©2016 Stock \#15032 and \#15053

On the Money: Math Activities to Build Financial Literacy in High School
©2016, Stock \#14589


Annual Perspectives in Mathematics Education 2016
©2016, Stock \#15198
Discovering Lessons for the Common Core Standards in Grades 9-12
©2016, Stock \#14588


NATIONAL COUNCIL OF
teachers of mathematics

Visit nctm.org/store for tables of contents and sample pages.
For more information or to place an order,
call (800) 235-7566 or visit nctm.org/store.

## MATHEMATICS TEACHING PRACTICES

## Principles to Actions: Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Principles to Actions: Ensuring Mathematical Success for All is an official position of the National Council of Teachers of Mathematics as approved by the NCTM Board of Directors, February 2014.

National Council of Teachers of Mathematics (NCTM). Principles to Actions: Ensuring Mathematical Success for All. Reston, Va.: NCTM, 2014

## NCTM PROCESS STANDARDS

## Problem Solving

Instructional programs from prekindergarten through grade 12 should enable all students to-

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems; and
- monitor and reflect on the process of mathematical problem solving.


## Reasoning and Proof

Instructional programs from prekindergarten through grade 12 should enable all students to-

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs; and
- select and use various types of reasoning and methods of proof.


## Communication

Instructional programs from prekindergarten through grade 12 should enable all students to-

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.


## Connections

Instructional programs from prekindergarten through grade 12 should enable all students to-

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.


## Representation

Instructional programs from prekindergarten through grade 12 should enable all students to-

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.


## Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

## 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3 Construct viable arguments and critique the reasoning of others.

 Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## CCSSM STANDARDS FOR MATHEMATICAL PRACTICE

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5+7 \times 3$, in preparation for learning about the distributive property. In the expression $x^{2}+9 x+14$, older students can see the 14 as $2 \times 7$ and the 9 as $2+7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3(x-y)^{2}$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

## 8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through ( 1,2 ) with slope 3 , middle school students might abstract the equation $(y-2) /(x-1)=3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1),(x-1)\left(x^{2}+x+1\right)$, and $(x-1)\left(x^{3}+x^{2}+x+1\right)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO). Common Core State Standards for Mathematics. Washington, D.C.: NGA Center and CCSSO, 2010.
http://www.corestandards.org

NOTES

NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

IS ALL AROUND US MATH IS ALL AROUND US

## Take Your Professional Development to a New Level of Excellence

## Developing Mathematical Ideas Professional Development Series <br> Number and Operations, Part 1: Building a System of Tens

Calculating with Whole Numbers and Decimals
BY DEBORAH SCHIFTER, VIRGINIA BASTABLE, AND SUSAN JO RUSSELL

Building a System of Tens is the first module in the seven-part Developing Mathematical Ideas Series. The complete module consists of a casebook for participants and an online facilitator's package that contains everything necessary to prepare for and lead the seminar, including access to the casebook content and classroom videos.

The DMI program engages participants in collaborative learning where they make sense of the content, recognize where and how their grade content is situated in the trajectory of learning from K-grade 8, build connections among different concepts, and analyze student thinking from a mathematical perspective. Teachers learn how to orient their instruction to specific mathematical goals and to develop a mathematics pedagogy in which student understanding is central.

The Building a System of Tens Casebook was designed as a key component of the modules. The thirty cases, written by teachers describing real situations and actual student thinking in their classrooms, provide the basis of each session's investigation into specific mathematical concepts and teaching strategies. Guided by the facilitator, participants explore the base-ten number structure, consider how that structure is exploited in multidigit computational procedures, and examine how basic concepts of whole numbers reappear when working with decimals.

The online Building a System of Tens Facilitator's Package includes an introduction, preseminar preparation for the facilitator, and all the readings, materials, and videos needed to lead the eight sessions of the seminar.

## Save 25\%! Use code Summer2016 when placing order. Offer expires 10/31/2016.

To Order: Call 800.235.7566 | Online: www.nctm.org/dmi

## NCTM Regional oㅡㅇ Conferences \& Expositions PHOENXI OCTOBER 26-28 PHILADELPHIA | OCT 31-NOV 2

PREMIER MATH EDUCATION EVENTS

## Innovate. Collaborate. Learn.

NCTM Regional Conference \& Expositions are an opportunity to share knowledge and learn with leaders in the field of mathematics education. Gain new strategies to unleash the mathematical mind of every student when you take advantage of superior mathematics resources right on your doorstep.

What you'll get:

- Innovative ideas you can immediately put to use.
- Updates on classroom best practices from recognized innovators.
- In-depth discussion into the latest education resources.
- Knowledge-sharing with like-minded peers.
- Interaction with the latest tools and products in the robust exhibit hall.

- Pk-12 classroom teachers
- Math coaches
- Administrators
- Math teacher educator
- Preservice teachers
- Math specialists

Join NCTM in either Phoenix or Philadelphia and discover the tools that will help you promote the mathematical habits of mind that will lead your students to college and career success.

Learn more at
nctm.org/regionals and follow us on






This certificate is presented to
in recognition of attendance and participation at the
NCTM 2016 Interactive Institute for Pre-K-Grade 5 Mathematics:
Connecting Number and Operations in the Classroom
Denerer, Colorado. July 21-23, 2016
(
NCTM

NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Connecting Number and Operations in the Classroom: An NCTM Interactive Institute for Pre-K-Grade 5 July 21-23, 2016
Denver, Colorado

Name of Provider: National Council of Teachers of Mathematics

## Educator's Name:

$\qquad$

## Description of Professional Development Activity:

This is a two-and-a-half-day institute sponsored by the National Council of Teachers of Mathematics. The focus of the institute is on number and operations. Participants attend four breakout workshops based on the grade band selected at registration. Participants also attend five keynote sessions.

Note: PD time earned should be the time actually spent in sessions and/or workshops.

| Date | Session <br> Type | Session Title | Presenter(s)/ <br> Facilitator's <br> Name(s) | Start/End <br> Time | PD Time <br> earned |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

I certify that the above-named educator accrued the indicated number of Professional Development hours.
Matthew Larson
President, National Council of Teachers of Mathematics
Please check with your state education agency and local administration to determine if these Institute hours can be used for professional development credits.


