

Atlanta | July 11–13, 2016

Engaging Students *in* Learning: Mathematical Practices

AN NCTM INTERACTIVE INSTITUTE FOR K–GRADE 8



PROGRAM WORKBOOK

NCTM
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

nctm.org

Welcome!

On behalf of the NCTM K–8 Institute Advisory Group, we welcome you to this interactive institute focusing on mathematical practices and teaching practices. This institute is one component of NCTM's ongoing initiative to help achieve the vision of reasoning and sense making as a part of the mathematics classroom everyday. This professional learning experience has been designed to help you engage students in learning mathematics by examining the Standards for Mathematical Practice, the NCTM Process Standards, and the eight effective teaching practices explained in NCTM's *Principles to Actions*, with guidance from recognized mathematics leaders as well as through thoughtful reflection and discussion with your peers.

The overarching goal of this institute is to explore classroom activities and examine instructional practices that develop powerful mathematical practices in all students. The two and a half days of the institute and the networks that you develop here are part of achieving that goal. We encourage you to take full advantage of this institute: participate in all plenary and breakout sessions, actively engage in deep dive sessions, and network with colleagues from throughout the United States and beyond. Then, at the end of the day, meet up with friends or family and enjoy the Atlanta area.

We wish to thank the staff at NCTM for helping us with the planning and logistics for the institute, marketing, registration, and on-site work. We also thank all the presenters for agreeing to participate and share their expertise, views, and insights. Finally, we thank everyone in attendance, and we hope that you will find the Institute helpful as you work to make reasoning and sense making through mathematical practices and processes a daily classroom experience for all students.

K–8 Institute Advisory Group

Kyndall Brown
University of California,
Los Angeles

Grace Kelemanik
Boston Teacher Residency Program,
Massachusetts

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National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-1502;
Telephone (703) 620-9840; Fax (703) 476-2970; E-mail nctm@nctm.org; Web www.nctm.org

Range of Activities

All presentations are open to all Institute participants. Admission is on a first-come, first-served basis. Reserving spaces in line or saving seats is not permitted. The following activities are available:

Keynote Sessions (60–75 minutes)—Well-known leaders in math education will address crucial topics related to and supporting the mathematical practices and process standards.

Breakout Workshops (90 minutes)—Math education practitioners will engage participants in hands-on activities and strategies for implementing these activities in the classroom. Rooms are set up with round tables for hands-on work. Choose from seven different workshops during each breakout workshop time slot.

Deep Dive Sessions (90–120 minutes)—Participants will engage in guided activities and facilitated discussions that address the CCSSM mathematical practices. Discussions on the mathematical practices will be rooted in a content strand. Attend deep dive sessions according to the content strands you selected when registering.

Program Updates

Program updates, including speaker updates, will be available at the Information Desk.

Materials Pickup & Information Desk

Located in the Atrium, the NCTM Information Desk is available to correct or replace badges and to offer general assistance. On-site registration for new attendees will not be available, and no payments will be collected at the meeting.

Materials Pickup & Information Desk Hours

Sunday, July 10:	4:00 p.m.–7:00 p.m.
Monday, July 11:	8:00 a.m.–4:30 p.m.
Tuesday, July 12:	8:00 a.m.–4:30 p.m.

You must wear your badge to enter all presentations.

By registering for the NCTM Interactive Institute, participants grant NCTM the right to use, in promotional materials, their likeness or voice as recorded on, or transferred to, videotape, film, slides, audiotapes, or other media.

For Your Child's Safety

Because of the size and nature of the 2016 K–8 Institute, this event is not an appropriate setting for children under 16 years of age. Your hotel concierge can recommend activities for children while you are attending the Institute. We appreciate your understanding and cooperation.

NCTM Book Display

The NCTM Bookstore, located in the Atrium, will feature NCTM publications with a focus on K–8 mathematics education. NCTM publications are for display purposes only; however, attendees can purchase books on-site and save 25 percent off the list price in addition to receiving free shipping. Books can be purchased after the meeting by using the special conference discount code K-8PRAC16; shipping fees will be an additional charge. Free shipping and the 25 percent discount apply only to on-site purchases from July 10 to July 13. This discount applies to all NCTM publications and is not limited to those on display. Offer expires August 31, 2016.

Book Display Hours

Sunday, July 10:	4:00 p.m.–7:00 p.m.
Monday, July 11:	8:00 a.m.–4:30 p.m.
Tuesday, July 12:	8:00 a.m.–4:30 p.m.
Wednesday, July 13:	8:00 a.m.–1:00 p.m.

Lost-and-Found

Attendees who have lost or found items may retrieve or turn them in at the Information Desk located in the Atrium. After the institute, all lost-and-found items will be turned over to hotel security.

Boxed Lunches

The networking lunch will take place in the Grand Ballroom. You will need to show your badge to pick up a lunch.

Wireless Internet Access

You will be able to access the Internet through a wireless connection in all meeting rooms by using the log-in information below:

Network name:	**Westin-MeetingRoom**
Password:	**nctm2016**

Social Media

Twitter: Find us on Twitter at #NCTMINST and, of course, at #NCTM.

Facebook: http://www.facebook.com/TeachersofMathematics

Handouts

Handouts and PDFs of the presentations will be available online within one to two weeks after the close of the Institute at **http://nctm.org/K_8prac16**.

Welcome Reception

Enjoy light hors d'oeuvres and a cash bar provided by the Westin Atlanta Airport in Candler on Monday, July 11, from 5:00 to 6:00 p.m.

Sunday, July 10

4:00 p.m.–7:00 p.m.	Materials Pickup

Monday, July 11

8:00 a.m.–4:30 p.m.	Materials Pickup
9:00 a.m.–10:15 a.m.	Opening Session: Timothy Kanold
10:30 a.m.–12:00 p.m.	Breakout Workshops
12:00 p.m.–1:00 p.m.	Boxed Lunches / Networking
1:00 p.m.–3:00 p.m.	Deep Dive Sessions
3:15 p.m.–4:15 p.m.	Keynote Session: Harold Asturias
5:00 p.m.–6:00 p.m.	Welcome Reception

Tuesday, July 12

8:30 a.m.–9:30 a.m.	Keynote Session: David Pugalee
9:45 a.m.–11:45 a.m.	Deep Dive Sessions
11:45 a.m.–12:45 p.m.	Boxed Lunches/Networking
12:45 p.m.–2:15 p.m.	Breakout Workshops
2:30 p.m.–4:00 p.m.	Breakout Workshops

Wednesday, July 13

8:30 a.m.–9:30 a.m.	Keynote Session: Robert Q. Berry, III
9:45 a.m.–11:15 a.m.	Deep Dive Sessions
11:30 a.m.–12:30 p.m.	Closing Session: Karen Karp

8:00 a.m.–4:30 p.m.

Materials Pickup—Atrium

9:00 a.m.–10:15 a.m.

Opening Session

Six Secrets of Highly Effective Mathematics Lessons—Every Day!

In this inspirational and interactive opening session, we will explore six research-affirmed lesson elements that inspire student learning of mathematics each and every day. Students' mathematical learning experiences must be meaningful and relevant and served by a high degree of active engagement! Let's uncover the secrets to great mathematics teaching, based on NCTM's Teaching and Learning Standards.

Timothy Kanold
tkanold@d125.org
The Mathematics Leadership Center in Chicago
Grand Ballroom

10:30 a.m.–12:00 p.m.

Breakout Workshops

Essential Understandings for K–Grade 2 Students: Composition and Decomposition

(K–Grade 2)

Composition and decomposition are two powerful ideas that are essential to foster in early elementary mathematics. The flexibility of being able to combine and partition units contributes to a student's mathematical success. This interactive workshop will explore foundational ideas of composing and decomposing in the context of number and operations, geometry, and measurement. Pedagogical implications will be explored highlighting the Standards for Mathematical Practice to "Attend to precision" (SMP 6) and "Construct viable arguments and critique the reasoning of others" (SMP 3).

Susie Katt
skatt@lps.org
Lincoln Public Schools, Lincoln, Nebraska
Atlanta Ballroom I and II

Concentration on Compensation (Not the Paycheck Kind!)

(K–Grade 2)

In this workshop, we will explore the underlying number sense necessary for students to understand the "Big Idea" of compensation and for them to use the powerful mental math strategies rooted in this concept. We will examine the trajectory of students' mathematical development and see how we can use a variety of resources including context-based units, quick images, number talks, games, and digital resources to provide opportunities for learning. We will also share strategies for turning classrooms into communities of mathematicians by implementing tasks that promote reasoning and problem solving, using and connecting mathematical representations, supporting productive struggle, and eliciting and using evidence of student thinking.

Susanna Stossel
sstossel@cathedral.org
Beauvoir, the National Cathedral Elementary School, Washington, D.C.
Atlanta Ballroom III and IV

Deriving Fraction Algorithms through Making Use of Structure & Regularity

(Grades 3–5)

Come join us for some fun with Play-Doh and fraction operations! Participants will move from the concrete to the pictoral to the abstract by (1) building an area model with Play-Doh, (2) drawing visual models, and (3) deriving the algorithms for multiplication and division of fractions. Conceptual understanding will be established first using the area model and number line in context. Participants will then use Standard for Mathematical Practice 7 (SMP 7) to identify a pattern or structure. Participants will use SMP 8 to observe calculations that repeat to then generalize with an algorithm. Mathematical practice 3 will be used to justify conclusions. The goals of this session include participants leaving with a greater understanding of (1) mathematical practices 3, 7, and 8; (2) multiplication and division of fractions content in CCSSM; and (3) methods to engage students in the content and classroom discourse.

Janna Canzone
jcanzone@uci.edu
University of California, Irvine
Grand Ballroom II, III and IV

To Think or Not to Think

(Grades 3–5)

"Did I get it right?" "Is this correct?" "Is this the right answer?" Questions like these are a sure sign that our students are too interested in being "answer-getters" and lack mathematical confidence. In this workshop, participants will complete activities for grades 3–5 designed to refocus students' thinking on the hows and whys of problem solving. With activities like these, students will develop self-check skills that will bring confidence and, ultimately, self-motivation.

Jason Chamberlain
jchamberlain@caruthers.k12.ca.us
San Joaquin Valley Math Project
Caruthers Elementary School, Caruthers, California
Grand Ballroom I

High Leverage Practices to Support Number Sense and Algebraic Reasoning

(Grades 6–8)

Do number sense routines really have a place in the middle school math classroom? Of course they do! Number sense routines engage our middle school students in algebraic thinking as they explore mathematical patterns and relationships, which is foundational for mathematical literacy beyond the walls of school. Join us as we explore high-leverage practices that provide opportunities for students to develop fluency, flexibility, and the ability to reason with numbers.

Janene Ward
janene.ward.75@gmail.com
UCLA Mathematics Project, Los Angeles, California
College Park

Help Students Understand Ratios and Proportional Relationships with Rich Activities

(Grades 6–8)

Understanding ratios and proportional relationships is an important journey for students. We will use rich contextual tasks and engaging activities to help students strengthen their understanding throughout their journey. Come explore the importance of making sense of problems, multiple representations, and the ability to reason abstractly and quantitatively about ratios and proportional relationships

Andrew Stadel
mr.stadel@estimation180.com
Tustin Unified School District, California
Dogwood/Gardenia

Capturing Quantities: An Instructional Routine to Foster SMP 2

(Grades 6–8)

Reasoning abstractly and quantitatively (SMP 2) is one of three math practices that define a type of mathematical reasoning that students can learn and apply. But how do we help them get there? Through math practice instructional routines! These routines describe a set of repeatable steps and behaviors that can help students focus on the mathematical reasoning characterized by a particular math practice. In this workshop, participants will learn about two foundational ideas and several key instructional questions that characterize SMP 2. Participants will then learn an instructional routine that builds students' capacity to reason quantitatively and abstractly (SMP 2). They will also learn how the Capturing Quantities routine leverages best practices, including NCTM teaching practices, to support struggling learners.

Grace Kelemanik
gracekelemanik@gmail.com
Boston Teacher Residency Program, Massachusetts
Jasmine/Magnolia

12:00 p.m.–1:00 p.m.

Networking Lunch

Grand Ballroom

1:00 p.m.–3:00 p.m.

Deep Dive Sessions

K–2 Number and Operations (Green Circle)—
Atlanta Ballroom I and II

K–2 Number and Operations (Yellow Circle)—
Atlanta Ballroom III and IV

3–5 Fractions/Rational Numbers (Red Circle)—
Dogwood/Gardenia

3–5 Fractions/Rational Numbers (Blue Circle)—
Grand Ballroom I

6–8 Ratio/Proportional Reasoning—
Grand Ballroom II, III and IV

6–8 Statistics and Probability—
College Park

3:15 p.m.–4:15 p.m.

Keynote Session

Linking Language and Learning

The focus of this session is discussing ways to provide each and every student with access and an opportunity to wrestle with, make sense of, and communicate about important mathematics. We will talk about what it means to ensure equitable instructional practices that provide students with access to the knowledge and skills that they will need to be successful in school and beyond.

Mathematically proficient students use academic language effectively to communicate their reasoning, justify their conjectures, and construct arguments. How do we support students as they develop academic language? We will discuss principles of design for instruction that attend to the connection of mathematics understanding and language development. We will examine a framework for understanding a language-rich, mathematically powerful classroom and a theory of action that will focus the design of instruction.

Harold Asturias
ha_@berkeley.edu
Lawrence Hall of Science, University of California, Berkeley
Grand Ballroom

5:00 p.m.–6:00 p.m.

Welcome Reception

Candler

8:30 a.m.–9:30 a.m.

Keynote Session

Writing and Mathematical Thinking: Grade-Level Strategies to Support Development of the Mathematical Practices

Students are increasingly asked to write as part of their mathematics experience. Writing in mathematics sustains the development of reasoning, communication, and connections while developing metacognitive thinking. Writing in mathematics supports both deep understanding of content and students' effective development of the mathematical practices. This session will highlight a plan for creating and implementing an effective mathematical writing program. Information will bridge the divide between language and mathematics instruction with user-friendly connections to theory and the standards, and ready-to-implement strategies for the classroom. Mathematics classroom–tested writing strategies will be used to demonstrate support of the eight mathematical practices by highlighting writing as a tool to develop these essential mathematical habits of mind.

David Pugalee
David.Pugalee@uncc.edu
University of North Carolina at Charlotte
Grand Ballroom

9:45 a.m.–11:45 a.m.

Deep Dive Sessions

K–2 Number and Operations (Green Circle)—
Atlanta Ballroom I and II

K–2 Number and Operations (Yellow Circle)—
Atlanta Ballroom III and IV

3–5 Fractions/Rational Numbers (Red Circle)—
Dogwood/Gardenia

3–5 Fractions/Rational Numbers (Blue Circle)—
Grand Ballroom I

6–8 Ratio/Proportional Reasoning—
Grand Ballroom II, III and IV

6–8 Statistics and Probability—***College Park***

11:45 a.m.–12:45 p.m.

Networking Lunch

Grand Ballroom

12:45 p.m.–2:15 p.m.

Breakout Workshops

Supporting Young Learners to Develop Understanding through Problem Solving

(K–Grade 2)

Young children have strong, innate problem-solving skills, although many primary students come up short when solving problems because they often rely on unproductive methods. Mathematics instruction in early elementary school must help students to develop an understanding of the problem-solving process and to reason abstractly and quantitatively through contextualizing/decontextualizing the situation (SMP 2). Attention should also be given to developing student dispositions that value cognitive disequilibrium, making mistakes, and perseverance (SMP 1). Participants will be encouraged to think deeply about intentional pedagogical strategies that develop young, confident problem solvers and to gather ideas to implement in their own classrooms.

Susie Katt
skatt@lps.org
Lincoln Public Schools, Lincoln, Nebraska
Atlanta Ballroom I and II

Building a Foundation for Later Grades: Addition and Subtraction

(K–Grade 2)

Developing a robust understanding of addition and subtraction begins in the early primary grades. This hands-on workshop will include analysis of addition and subtraction problem types and exploration of different ways to reason about addition and subtraction situations. A young mathematician's ability to contextualize, decontextualize, and model additive relationships supports flexibility and fluency with operations. Participants will discuss how such understanding better prepares students for the more complex mathematics they will encounter in subsequent grades.

Delise Andrews
dandrews@lps.org
Lincoln Public Schools, Lincoln, Nebraska
Atlanta Ballroom III and IV

Raising Money for Children's Hospital—A Mathematical Modeling Task

(Grades 3–5)

In order to raise $1,000,000 for Children's Hospital, should a toy store ask its customers to donate whole dollar amounts of $1, $5, or $10 or request that customers donate the change needed to increase their total bill to the next full dollar amount? Join us for an authentic modeling task where participants will engage in real-world problem solving. This will be accomplished through making assumptions, researching additional needed information, formulating a plan, and reporting out their findings. The goals of this session include participants leaving with a greater understanding of: (1) math practice 4, (2) how to implement a modeling task related to decimal operations, and (3) methods to engage students in the content and classroom discourse.

Janna Canzone
jcanzone@uci.edu
University of California, Irvine
Grand Ballroom II, III and IV

Seeing the World Through Mathematics & Social Justice

(Grades 3–5)

In *Rethinking Mathematics* (2013), Gutstein and Peterson assert, "Students can recognize the power of mathematics as an essential analytical tool to understand and potentially change the world, rather than merely regarding math as a collection of disconnected rules to be rotely memorized and regurgitated." Come to this session to engage in a mathematics task that prompts students to devise their own approach and strategy to investigate the world around them; communicate their mathematical findings; and take an active role in understanding, engaging in, and changing the world.

Carolee Koehn Hurtado
koehn@gseis.ucla.edu
UCLA Mathematics Project, Los Angeles, California
Grand Ballroom I

Extending Algebraic Thinking and Reasoning: Let's Tile A Pool!

(Grades 6–8)

In this session, we will build on our exploration of mathematical patterns and relationships as we tile a pool. Through this engaging activity, students in grades 6–8 are challenged to describe, represent, and generalize their mathematical thinking in a real-world context. Participants will leave this interactive session with ideas of how to implement effective tasks that promote algebraic reasoning and problem solving in the middle school classroom.

Janene Ward
janene.ward.75@gmail.com
UCLA Mathematics Project, Los Angeles, California
College Park

Diagrams as Problem-Solving and Communication Tools for English Learners

(Grades 6–8)

Creating and analyzing mathematical diagrams can support students' mathematical thinking and their mathematical communication in line with the mathematical practices. In this session, we will explore how to support English language learners (ELLs) to use diagrams for problem solving in ratio and proportion contexts. We will discuss language support strategies that can be integrated into work with diagrams and will consider implications for tailoring those strategies to the needs and strengths of students. Participants will have opportunities to engage with diagramming, analyze example ELL student work, and consider instructional decisions about how to support ELLs in the classroom.

Johannah Nikula
jnikula@edc.org
Education Development Center, Waltham, Massachusetts
Dogwood/Gardenia

Teach Structural Thinking within Expressions and Equations, while Developing Precision in Language

(Grades 6–8)

The Standards for Mathematical Practice, along with the NCTM Process Standards, raise the expectations for classroom practices and student learning. In particular, CCSSM emphasizes mathematical structure in both content and practice standards. Although the articulation of such thinking is relatively new, teachers do not need new curriculum to develop structural thinking in their students. Structural thinking lies in three actionable steps—chunk, change, and connect. Participants will experience and learn an instructional routine, Connecting Representations. By design, it provides access to a wide range of learners, integrating high-leverage, research-based pedagogies to develop structural thinking all students.

Amy Lucenta
amylucenta@gmail.com
BPE Boston Teacher Residency, Boston, Massachusetts
Jasmine/Magnolia

2:30 p.m.–4:00 p.m.

Breakout Workshops

"That one's an upside-down triangle!"

(K–Grade 2)

In this workshop, we will focus on moving beyond children's naming of shapes to a true understanding of the connection between directional and Euclidian geometry and see how this relationship can provide a meaningful foundation for coding. We will examine the developmental trajectory of students' understanding of geometry and see how we can use a variety of resources including context-based units, quick images, games, and digital resources to provide opportunities for learning. We will also share strategies for turning classrooms into communities of mathematicians by implementing tasks that promote reasoning and problem solving, facilitating meaningful discourse, posing purposeful questions, and supporting productive struggle.

Susanna Stossel
sstossel@cathedral.org
Beauvoir, the National Cathedral Elementary School, Washington, D.C.
Atlanta Ballroom I and II

Building a Foundation for Later Grades: Number and Base Ten

(K–Grade 2)

A deep understanding of the structure of our number system is a fundamental building block for developing operations in base ten. This hands-on workshop will explore ways to bring students' attention to patterns in place value. Participants will discuss how deep understanding of place value relationships in the early grades can build flexibility with number concepts, which in turn better supports fluency and flexibility with greater numbers and base-ten operations in later grades.

Delise Andrews
dandrews@lps.org
Lincoln Public Schools, Lincoln, Nebraska
Atlanta Ballroom III and IV

?gnihtynA ecitoN uoY oD

(Grades 3–5)

What motivates students to understand math? A number of factors can play a part, but the most effective and longest-lasting stimulus is internal. Tuning students in to the exciting world of pattern-based mathematics will turn on this self-motivation. Once they begin looking for patterns, they will find them everywhere. In this workshop, participants will complete activities for grades 3–5 designed to draw student interest into the fascinating world of patterns. With activities like these, it's difficult to see how anyone could be a math hater.

Jason Chamberlain
jchamberlain@caruthers.k12.ca.us
San Joaquin Valley Math Project
Caruthers Elementary School, Caruthers, California
Grand Ballroom II, III and IV

Fair Share: Building on Students' Experiences to Support Understanding of Fractions

(Grades 3–5)

How might students' experiences with addition and multiplication of whole numbers, geometrical shapes, and problem solving support fractions concepts? This session will engage participants in tasks and reflection about how to support children's understanding of fractions by building on prior experience and students' thinking.

Carolee Koehn Hurtado
koehn@gseis.ucla.edu
UCLA Mathematics Project, Los Angeles, California
Grand Ballroom I

Give Students Engaging Tasks to Make Sense of Geometry

(Grades 6–8)

Making sense of geometric properties and constructing viable arguments is an important journey for students. We will use concrete examples and participate in engaging tasks that help students better understand geometric relationships and strengthen their ability to articulate their reasoning while modeling with mathematics and problem solving.

Andrew Stadel
mr.stadel@estimation180.com
Tustin Unified School District, California
College Park

Understanding and Planning for Mathematical Practices in Geometry Lessons

(Grades 6–8)

This session will explore mathematical thinking and communication in geometry contexts that reflect the Standards for Mathematical Practice, focusing particularly on making sense of problems and persevering in solving them (SMP 1), using appropriate tools strategically (SMP 5), and looking for and making use of structure (SMP 7). Participants will engage with a rich geometry task to examine what mathematical practices look like in a geometric context. They will then consider implications for planning goals related to the mathematical practices in geometry, instructional questions, and how to support students' productive struggle in geometry. Questions focused on the mathematical practices will be shared as a resource.

Johannah Nikula
jnikula@edc.org
Education Development Center, Waltham, Massachusetts
Dogwood/Gardenia

Teach Structural Thinking within Statistics and Probability, while Developing Precision in Language

(Grades 6–8)

The Standards for Mathematical Practice, along with the NCTM Process Standards, raise the expectations for classroom practices and student learning. In particular, CCSSM emphasizes mathematical structure in both content and practice standards. Although the articulation of such thinking is relatively new, teachers do not need new curriculum to develop structural thinking in their students. Structural thinking lies in three actionable steps—chunk, change, and connect. Participants will experience and learn an instructional routine, Connecting Representations. By design, it provides access to a wide range of learners, integrating high-leverage, research-based pedagogies to develop structural thinking in all students.

Amy Lucenta
amylucenta@gmail.com
BPE Boston Teacher Residency, Boston, Massachusetts
Jasmine/Magnolia

8:30 a.m.–9:30 a.m.

Keynote Session

Formative Assessment: Eliciting Evidence That Moves Learning Forward

This keynote session uses Dylan Wiliam's framework to focus on three key formative assessment strategies that support the Common Core Standards for Mathematical Practice: (1) engaging students in discussion, activities, and tasks that elicit evidence of learning; (2) eliciting feedback that moves learning forward; and (3) activating learners as resources for one another.

Robert Q. Berry, III
rqb3e@vrirginia.edu
University of Virginia
Grand Ballroom

9:45 a.m.–11:15 a.m.

Deep Dive Sessions

K–2 Number and Operations (Green Circle)—
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K–2 Number and Operations (Yellow Circle)—
Atlanta Ballroom III and IV

3–5 Fractions/Rational Numbers (Red Circle)—
Dogwood/Gardenia

3–5 Fractions/Rational Numbers (Blue Circle)—
Grand Ballroom I

6–8 Ratio/Proportional Reasoning—
Grand Ballroom II, III and IV

6–8 Statistics and Probability—
College Park

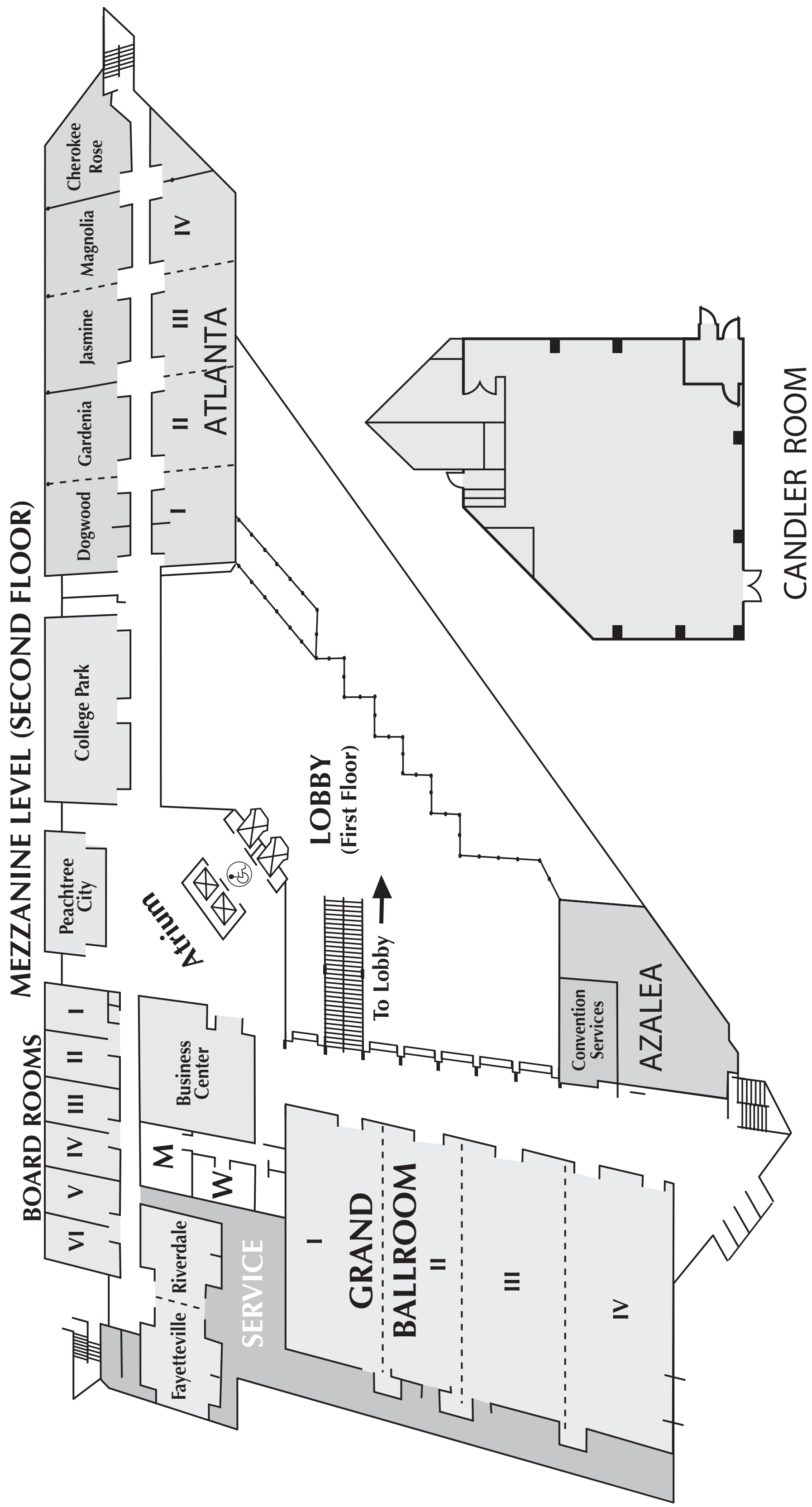
11:30 a.m.–12:30 p.m.

Closing Session

Ideas for Interventions and Assessment Strategies in the Mathematics Classroom: Teaching Students with Disabilities

Learn about foundational strategies and conceptual approaches, such as the research-based CSA (concrete–semi-concrete–abstract) model, for effective mathematics teaching for students with disabilities. Explore possible interventions and assessment strategies, including diagnostic interview.

Karen Karp
kkarp1@jhu.edu
Johns Hopkins University
Grand Ballroom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(First Floor)
To Lobby
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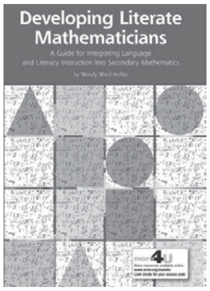

Developing Literate Mathematicians: A Guide for Integrating Language and Literacy Instruction into Secondary Mathematics

BY WENDY WARD HOFFER

How can we integrate literacy instruction authentically into mathematics content to support mathematical understanding? Busy secondary mathematics teachers who seek to respond to the needs of their students and the demands of the Common Core State Standards will welcome this book, which offers lively classroom examples, usable research, and specific ideas and resources. Enrich your students' understanding of mathematics by attending to reading, vocabulary, discourse, and writing through a workshop model.

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More Lessons Learned from Research, Volume 2: Helping All Students Understand Important Mathematics

EDITED BY EDWARD A. SILVER AND PATRICIA ANN KENNEY

Applying research to strengthen teaching practice and ensure students' success in mathematics

More than seventy years of research point to the importance of teaching mathematics for understanding. Successful students actively construct understanding rather than passively receive knowledge. Implications of this fundamental lesson from research are explored in different ways through twenty-four chapters presented in this book. Chapters cover investigations of a wide range of topics, approaches, and settings, and mathematics teachers at all levels will find examples of research that are relevant to the challenges they face.

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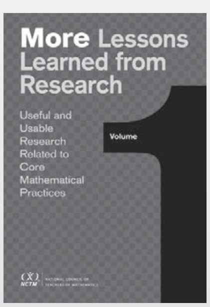

DON'T MISS!
More Lessons Learned from Research, Volume 1

EDITED BY EDWARD A. SILVER

Helps to link classroom teachers to all that original research has to offer

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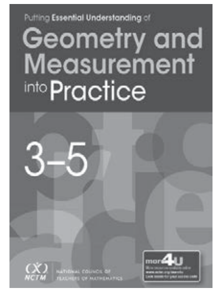
All books available as eBook.

Putting Essential Understanding of Geometry and Measurement into Practice in Grades 3–5



BY KATHRYN CHVAL, JOHN LANNIN, AND DUSTY JONES

KATHRYN CHVAL, VOLUME EDITOR

BARBARA J. DOUGHERTY, SERIES EDITOR

Do your students have "concept images" that limit their ideas of shapes to specific examples, oriented in particular ways? Do they confuse the size of an angle with the length of the rays in a drawing of an angle? This book demonstrates how to use multifaceted knowledge to address the big ideas and essential understandings that students must develop for success with geometry and measurement—not only in their current work, but also in higher-level mathematics and a myriad of real-world contexts.

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Problem Solving in All Seasons, Grades 3–5

BY KIM MARKWORTH, JENNI MCCOOL, AND JENNIFER KOSIAK

Holidays and seasonal activities offer perfect backdrops for mathematical tasks that can be related to other topics and themes in the classroom. This book delivers thirty-six appealing, real-world mathematical tasks, arranged in grade-level order, to engage young learners in problems tied to the Common Core and designed to allow children to participate in the Common Core Standards for Mathematical Practice. Each task includes a complete implementation guide, and handouts and ancillary materials can be accessed online. This is your all-in-one practical handbook for problem solving in the primary years.

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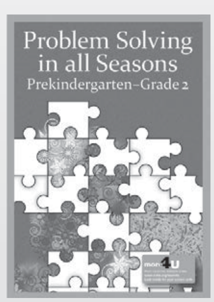

DON'T MISS!
Problem Solving in All Seasons, Pre-K–Grade 2

BY KIM MARKWORTH, JENNI MCCOOL, AND JENNIFER KOSIAK

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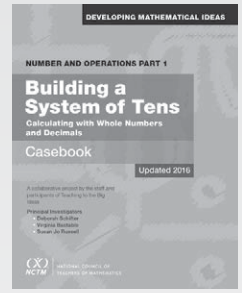
ADDITIONAL NEW TITLES



Developing Mathematical Ideas: Building a System of Tens, Casebook and Facilitators Guide

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On the Money: Math Activities to Build Financial Literacy in High School

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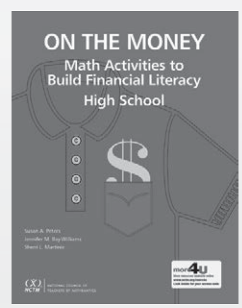

Annual Perspectives in Mathematics Education 2016

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Discovering Lessons for the Common Core Standards in Grades 9–12

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K–Grade 2 Number & Operations

Session 1: Place-Value Structure of Numbers

"Why isn't fifty-ten the number after fifty-nine?" Explore the ways young children make sense of spoken and written numbers. Using teacher-written narratives describing events from their own classrooms and digging into the mathematics for ourselves, we will analyze the mathematical understandings necessary to make sense of multidigit numbers. The session will also include opportunities to consider the implications for K–grade 2 teachers of the NCTM Effective Mathematics Teaching Practices and the Standards for Mathematical Practice from the Common Core.

Session 2: Making Sense of Addition Strategies

"First I splitted it into tens and ones." Examine how the strategies that children develop for addition call on the place-value structure of the base-ten number system. Using teacher-written narratives and video clips of students explaining their approaches, we will analyze strategies for addition and explore the mathematical concepts that underlie them. The session will also include opportunities to consider the implications for K–grade 2 teachers of the NCTM Effective Mathematics Teaching Practices and the Standards for Mathematical Practice from the Common Core.

Session 3: Making Sense of Subtraction Strategies

"To find 40 – 18, first I thought about how many I had and how many more I needed to get to 40." Examine how the strategies that children develop for subtraction call on the place-value structure of the base-ten number system as well as the relationship between addition and subtraction. Using teacher-written narratives and video clips of students explaining their approaches, we will analyze strategies for subtraction and explore the mathematical ideas that underlie them. The session will also include opportunities to consider the implications for K–grade 2 teachers of the NCTM Effective Mathematics Teaching Practices and the Standards for Mathematical Practice from the Common Core.

Facilitators

Virginia Bastable
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Mount Holyoke College, South Hadley, Massachusetts

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Grades 3–5 Fractions/Rational Number

Session 1: Understanding the Number Line: How Linear Measurement Extends from Whole to Rational Numbers

Why do we have third graders place fractional values on the number line? What do we hope they will gain from this task? This session will argue that because the number line is abstract, for students to understand a fraction on the number line, they must (1) have a robust understanding of finite linear measurement, (2) have experiences working on complex whole-number tasks on the number line, and (3) be able to connect other diagrammatic and contextual representations of rational numbers to the number line. Participants will explore tasks that use linear measurement to develop in students an early understanding of fractions. This understanding will also be used to think of how the measurement construct connects to the idea of "unit" and the multiple interpretations of fractions. This session will focus on the first Big Idea from NCTM's *Essential Understanding* book on fractions and on Mathematics Teaching Practice 6 (Build procedural fluency from conceptual understanding).

Session 2: Is 3 × 5 the same as 5 × 3? Using Precise Notation to Explore the Structure of Multiplication

Why does CCSSM specifically define "$a \times b$" as "a copies of b"? Although the whole-number implications may seem thin, the rational-number implications are somewhat important. How can you model and find the product of 6 copies of 1 1/2? Does the model of 1 1/2 copies of 6 look the same? Students who discover that these products are the same through models and contexts can actually be left in awe of commutativity—not taking it as a predefined quality of multiplication. Using linear measurement diagrams and the number line, participants will look at a progression of multiplication problems with increasingly complex structure. They will then discuss how students can move from one step to the next. After analyzing the structure of these products, participants will consider implications for early instruction of multiplication. This session will focus on the fourth Big Idea from NCTM's *Essential Understanding* book on fractions and on Mathematics Teaching Practice 3 (Use and connect mathematical representations).

Session 3: From Interpretation to Algorithms: Using Contexts to Develop Two Different Fraction-Division Algorithms

What does it mean to define division as "the inverse of multiplication"? When can we interpret $a \div b$ as the solution to ___ $\times b = a$ and when as $b \times$ ___ $= a$? Participants will look at a variety of contexts that can lead students to a robust understanding of contextual and noncontextual rational-number division. They will then look at how the different interpretations (when developed through sense making) lead to two different algorithms—either the common-denominator algorithm or the invert-and-multiply algorithm. Participants will see why these are generalizations of student-generated processes. This understanding will then be used to consider how division can be introduced in earlier elementary grades. Specifically, participants will look at ways of developing symbolic notation for division. We will also look at strategies of how to help students decide between partitive and quotative division, even in noncontextual (whole-number and rational number) situations. This session will focus on the fourth Big Idea from NCTM's *Essential Understanding* book on fractions and on Mathematics Teaching Practice 1 (Establish mathematics goals to focus learning).

Facilitators

Mina Kim
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Grades 6–8 Ratio/Proportional Reasoning

Session 1: Developing Pedagogical Content Knowledge

We will take a look at the Big Idea and essential understandings for ratios, proportions, and proportional reasoning and how these understandings can support our content expertise and structure our instructional strategies. We will examine the four components of pedagogical content knowledge and use them as a framework for each task specifically and for the unit as a whole. We will do a few tasks and examine students' work as we proceed toward the big idea of ratios and proportions.

Session 2: Working toward Essential Understandings 3 and 7

There are ten essential understandings for ratios, proportions, and proportional reasoning, but we will focus on four of these understandings (two on one day and two on the next). We will work toward Essential Understanding 3 by examining tasks that have quantitative attributes (direct measurements) as well as relational attributes (probability and speed). We will also work toward Essential Understanding 7 by examining students' reasoning and understanding of equivalent ratios, proportional relationships, and the relationship between multiplicative comparisons and composed units. We will work through some tasks that challenge our own thinking about these attributes.

Session 3: Working toward Essential Understandings 8 and 9

We will work toward Essential Understanding 8 by defining the terms ratio and rate and finding ways to help students develop their own conceptions on which they may build future understandings. Essential Understanding 9 states, "Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems." We will work toward this understanding by highlighting students' work that demonstrates how they can "reason abstractly and quantitatively." We will wrap up our session by looking at the key concepts in ratios and proportions that are covered before and after grades 6–8.

Facilitator

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Mesa Union Junior High School, Somis, California

Grades 6–8 Statistics and Probability

Session 1: Data Variability

This workshop will start by having participants collect and record data about themselves (e.g., height, shoe size, distance from home, number of TVs, cubit length) on chart paper posted around the room. After the data has been collected, the participants will study the statistical reasoning process as outlined in the Guidelines for Assessment and Instruction in Statistics Education (GAISE). This workshop will emphasize the statistics standards in the sixth-grade Common Core standards; particularly, analyzing a set of data that has a distribution that can be described by its center, spread, and overall shape. The context for collecting the data is measuring each person's cubit, the length of the forearm from the elbow to the end of the middle finger. These data will be recorded and used to create a human box-and-whisker-plot. Participants will find the minimum, maximum, median, upper quartile, and lower quartile of the data. Participants will calculate the inner quartile range and determine if the data has any outliers. If enough participants are present, box-and-whisker plots will be made for each gender and compared. The participants will respond to a number of questions about the data distributions. The participants will discuss do an idea wave to review the important ideas in the activity and support language development.

Session 2: Investigating Chance

This workshop will provide an opportunity for participants to explore theoretical and experimental probability. The participants will play two different two-dice games. In the first game, two dice are rolled. If the sum is even, player 1 gets a point. If the sum is odd, player 2 gets a point. Participants will play at least twenty rounds of the game and record their results in a table. The participants will calculate the experimental probability for an even sum and an odd sum. In the second game, two dice are rolled. If the product is even, player 1 gets a point. If the product is odd, player 2 gets a point. Participants will play at least twenty rounds of the game and record their results in a table. The participants will calculate the experimental probability for an even sum and an odd product. Next, the participants will analyze the games by looking at the possible outcomes for each game and calculating the theoretical probability for each outcome. The participants will connect this activity to the GAISE statistical reasoning process and discuss the implications for classroom implementation.

Session 3: Investigations Bi-Variate Data

In the first workshop, the participants recorded their height (in inches) and their shoe size in a table. Participants will use the data height and shoe size that was collected in the first workshop to create a scatterplot. The scatterplot will be used to create a line of best fit for the data. The participants will calculate the equation of the line of best fit and then will use the equation of the line of best fit to calculate their shoe size given their height in inches. Participants will compare their results to determine whose equation did the best job of calculating shoe size from height. Participants will use their equations to calculate the shoe sizes of the world's tallest man and woman. The participants will have a discussion about whether using height as a predictor of shoe size is reasonable. The session will end with an idea wave on scatterplots.

Facilitator

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University of California, Los Angeles

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MATH IS ALL AROUND US

Take Your Professional Development to a New Level of Excellence

Developing Mathematical Ideas Professional Development Series

Number and Operations, Part 1: Building a System of Tens

Calculating with Whole Numbers and Decimals

BY DEBORAH SCHIFTER, VIRGINIA BASTABLE, AND SUSAN JO RUSSELL

Casebook: NCTM Stock No. 15032
List: $40.95 | **Member: $32.76**

Facilitator's Guide: NCTM Stock No. 15053
List: $62.95 | **Member: $50.36**

Building a System of Tens is the first module in the seven-part **Developing Mathematical Ideas Series**. The complete module consists of a casebook for participants and an online facilitator's package that contains everything necessary to prepare for and lead the seminar, including access to the casebook content and classroom videos.

The DMI program engages participants in collaborative learning where they make sense of the content, recognize where and how their grade content is situated in the trajectory of learning from K–grade 8, build connections among different concepts, and analyze student thinking from a mathematical perspective. Teachers learn how to orient their instruction to specific mathematical goals and to develop a mathematics pedagogy in which student understanding is central.

The ***Building a System of Tens Casebook*** was designed as a key component of the modules. The thirty cases, written by teachers describing real situations and actual student thinking in their classrooms, provide the basis of each session's investigation into specific mathematical concepts and teaching strategies. Guided by the facilitator, participants explore the base-ten number structure, consider how that structure is exploited in multidigit computational procedures, and examine how basic concepts of whole numbers reappear when working with decimals.

The online ***Building a System of Tens Facilitator's Package*** includes an introduction, preseminar preparation for the facilitator, and all the readings, materials, and videos needed to lead the eight sessions of the seminar.

Here is what educators are saying about the Developing Mathematical Ideas Series:

"Our teachers love DMI seminars. The seminars provide them with an opportunity to dig deeply into important mathematics content … teachers here say that it was what they learned in our DMI seminars that helped them transform their teaching."

LINDA RUIZ DAVENPORT
Director of K–12 Mathematics
Boston Public Schools

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Worthwhile Mathematical Tasks

The teacher of mathematics should pose tasks that are based on—

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences; and
- knowledge of the range of ways that diverse students learn mathematics—

and that

- engage students' intellect;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions; and
- promote the development of all students' dispositions to do mathematics.

National Council of Teachers of Mathematics (NCTM). *Professional Teaching Standards.* Reston, Va.: NCTM, 1991

***Principles to Actions:* Mathematics Teaching Practices**
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Principles to Actions: Ensuring Mathematical Success for All is an official position of the National Council of Teachers of Mathematics as approved by the NCTM Board of Directors, February 2014.

National Council of Teachers of Mathematics (NCTM). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, Va.: NCTM, 2014

NCTM PROCESS STANDARDS

Problem Solving

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems; and
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs; and
- select and use various types of reasoning and methods of proof.

Communication

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

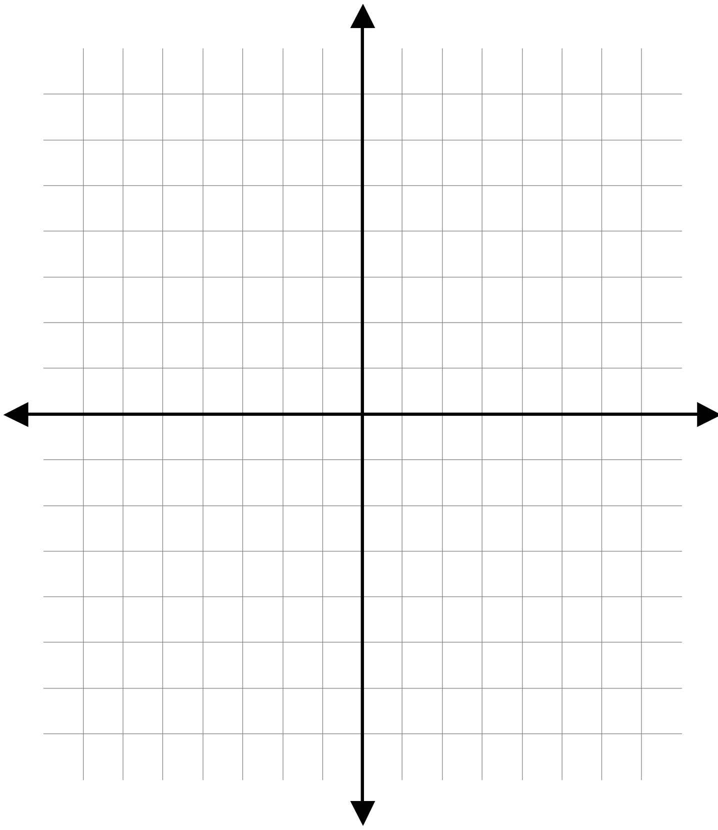
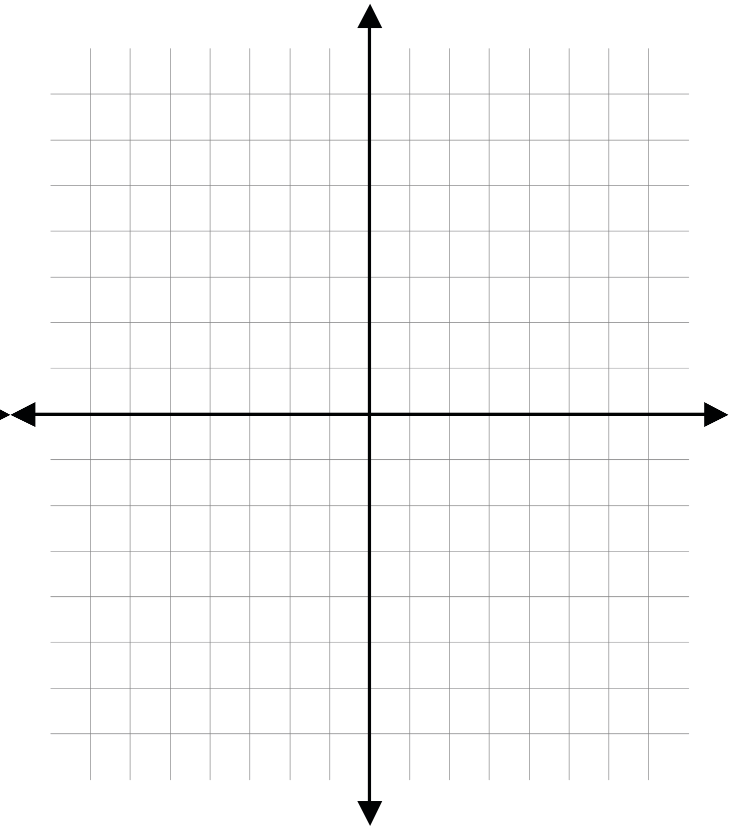
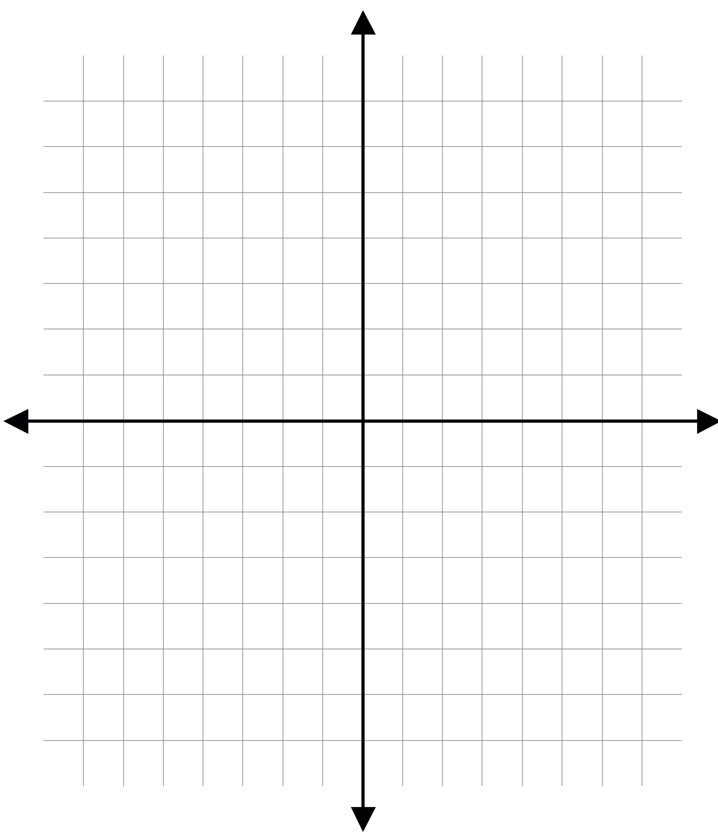
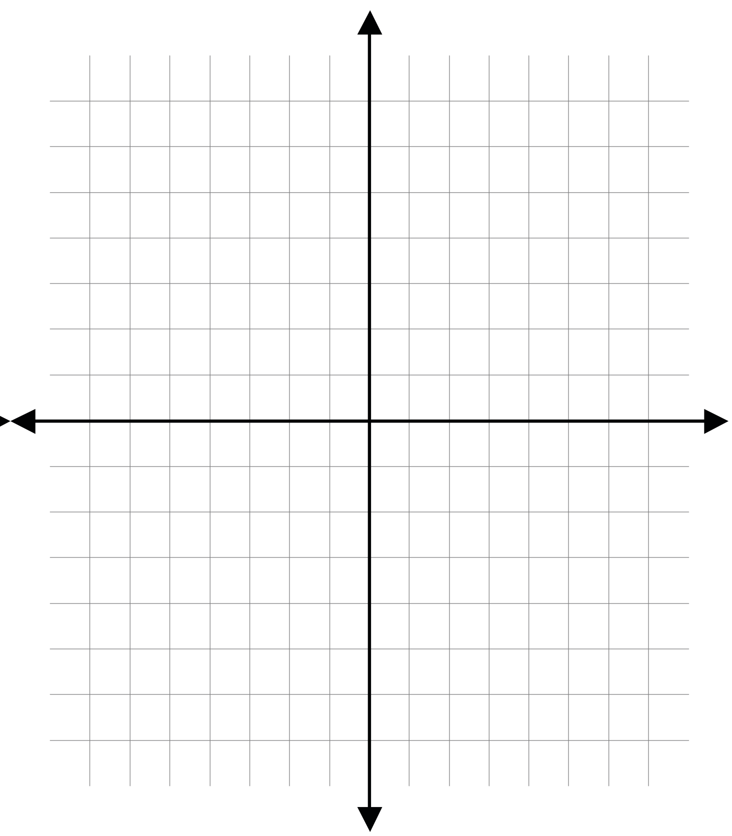
Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

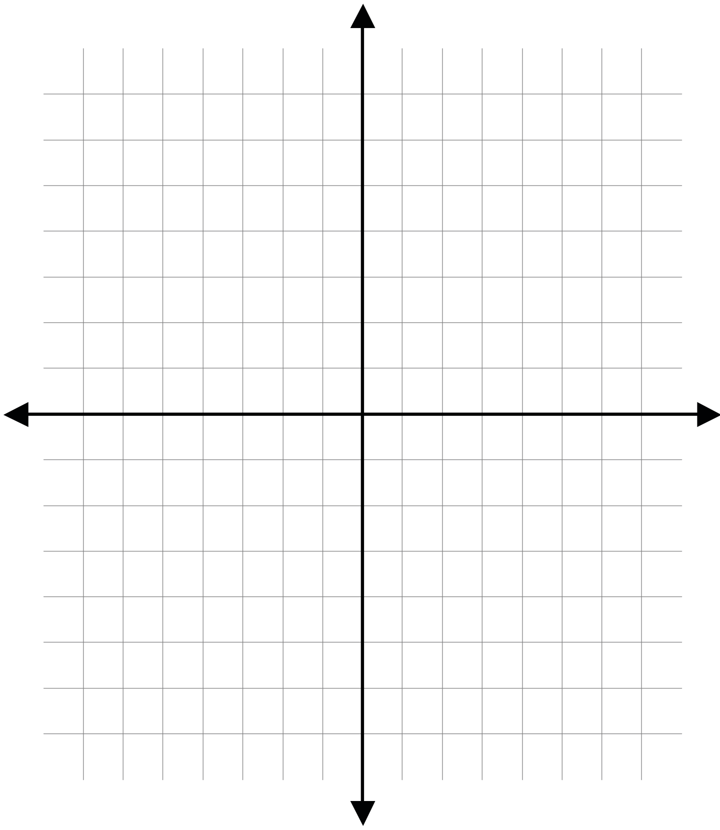
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

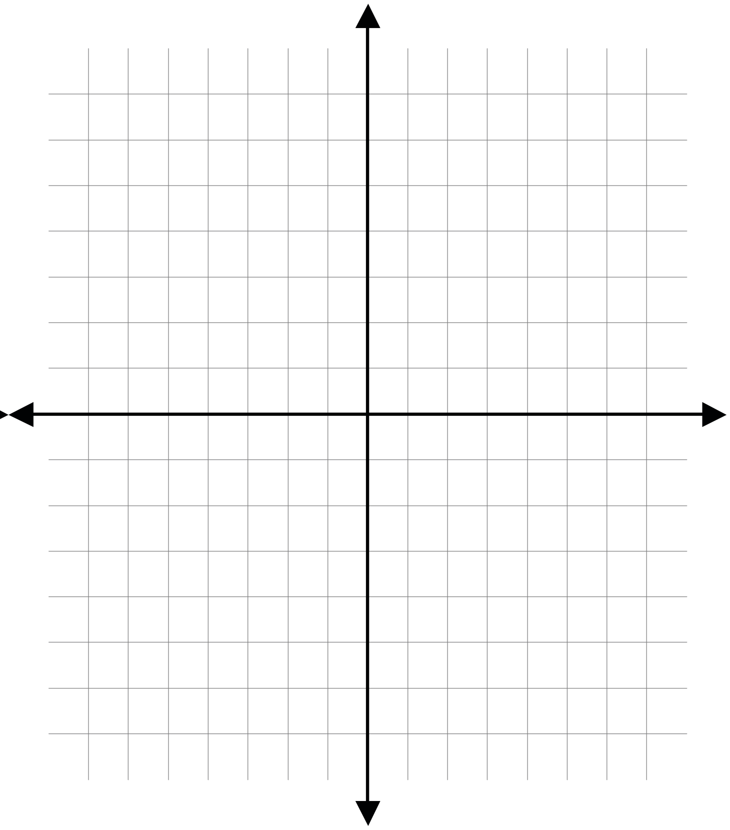
The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

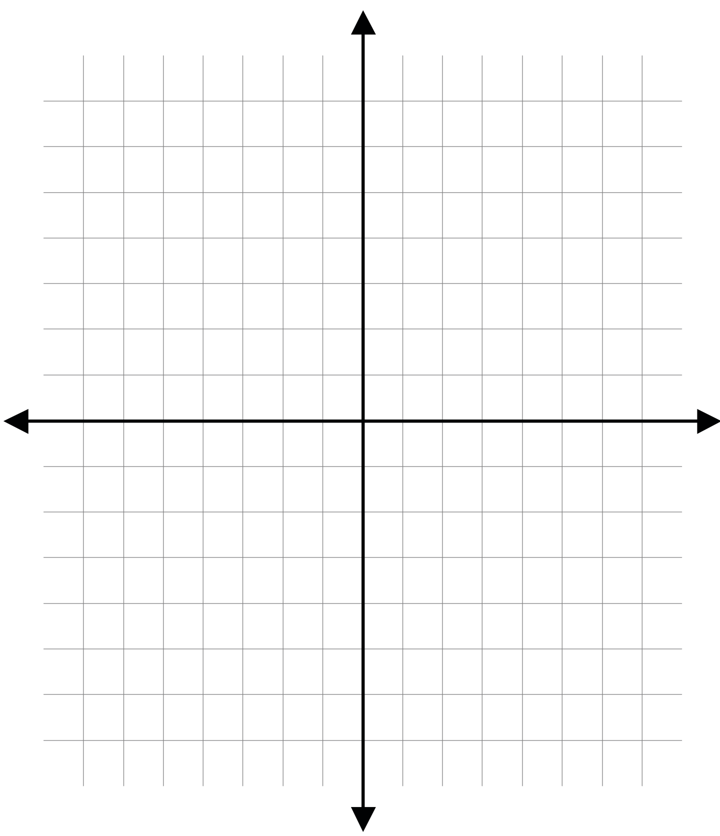
In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

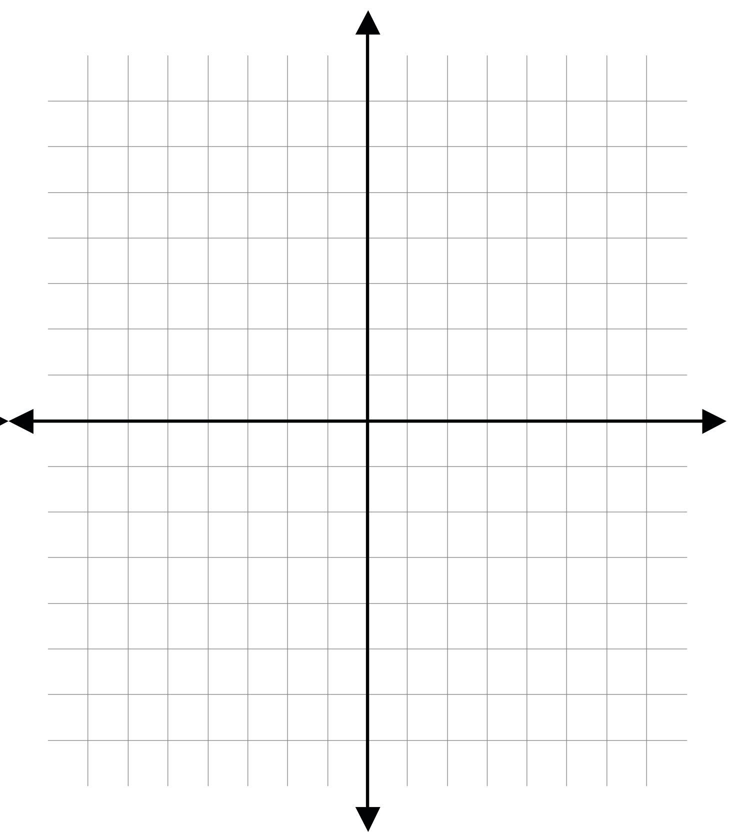
National Governors Association Center for Best Practices and the Council of Chief State School Officers (NGA Center and CCSSO). *Common Core State Standards for Mathematics.* Washington, D.C.: NGA Center and CCSSO, 2010. http://www.corestandards.org









Graph Paper 1 cm

NAME ____________________________

Graph Paper 1 cm

NAME ____________________________

ILLUMINATIONS
Resources for Teaching Math

Graph Paper 1 cm

NAME ______________________________

Graph Paper 1 cm

NAME ____________________________

NOTES

NOTES

NCTM®

NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

This certificate is presented to

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in recognition of attendance and participation at the
NCTM 2016 Interactive Institute on K–Grade 8 Mathematics—
Engaging Students in Learning:
Mathematical Practices

Atlanta, Georgia • July 11–13, 2016

Matthew Larson
President, NCTM

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NCTM Interactive Institute on
K–Grade 8 Mathematics
July 11–13, 2016
Atlanta, Georgia

Name of Provider: National Council of Teachers of Mathematics

Educator's Name: ____

Description of Professional Development Activity: This is a two-and-a-half-day Institute sponsored by the National Council of Teachers of Mathematics. The focus of the Institute is on the CCSS Mathematical Practices, NCTM's Process Standards, and the eight effective practices outlined in *Principles to Actions*. Participants will attend three deep dive session groups based on the strand they selected at registration. Participants will select three out of twenty-one breakout workshops and will attend five keynote sessions.

Note: PD time earned should be the time actually spent in sessions and/or workshops.

Date	Session Type	Session Title	Presenter(s)/ Facilitator's Name(s)	Start/End Time	PD Time earned
TOTAL Professional Development Hours Accrued:					

I certify that the above named educator accrued the indicated number of Professional Development hours.

Matthew Larson
President, National Council of Teachers of Mathematics

Please check with your state education agency and local administration to determine if these Institute hours can be used for professional development credits.