



Guiding Principles for Mathematics Curriculum and Assessment

A national curriculum for school mathematics is a topic of growing interest at state, national, and policy levels. The development of a common national curriculum and assessment in mathematics should be driven by the following basic principles for designing an excellent curriculum to avoid the risk of producing a negotiated list of standards that is merely an intersection of those that are currently addressed in each of the 50 states. Therefore, NCTM recommends the following guiding principles for the potential development of any set of common curricular expectations and assessments across the nation.

The National Council of Teachers of Mathematics (NCTM) ushered in the standards era with the 1989 publication of *Curriculum and Evaluation Standards for School Mathematics*, which was updated in 2000 as *Principles and Standards for School Mathematics. Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006) addressed the related issues of curricular focus and coherence, which are at the heart of the growing call for common standards. The forthcoming *Focus in High School Mathematics: Reasoning and Sense Making* (2009) will address mathematics education in high school. The following guiding principles are adapted from these NCTM publications.

A curriculum is more than a collection of activities: It must be coherent, focused on important mathematics, and well articulated across the grades.

Focus and coherence: Mathematics consists of different topical strands, such as algebra and geometry, but the strands are highly interconnected. A coherent curriculum effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on or connect with other ideas, thus enabling students to learn with understanding, develop skill proficiency, and solve problems.

Important mathematics: A mathematics curriculum should focus on mathematics content and processes that are important and worth the time and attention of students. Mathematics topics may be important for different reasons, such as their utility in developing other mathematical ideas, in linking different areas of mathematics and in preparing students for college, the workforce, and citizenship.

Articulation across grades: Learning mathematics involves accumulating ideas and building successively deeper and more refined understanding. A school mathematics curriculum should provide a road map that helps teachers guide students to increasing levels of sophistication and depths of knowledge. Such guidance requires a well-articulated curriculum so that teachers at each level understand the mathematics that has been studied by students at the previous level and what is to be the focus at successive levels.

Any national mathematics curriculum must emphasize depth over breadth and must focus on the essential ideas and processes of mathematics. The generally long lists of state and local school district curricular expectations have led to teaching too much too quickly with far too little depth.

The most important challenge for any mathematics curriculum is to be focused in scope and not simply a long list of disconnected expectations. An effective curriculum is focused, delves deeply into each topic and concept, and is coherent across grades.

Mathematical literacy emerges from, among other foundational understandings, a mature sense of number that includes an understanding of place value and comfort with estimating; a data sense that recognizes outliers and misinterpretation of data; a spatial sense that links two- and three-dimensional objects; and a symbol sense that results in algebraic representations that enable generalizations and predictions. Rather than long lists of skills, these concepts must be the foundation of any set of national mathematics curriculum. NCTM's *Curriculum Focal Points* is a starting point for identifying these big ideas and their translation into a high-quality curriculum.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. Learning mathematics with understanding is essential.

Psychological and educational research on the learning of complex subjects such as mathematics has solidly established the important role of conceptual understanding in the knowledge and activity of persons who are proficient. Being proficient in a complex domain such as mathematics entails being able to use knowledge flexibly, appropriately applying what is learned in one setting to another. The union of factual knowledge, procedural proficiency, and conceptual understanding enhances all three components, making the resulting learning usable in powerful ways.

If a voluntary national mathematics curriculum is developed, the topics studied in that curriculum must be taught and learned in an equitable manner in a setting that ensures that problem solving, reasoning, connections, communication, and conceptual understanding are all developed simultaneously along with procedural fluency.

Problem Solving

Problem solving means engaging in a task for which the solution method is not known in advance. To find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort. They should then be encouraged to reflect on their thinking. Problem solving is an integral part of all mathematics learning.

Reasoning and Proof

Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena. Those who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask whether those patterns are accidental or whether they occur for a reason; and they conjecture and prove.

Ultimately, a mathematical proof is a formal way of expressing particular kinds of reasoning and justification.

Being able to reason is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and at all grade levels, students should recognize and expect that mathematics makes sense. Building on the considerable reasoning skills that children bring to school, teachers can help students learn what mathematical reasoning entails.

Communication

Communicating mathematical thinking and reasoning is an essential part of developing understanding. It is a way of sharing and clarifying ideas. Through communication, ideas become objects of reflection, refinement, and discussion and often require adjustments of thinking. The communication process also helps build meaning and permanence for ideas and makes them public. When students are challenged to think and reason about mathematics and communicate the results of their thinking with others, they learn to be clear and convincing in their verbal and written explanations. Listening to others explain gives students opportunities to develop their own understanding. Conversations in which mathematical ideas are explored from multiple perspectives help learners sharpen their ability to reason, conjecture, and make connections.

Connections

Too often individuals perceive mathematics as a set of isolated facts and procedures. Through curricular and everyday experiences, students should recognize and use connections among mathematical ideas. Of great importance are the infinite connections between algebra and geometry. These two strands of mathematics are mutually reinforcing in terms of concept development and the results that form the basis for much advanced work in mathematics as well as in applications. Such connections build mathematical conceptual understanding based on interrelationships across earlier work in what appear to be separate topics.

In addition, students should recognize and apply mathematics in contexts outside mathematics. Students need experiences applying mathematics concepts and representations to describe and predict events in almost all academic disciplines, as well as in the workplace as we develop a fully informed citizenry.

As stated above, a potential national curriculum must include important mathematics. Content should include the following key content areas.

Number and Operations with Procedural Fluency

Proficiency with number and operations requires the deep and fundamental understanding of counting numbers, rational numbers (fractions, decimals, and percents), and positive and negative numbers, beginning in the elementary and middle grades. This understanding is extended to other number systems. Students must demonstrate understanding of numbers and

relationships among numbers with a focus on the place-value system. Students must develop understanding of number operations and how they relate to one another.

Written mathematical procedures—computational procedures in the elementary grades and more symbolic algebraic procedures as students move into the secondary level—continue to be an important focus of school mathematics programs. Equally important is the ability to be comfortable and competent with estimation and mental math. As students develop number sense, they acquire abilities to estimate and perform mental calculations quickly and proficiently. Students should become proficient at using mental math shortcuts, performing basic computations mentally, and generating reasonable estimates for situations involving size, distance, and magnitude.

Algebra

Algebra is more than a set of procedures for manipulating symbols. Algebra provides a way to explore, analyze, and represent mathematical concepts and ideas. It can describe relationships that are purely mathematical or ones that arise in real-world phenomena and are modeled by algebraic expressions. Learning algebra helps students make connections in varied mathematical representations, mathematics topics, and disciplines that rely on mathematical relationships. Algebra offers a way to generalize mathematical ideas and relationships, which apply to a wide variety of mathematical and nonmathematical settings.

Algebraic concepts and skills should be a focus across the pre-K–12 curriculum. The development of algebraic concepts and skills does not occur within a single course or academic year. An understanding of algebra as a topic is a course of study. As a collection of mathematical understandings develops over time, students must encounter algebraic ideas across the pre-K–12 curriculum. At the elementary school level, teachers help students be proficient with numbers, identify relationships, and use a variety of representations to describe and generalize patterns and solve equations. Secondary school teachers help students move from verbal descriptions of relationships to proficiency in the language of functions and skill in generalizing numerical relationships expressed by symbolic representations.

Algebra readiness is determined not at a prescribed grade level but when students exhibit demonstrable success with prerequisite skills. Only then should students focus explicitly and extensively on algebra, either in a course called Algebra 1 or within an integrated mathematics curriculum. Exposing students to such coursework before they are ready often leads to frustration, failure, and negative attitudes toward mathematics and learning. The appropriate prerequisite knowledge for a first formal algebra course is outlined in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*.

Geometry and Measurement

Geometry is a natural place for the development of students' reasoning and justification skills, culminating in work with proof in the secondary grades. Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving. Geometric ideas are useful in representing and solving problems in other

areas of mathematics and in real-world situations, so geometry should be integrated with these other areas. Geometric representations can help students make sense of area and fractions; histograms and scatter plots can give insights about data; and coordinate graphs can serve to connect ideas in geometry and algebra.

The study of measurement is important in the mathematics curriculum from prekindergarten through high school because of the practicality and pervasiveness of measurement in so many aspects of everyday life. The study of measurement offers an opportunity for learning and applying other mathematics, including number operations, geometric ideas, statistical concepts, and notions of function. It highlights connections within mathematics and connections between mathematics and areas outside mathematics, such as social studies, science, art, and physical education.

Data Analysis, Statistics, and Probability

Students should have experience in formulating questions, designing simple surveys and experiments, gathering and representing data, and analyzing and interpreting these data in a variety of ways. They need to explore variability by knowing and using basic measures of data spread and center, be able to describe the shape of data distributions, and be able to make inferences and draw conclusions based on information from samples of populations. They need to be able to compute probabilities of simple and compound events and to create simulations that can estimate probabilities for events.

A natural link exists between data analysis in statistics and algebra. Students' understanding of graphs and functions can both enhance and be enhanced by tackling problems that involve data analysis and statistics in authentic situations. Basic ideas of probability form the underpinnings of statistical inference. Probability is also linked to other mathematical content areas such as counting techniques (number and operation), ratios of areas and volumes (geometry), and relationships between functions and the area under their graphs (algebra, data analysis).

Summary

These guiding principles should be integral in the development of any curriculum for mathematics education. Equally important, any curriculum must be linked to assessments based on standards. A curriculum should provide a rich, connected learning experience for students while adding coherence to the standards, and standards must align with the curriculum rather than be separate, long lists of learning expectations. Alignment and coherence of these three elements—curriculum, standards, and assessment—are critically important foundations of mathematics education.

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