Using a Publication as a Professional Development Experience

Title: Growing Professionally
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Overview:
The following enhancement can be used with the NCTM publication *Growing Professionally: Readings from NCTM Publications for Grades K-8* in professional learning communities or in other professional development venues. Each section in the enhancement parallels the sections in the book in Part 3. The book also includes two other sections that may be useful in your planning - Part 1: Ideas for Using Articles in Professional Development and Part 2: Articles about Growing Professionally. For your convenience, the pages listed below reflect those in the book, not those in the original journal.

Growing Professionally, Part 3: Articles for Use in Professional Development

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Section: Lifelong Learning

Articles:
Four Teacher-Friendly Postulates for Thriving in a Sea of Change, Leinwand, S., *Mathematics Teacher*, September 1994 – pp. 87-95

Never Say Anything a Kid Can Say! Reinhart, S. *Mathematics Teaching in the Middle School*, April 2000 – pp. 89-94

Redefining Success in Mathematics Teaching and Learning, Smith, M. S., *Mathematics Teaching in the Middle School*, February 2000 – pp. 95-101

Signposts for Teaching Mathematics through Problem Solving, Hiebert, J. Chapter 4 of *Teaching Mathematics through Problem Solving* by Lester, F. and Charles, R., 2003. pp. 102 – 107.

I. Preparing to Use the Articles
One definition of lifelong learning is a framework that encompasses learning throughout the life cycle, from birth to grave and in different learning environments, formal, non-formal and informal. Discuss this definition and as it relates to learning.

II. Working through the Articles
Learning is a great activity. It expands your viewpoint and can give you new knowledge you can use to improve your teaching. Which of Leinwand's provocative postulates in addressing change in mathematics education can help you improve your teaching?

What is similar between Leinwand's Postulates and Reinhart's Fundamental Flaw?

How can we rethink our beliefs about the learning goals we set for students and how students can best achieve these goals?

Why does traditional “teaching and telling” give teachers a sense of efficacy?

It is unreasonable to ask a professional to change much more than 10 percent a year, but it is unprofessional to change less than 10 percent a year?

Agree or Disagree: The best questions are open ended.

What does it mean for students to be successful in your mathematics class?

III. Keeping the Thinking Going

Create a to-do list with three tasks you can accomplish with respect to mathematics teaching. Also try to create a “to-learn” list. On it you can write ideas for new areas of study in mathematics teaching and learning. Whatever motivates you, write it down!

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Section: Windows and Mirrors

Articles:

Helping English-Learners Develop Computational Fluency. Bresser, R., *Teaching Children Mathematics*, February 2003, pp. 111-117.

Mathematical Notations and Procedures of Recent Immigrant Students. Perkins, I. & Flores, A. *Mathematics Teaching in the Middle School*, February 2002, pp. 118-124.

Building Responsibility for Learning in Students with Special Needs. Karp, K. & Howell, P., *Teaching Children Mathematics*, October 2004, pp. 125-134.

Differentiating the Curriculum for Elementary Gifted Mathematics Students. Wilkins, M.M., Wilkins, J.L.M., & Oliver, T. *Teaching Children Mathematics*, August 2006, pp. 136-144.

I. Preparing for Using the Articles

Windows and mirrors bring to our attention the need for us to consider equity issues in mathematics education, specifically, issues of diversity in ethnicity, language, social class, gifted students, and students with special needs.

"All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence." (NCTM Principles and Standards, 2000, p.5)

“Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for *all* students” (NCTM Principles and Standards, 2000, p. 12).

Discuss what the quotations above mean to you as a teacher of mathematics.

II. Working through the Articles

Article: *Helping English-Learners Develop Computational Fluency* – BresserB
Because equity requires accommodation of differences to promote access and attainment, how can you create and facilitate classroom discussions so that English learning students equally participate?

Discuss how the specific strategy, "Think-Pair-Share" promotes student reflection and communication for access to everyone's ideas.

What are various ways that mathematical ideas can be represented in order to support student development in both mathematical communication and conceptual learning?

Article: *Mathematical Notations and Procedures of Recent Immigrant Students* - Perkins & Flores How can you become more accepting of a variety of algorithms and notation that differ from their own?

How can you validate students' previous schooling experiences both linguistically and mathematically?

How does acknowledging all students' background and experiences in mathematics, especially including students who are recent immigrants, and/or English learners, play a role in establishing an equitable learning environment?

Article: *Building Responsibility for Learning in Students with Special Needs* - Karp & Howell Discuss how a learning disability in one or more areas of underlying learning skills (memory, self-regulation, visual processing, & motor skills) creates a roadblock between the student and learning mathematics.

Discuss the four components of instructional individualization for students with special needs with respect to equity and promoting access and attainment for *all* students in mathematics.

Discuss the two myths described on p. 126. How can regular education teachers come to realize that students with disabilities require different learning conditions and methods than regular education students?

Discuss how you could or should build responsibility for learning mathematics in all students, and especially for students with special needs.

Article: *Differentiating the Curriculum for Elementary Gifted Mathematics Students* - Wilkins, Wilkins, & Oliver
Discuss the ideal goals for differentiating the curriculum for gifted students in mathematics.

Discuss the creation and selection of activities for a matrix on a mathematics topic similar to the Mathematics Investigation Center described in the article. Brainstorm the activities that you would include for your topic or theme. Consider the recommendations made on p. 140.

Discuss the development of independence, semi-independence, and responsibility for learning mathematics in all students, and especially for gifted students of mathematics.

III. Keep the Thinking Going

Reflect on each of the 4 articles that focus on various issues of equity. What are some similarities and differences in the various perspectives of each article on issues of equity?

Reflect on the notion that we need windows to see the diverse experiences of all our students so that we may be inclusive in our curriculum and teaching of mathematics. Discuss any new perspectives that you may have gained.

Reflect on the notion that we need mirrors to reflect our own reality. Write down any topics in which you see reflection of yourself in mathematics education that you found in the articles.

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Section: Meaningful Mathematics Tasks

Articles:
Selecting and Creating Mathematical Tasks: From Research to Practice. Smith, M. S. & Stein, M. K. *Mathematics Teaching in the Middle School*, February 1998 – pp. 147-153.

Turning Traditional Textbook Problems into Open-Ended Problems. Kabiri, M. S. & Smith, N. *Mathematics Teaching in the Middle School,* November 2003 – pp. 154-161.

The Role of Textbooks in Implementing the Curriculum Principle and the Learning Principle. Reys, B. & Bay-Williams, J. M. *Mathematics Teaching in the Middle School,* October 2003 – pp. 162-166.

I. Preparing for Using the Articles

Rational for use: This coherent set of articles could be used over a period of time to help teachers develop and use mathematical tasks that require high-level mathematical reasoning of their students in their classrooms. Since they all appeared in the *Mathematics Teaching in the Middle School* journal, the examples are at that level. However, the concept of creating cognitively demanding tasks that are more open than typical textbooks problems can be applied at any level.

II. Working through the Articles

The following plan starts with the first article, move to the third article in the book, and then concludes with the second article. It assumes a group of teachers is meeting regularly for 1 to 1½ hours as a professional learning community (see also the Groth article on using mathematics tasks for professional development on pp. 70-75). The leader can adjust the pace of the procedures to fit the needs of the group.

Article: *Selecting and Creating Mathematical Tasks: From Research to Practice* - Smith & Stein

Pre-reading set-up:
Make a group of cards for each 2-3 participants, putting each task in Fig. 1 on a separate card; make a copy of Fig. 2 (Levels of Demands) for each participant.

Select two opening tasks that address the same content—one that has a low cognitive demand and the other that has high cognitive demand, make projection copies and copies for each teacher. Some sample problems of this type are in article 2 and the introduction to the Smith, et al. book in the reference list below.

Questions to pose before and during the process:

1. (Overarching—prior to the opening tasks) What are the characteristics of mathematical tasks that help students develop their capacity to think, reason, and problem solve?
2. (After completing the opening tasks) What mathematics content is needed by students to solve each of the tasks?
3. (After distributing the Levels of Demands handout) Where would each of the tasks fit and why?

4. (After looking at Fig. 3) How can a task with lower-level demands be revised to become a task with higher-level demands?

Reading the Article:
- Ask Question 1 and post the participants' responses.
- Have participants do the opening tasks in pairs.
- Pose Question 2. Use think-pair-share to discuss the differences in cognitive demands these tasks would make on their students.
- Distribute the handout, Levels of Demand, and ask Question 3.
- Distribute a set of Task Cards (from Fig. 1) to groups of 2-3 participants. Ask participants to place each task in the appropriate Level of Demand.
- Read the article and discuss similarities and differences as to where the tasks were placed by participants and as suggested in the article.
- Discuss the illustrations in Fig. 3 of the same content written at different levels of cognitive challenge.
- Have each group select a different topic and write illustrative tasks for each cognitive level. Share with the entire group. (If time does not allow for this, make this an assignment to do outside of the group and to bring back to the next meeting.)

Article: *The Role of Textbooks in Implementing the Curriculum Principle and the Learning Principle* - Reys & Bay-Williams

Note that this is the third article in the book, but it is to be used second in this suggested sequence. However, the procedure below could be used in a stand-alone session.

Rationale for use: This is a short article with one pair of tasks that deal with the same content (volumes of cylinders and cones) presented at different cognitive levels in different textbooks. It could provide an opportunity to revisit the content of the first article.

Pre-reading set-up:
Participants should have access during the session to the *Principles and Standards of School Mathematics (PSSM)* (2000), either electronically or in the print version. This article, contrasted with the first article, brings in the connection to *PSSM*, especially the Curriculum Principle (pp. 14-16) and Learning Principle (pp. 20-21). In addition, make a chart of the bulleted items listed at the bottom of p. 165 and the top of p. 166. Having the Levels of Demands Chart from the previous article on hand might also be useful.

Overarching question: What criteria can be used to inform the selection of textbooks that are aligned with NCTM's Teaching and Learning Principles?

Other questions may depend on whether the group is involved in a current textbook selection process or not:
Have the teachers in your school articulated the central mathematical ideas at each grade level? How does a given textbook align with these ideas?
Can you find tasks in our textbook that have the kinds of cognitive challenge we discussed in previous sessions?
How can teachers be supported in the use of materials that focus on mathematical thinking and problem solving?

Reading the Article:
- Divide your group in two, and have each group read and report on a Principle, using one of the strategies given on p. 10 of *Growing Professionally*.

- If participants have previously worked out the tasks in Figures 1 and 2, review their work. If not, have each group work out one of the tasks and share the solution processes. Connect to the Levels of Demands Chart from the previous article, if appropriate.
- Refer to the chart of steps in choosing high-quality textbooks and pose questions appropriate to your situation (choosing textbooks or working with previously selected textbooks).

A cautionary note—regardless of the selected curriculum, teachers can make the kinds of modifications suggested in the final article so that their students are challenged to learn with understanding. This should not be a time to voice concerns about their current situation.

Article: *Turning Traditional Textbook Problems into Open-Ended Problems* - Kabiri & Smith

Please note this is the second article in this section of the book but it is to be used third in this plan. The procedure below could be used in a stand-alone session as well.

Rationale for use: This article has specific suggestions and illustrations in different content strands for turning traditional textbook problems into open-ended problems.

Pre-reading set-up:
Participants should bring their math textbooks from their classrooms with them to this PD session. Plan ahead for sharing the open-ended problem that the participants write. If they are using laptop computers, the newly written problems could be e-mailed to a group list, for example. If they are done on chart paper, the problems could be collected and typed up for distribution.

Overarching question: How can traditional textbook problems be modified so that their level of cognitive demand is increased?

Before reading the article, have participants do a “quick write” on their interpretation of what is meant by an open-ended problem. Have them share with a partner and then elicit some general sharing.

Reading the article:
- Turn to the article and note the description it gives on the first page of open-ended problems. Compare to the groups’ previous responses.
- Break into 5 groups and assign each group a different problem set (figures 1, 3, 5, 6, 7). For a large number of participants, have groups in multiples of 5 so that no group is larger than 4 people. Have each group begin by working through the problems given. Then have each group find a traditional problem in their textbook in the same content strand and develop a related open-ended problem.
- Have each group share its open-ended problem and the original textbook problem on which it is based.
- Select one of the open-ended problems that all will have their students work out. Each participant will bring representative samples of anonymous student work to the next PD session.
- At the next meeting have teachers share and review the student work for a few minutes. Chart the insights and questions they have.
- Refer back to the Teaching and Learning Principles and discuss how giving students open-ended problems relates to them.
- It might be helpful to refer to the findings on task implementation in the Smith article in the references to encourage teachers to maintain the level of cognitive challenge for their students when implementing more demanding tasks.

- Have teachers develop additional open-ended problems for a content area that they will be teaching prior to the next meeting.
- This process could be iterative, focusing on different content areas as the year progresses. The collected open-ended problems could be gathered and made available for others in the school/district.

III. Keep the Thinking Going

After each professional development session, have a brief reflection time where participants write and share about how they plan to use the information and what questions or needs they have about implementation.

After teachers have given students higher cognitive demand problems, ask: What have you learned about your students as a result of implementing these tasks? What have you learned about the mathematics you teach?

If revision of a task is suggested, ask teachers to share the revised version.

Maintain a collection of the revised open-ended tasks, organized by content area, so that they can be used in the future.

References to other NCTM Publications:

1. Smith, Margaret S., Victoria Bill, and Elizabeth K. Hughes (2008). Thinking through a lesson: successfully implementing high-level tasks. *Teaching Children Mathematics*. 14 (3), 132 - 138.

2. Stein, Mary Kay, Smith Margaret Schwan, Henningsen, Marjorie A., Silver, Edward A. (2000) *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development*.

3. Whitin, Phyllis (2004). Promoting problem-posing explorations. *Teaching Children Mathematics*. 11 (4), 180 - 186.

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Section: Classroom Discourse

Articles:

Assessment and Accountability: Strategies for Inquiry-Style Discussions. Beto, R. *Teaching Children Mathematics*, May 2004 – pp. 169-174.

Strategies for Advancing Children's Mathematical Thinking. Fraivillig, J., *Teaching Children Mathematics*, April 2001 – pp. 175-180.

Questioning our Patterns of Questioning. Herbel-Eisenmann, B. A. & Breyfogle, M. L., *Mathematics Teaching in the Middle School*, May 2005 – pp. 181-187.

Discourse that Promotes Conceptual Understanding. Kazemi, E. *Teaching Children Mathematics,* March 1998 – pp. 188-194.

I. Preparing for Using the Articles

Rationale for articles: For too long we have put talking aside and used technology and textbooks in its place. Mathematics to students, often times, is just getting the right answer. Yet it's more than that. When teachers facilitate discourse in mathematics, they provide support for students' thinking and connect everyday language with the more specific language of mathematics. When

teachers increase the discourse in mathematics, they see positive changes in students' attitudes and ability to explain and understand mathematics.

Article: *Assessment and Accountability: Strategies for Inquiry Style Discussions* - Beto

II. Working through the Article

Pre-reading questions:
Ask teachers to briefly reflect on the use of discourse in their classroom. How often is it used? Have they been successful with discourse? What has inhibited the use of discourse up to now?

Read the article.

III. Keep the Thinking Going

Reflection and Discussion questions:

- Have you tried any of the strategies discussed in the article? How did they work?
- What would be the advantages to you and to the students of using inquiry-based discussions?
- What specific challenges can you think of in trying to implement inquiry-style discussions?
- As a group, brainstorm: How can you overcome the challenges you may face in facilitating inquiry-based discussions?
- What are some considerations you need to think about when using inquiry-style discussions with children working as partners, in small groups, and as a whole group? How do you think each format works differently from one another?
- How do you think inquiry-style discussion would work in a classroom of varied ability? What might your do to differentiate?
- Obviously, an inquiry-approach takes more instructional time. What are some ways you can organize your planning to fit inquiry-style discussions into an already very busy day?
- What are two "next steps" that you can try in your classroom as a result of reading this article? Please share some artifacts (e.g., student work, etc.) at the next meeting and talk about your action plan and experiences.

Article: *Strategies for Advancing Children's Mathematical Thinking* - Fraivillig

II. Working through the Article

Pre-reading questions:
How can teachers foster problem solving skills in children? How can teachers advance children's thinking while students are engaged in mathematical inquiry?

Read the article.

1. Paragraph 3 gives an example of a closed question. Do you ask more closed questions or the other type of questions? Why do you think this is true?
2. How do you establish the environment needed for the ACT framework to succeed?
3. What new areas of growth would you anticipate seeing in your students if you implemented the ACT framework?
4. How is the idea of eliciting students' solutions method similar to what you do in language arts? Can these strategies also be effective in mathematics? Why or why not?

III. Keep the Thinking Going

Ask teachers to carefully plan for a specific lesson. They should spend time identifying the important mathematical concepts. They will need to think about misconceptions that might occur and plan for the specific talk moves they will make.

Ask participants to keep a record of how the lesson unfolded.

Reflect on the lesson giving thought to why or why it wasn't successful.

Article: *Questioning Our Patterns of Questioning* - Herbel-Eisenmann & Breyfogle

II. Working through the Article

As mentioned in the side bar of the article, before reading the text, share each of the vignettes, asking teachers to comment on the expectations of the teacher, the style of the discourse, and the opportunities for learning. Ask teachers to take on the task, described in the article, of turning a funneling example into a focusing example.

Read the article.

1. Think of a specific mathematical concept that you teach where you think you facilitate in a funneling manner. Write out an example of a dialogue.
2. Change the dialogue into one in which you are "focusing."

III. Keep the Thinking Going

Ask for volunteers to be videotaped as described in the final section of the article, "Examining Management Strategies." Use as a discussion for change.

Article: *Discourse that Promotes Conceptual Understanding* - Kazemi

II. Working through the Article

Read the article.

What do you believe about how a teacher can stimulate deep thinking and promote great conceptual understanding in mathematics?

III. Keep the Thinking Going

Explore one of the questions the author of the article listed under "Action Research Ideas". Be prepared to share your experiences with the group at the next meeting.

IV. Bringing the Collection of Classroom Discourse Articles Together

What ideas were presented in the collection of articles in this section on Classroom Discourse that will help you engage students in serious mathematical discussions in your classroom? Which ideas are you willing to try immediately? Make a plan to try one or more of these ideas in your classroom and keep a journal of your successes and challenges.

If a second meeting is held with the group, individuals can share their experiences and then update their plans to support student thinking and learning through classroom discourse.

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Section: Challenging Students

Articles:
Sometimes Less is More, Buschman, L., *Arithmetic Teacher*, March 1994 – pp. 197-200.

Isn't That Interesting! Buschman, L., *Teaching Children Mathematics*, August 2005 – pp. 201-206.

Is a Rectangle a Square? Developing Mathematical Vocabulary and Conceptual Understanding. Renee, C., *Teaching Children Mathematics*, January 2004 – pp. 207-213.

Using Counterintuitive Problems to Promote Student Discussion. Maylone, N., *Mathematics Teaching in the Middle School*, April 2000 – pp. 214-218.

I. Preparing for Using the Articles

Rationale: The four articles in this section address the issues encountered by teachers as they challenge and support students in learning mathematics. There are many decisions a teacher must make from choosing the appropriate task to facilitating the classroom discussion. All of these decisions are done to help students growing mathematically while developing the behaviors attributed to mathematicians.

Article: *Sometimes Less is More* – Buschman
The article examines how second grade students approach problems when examples are either given to follow or are not given. The variety of student characteristics evolving from each condition is listed.

II. Working through the Article

Read the article.

Facilitate discussion of the questions, recording each participants' key points on chart paper (you can also consider using the format on p. 8):

Consider your students' motivations for completing mathematics tasks. Are they focused on finding answers or making sense of mathematical ideas? What can you do to encourage students to reflect on their own thinking as well as on mathematical structures?
If problem solving is the goal of lesson, do competed examples hinder student thinking?

Discuss the following questions as a group:
The author gives two lists of student behavior to describe when students were not given a solution example to follow and when given a solution example to follow. With which student behavior do you agree or disagree? Why? What characteristics can you add to either list?
What set of tools and strategies have your students developed to help them solve unfamiliar problems?

How do you encourage students to take intellectual risks in the mathematics classroom? In what ways can you tell whether students are engaged in genuine problem solving rather than completing routine exercises?

III. Keep the Thinking Going

Replicate the experiment with problem(s) decided on by the group. What student behaviors

matched those on the lists from the article? How does this impact your thinking about the teaching and learning of mathematics?

Challenge students to solve an open-ended problem in as many ways as possible. Did they use as many methods as you would like? How will you enhance their problem solving repertoire?

Article: *Isn't That Interesting!* – Buschman
The article expects students to solve complex problems in ways that make sense to them. The dilemma for teachers is how and when to intervene.

In advance of the session, prepare a handout for the participants with the following problems from the article: Pencil Problem (p. 210), Circus Problem (p. 202), Ghost Problem (p. 203), and Age Problem (p. 204).

II. Working through the Article

Read the article.

Assign pairs of participants to solve one of the problems from the handout in various ways that students might approach the problem, also noting difficulties/barriers students might have with the problem. Share each group's work with the large group.

Discuss the following questions as a group:
How do prior discussions relate to the students' work in the article?
What surprised you?
What concerns did the article elicit?
When is direct instruction used in problem solving?
When are leading questions used in problem solving? What leading questions help students in problem solving?
What decisions must teachers consider when having students present their solutions to the rest of the classroom?

III. Keep the Thinking Going

Reflect on the following questions as they impact your classroom:
In what ways are **all** students in the classroom being challenged with mathematical tasks that maintain their interest and help them see a variety of mathematical perspectives?
What obstacles might prevent you from holding high expectations for all students?
How do you work to overcome those obstacles?

Article: *Is a Square a Rectangle?* – Renee
The article suggests ways to challenge students' understanding while facilitating discourse.

II. Working through the Article

Each pair/group of participants will need highlighters and sticky notes/note cards.
Each participant reads the article, highlighting essential points.

Use technique "Significant SenTENces" (p. 10) to discuss article.
In pairs, select ten essential points that were highlighted be either one or both of the partners.
Each essential point is paraphrased onto a sticky note/note card. These cards are prioritized.

Make a larger group by combining pairs. The groups report similarities and differences in the essential points listed on the sticky notes/note cards.

Discuss the following questions as a group:

Limitations, decisions, missed opportunities, and mathematical discourse are part of any classroom. Discuss how the article talks about these practices.

How do these practices impact your classroom?

III. Keep the Thinking Going

Design an action research opportunity to investigate how well students in your classroom are being challenged in mathematics.

Article: *Using Counterintuitive Problems to Promote Student Discussion* – Maylone

The article offers six examples of challenging tasks that lead to rich classroom discussion.

Prepare a handout with five problems from the article: Busy Lawyers (p. 215), Why Isn't Pat Broke? (p. 216), The Raffle (p. 216), Rolling the Block (p. 217), and The Careless Cat (p. 217).

II. Working through the Article

Assign each group of participants one of the five problems from the handout. Each group is to prepare a poster displaying approaches to solving their given problem along with possible confusion points encountered in the problem.

Display the posters and conduct a gallery walk so that everyone has an opportunity to view the problem, solution, and confusion points of all five problems. Discuss comments as they relate to the posters.

Read the article. Each group compares the discussion from the article with their poster.

Discuss how to facilitate the classroom discourse of each problem with their students.

Reflection question to summarize session:
If students are to be encouraged to struggle with and make sense of mathematical ideas, how should that focus be reflected in classroom discourse?

III. Keep the Thinking Going

Collect other problems that promote momentary confusion and dissonance for students.

Plan a collaborative lesson plan with the participants using one of the problems from the article. Concentrate on how to conduct the classroom discourse. After implementing the lesson plan, reflect on its impact in the classroom.

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Section: Deepening Understanding

Articles:

Relational Understanding and Instrumental Understanding, Skemp, R., *Mathematics Teaching in the Middle School*, September 2006 (reprint) – pp. 221-230.

Learning Strategies for Addition and Subtraction Facts: The Road to Fluency and the License to Think, Buchholz, L., *Teaching Children Mathematics*, March 2004 – pp. 231-236.

Using Language and Visualization to Teach Place Value. Cotter, J., *Teaching Children Mathematics*, October 2000 – pp. 237-245.

Meaning and Skill - Maintaining Balance. Brownell, W. A., *Teaching Children Mathematics*, February 2003 (reprint) – pp. 246-254.

Multiplying Fractions. Wu, Z., *Teaching Children Mathematics*, November 2001 – pp. 255-259.

Developing Algebraic Reasoning through Generalization. Lannin, J. K., *Mathematics Teaching in the Middle School*, March 2003 – pp. 260-267.

Three Balloons for Two Dollars: Developing Proportional Reasoning. Langrall, C. W. & Swafford, J., *Mathematics Teaching in the Middle School*, December 2000 – pp. 268-276.

I. Preparing for the Using Articles in this Section

Rationale for Use: This set of seven articles was chosen to help teachers understand the difference between knowing and understanding, and that the development of a deep understanding of mathematics facilitates learning, increases students' abilities to transfer knowledge, and improve the chances that knowledge is retained.

Article: *Relational Understanding and Instrumental Understanding*— Skemp

II. Working through the Article

Have the participants discuss the difference between relational and instrumental understanding. Are there other terms we use to describe relational understanding? Instrumental understanding?

Come up with some examples of relational and instrumental teaching in math classrooms. Which type do you use in your classroom?

Discuss when relational understanding is necessary, and when instrumental understanding is sufficient.

Does it matter that we have two types of understanding? Is one kind better than the other?

Since we have these two differing types of understanding, it is conceivable that a mismatch could possibly arise in the classroom. What happens when a mismatch arises? Is one type of mismatch more detrimental than another?

How can we determine if a student has attained relational understanding of a concept?

How do the findings of the TIMSS Studies relate to this article?

III. Keep the Thinking Going

Gather and review your school/district data. Analyze the assessments and data to determine whether or not items appropriately assess relational and instrumental understanding. How do the findings of your school/district data relate to this article?

Article: *Meaning and Skill - Maintaining Balance* — Brownell

II. Working through the Article

Break participants into groups to discuss these questions:
What is the purpose of teaching arithmetic in elementary school?
How does Brownell define the dilemmas in meaningful arithmetic instruction?

Are there any similarities between this article and the Skemp article?
Explain the difference between habituation and meaningful habituation.
Describe the differences between repetitive and varied practice in both what the learner does and produces.
What does Brownell suggest be done to correct the upset of balance in meaning and skill?

III. Keep the Thinking Going

Via grade-level teams, review an arithmetic lesson in your curriculum materials. Engage in discourse about whether or not it adequately balances meaning and skill. If not, what adaptations or modifications would you need to make so that meaning and skill are well balanced?

Article: *Multiplying Fractions* – Wu

II. Working through the Article

Discuss the following questions:
What are some of the problems that can arise when multiplication of fractions is taught using only the repeated addition model?
What are some of the problems that can result from using pictorial representations for fractional word problems?
Why is the concept of the "whole" confusing when teaching fractional concepts?

III. Keep the Thinking Going

Discuss how you might help students develop a coherent understanding of real-life context problems involving fractions. Design a mini-lesson on a particular fraction concept situated in a real-life context.

Article: *Learning Strategies for Addition and Subtraction Facts: The Road to Fluency and the License to Think* — Buchholz

II. Working through the Article

Ask participants to describe in writing their own journey for helping students with the acquisition of the basic addition and subtraction facts. Next, have participants compare their journey to Buchnolz's journey.

Via whole group, ask participants to individually reflect on the experiences that shaped their beliefs about students internalizing and memorizing facts such as addition and subtraction?

Next, discuss the following questions:
What does it really mean to internalize and memorize facts?
How do we help students with the learning process for internalization and memorization?
Why do we really care that students learn their facts?
Why does it matter that students learn their facts?

III. Keep the Thinking Going

If we believe that computational fluency and some level of atomicity of addition and subtraction facts are important, what are the essential strategies for learning them? Generate a list of essential strategies for learning facts with fluency and atomicity. Compare how these strategies correlated to your school/district learning standards and/or curriculum materials.

Article: *Using Language and Visualization to Teach Place Value* – Cotter

II. Working through the Article

In whole group, randomly assign new names to the numbers one through ten. For example, (1) Plop, (2) Bang, (3) Pizzazz, (4) Rump, (5) Loop, (6) Hobble, (7) Messing, (8) NeNe, (9) Ohio and (10) Seize. Introduce this naming system for numbers. Ask participants to repeat after you or count to 10 themselves using the new system. Note: Most participants will struggle with this activity. Now hold up 3 fingers and ask participants to name the specific number being shown. Continue on with other numbers. This activity will help participants see the complexity of the language of numbers and more particularly in the way in which we name our numbers through the lens of young children.

Next, begin a discussion with the following questions:
How might we better facilitate students making sense of our base 10 numbers naming system?
What [other types] of learning strategies might better help students make sense of our number naming system?
What learning strategies do you already use to help your students make sense of our base 10 naming systems? How does this strategy relate to the Cotter's research?
How can we help students recognize and use patterns to understand our number system?

III. Keeping the Thinking Going

"Young children are capable of adding and subtracting and performing other mathematical tasks before they develop accurate counting skills. Counting need not be the basis of arithmetic" (p. 244). Discuss what might inhibit some teachers from giving children flexibility to explore addition and subtraction concepts without having solidified counting skills.

Article: *Developing Algebraic Reasoning through Generalization* – Lannin

II. Working through the Article

Give an example of a simple mathematical situation (number of people, number of eyes) that can be generalized. Via whole group, ask participants to make generalizations about the situation. Next identify a list of attributes that might constitute an efficient and valid generalization for any mathematical situation. Be sure to discuss what characteristics are important to facilitate generalization.

Ask the participants to complete the Cube Sticker Problem if possible without using any algebra. Encourage participants to try several different strategies. Post the various strategies and determine/label the type of strategy (Counting, Recursion, Whole-Object, Contextual, Guess and Check, Rate-Adjust) applied. Decide whether or not the strategies are efficient and valid for making generalizations. In pairs/groups, choose a strategy, describe it in written words, translate the written words to meaningful symbols and then translate the symbols into a rule.

III. Keep the Thinking Going

As a whole group, discuss how to help students link their rules to the context of a mathematical situation.

Article: *Three Balloons for Two Dollars: Developing Proportional Reasoning* – Langrall & Swafford

II. Working through the Article

Prior to reading and discussing this article for professional learning, distribute the Three Balloons for Two Dollars problem to the participants. Ask them to try the problem with their students. Note:

It would be great if this problem can be seamlessly integrated into the study of proportional reasoning. Ask participants to bring sample student solution strategies. In small groups, the participants will discuss and assess student solution strategies categorizing them as Level 0, Level 1, Level 2 or Level 3.

Next review and discuss the four different types of proportion problems (e.g. Part-Part-Whole; Associated Sets, Well-Known Measures, and Growth). Next, distribute sample proportion problems in a unit of study and ask participants to sort them by type.

Discuss the four essential components of proportional reasoning using the guiding questions below:

Discuss the underlying concepts and skills [or fundamental components] for developing proportional reasoning in elementary mathematics.

- How can we help students recognize the difference between absolute and relative change?
- How can we help students know when does it make sense to use a ratio?
- Describe the difference between covariant and variant. Explain the relationship of invariant and covariant. Give examples.
- Explain the unitizing process.

Discuss how proportional reasoning builds the foundation for high school mathematics. Give some examples across the content topics situated in real-life context (e.g. Algebra).

How can we help students make connections and transitions from using informal non-proportional reasoning?

What would this transition look like in the scope and sequence of curriculum and instruction in your classroom?

III. Keep the Thinking Going

With a partner, design an overview for a mini-unit of study for developing proportional reasoning. Be sure to include a variety of types of proportional reasoning as well as the four components essential for proportional reasoning.

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Section: Misconceptions

Articles:

Balancing Act: The Truth Behind the Equals Sign. Mann, R. *Teaching Children Mathematics*, September 2004 – pp. 279-283.

Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them. Baroody, A., *Teaching Children Mathematics*, August 2006 – pp. 284-294.

The Harmful Effects of Algorithms in Grades 1 – 4. Kamii, C. & Dominick, A. Chapter 17 of *The Teaching and Learning of Algorithms in School Mathematics: 1998 Yearbook of the NCTM*, edited by Morrow, L. – pp. 295-304.

Mean and Median: Are They Really So Easy? Zawojewski, J. & Shaughnessy, M. *Mathematics Teaching in the Middle School*, March 2000 – pp. 304-308.

I. Preparing for Using the Articles in this Section

Pre-reading activity:
Think-pair-share to discuss the following questions and share with the larger group:
What are misconceptions? Provide examples from your classroom experience.
How did you address the misconceptions mentioned in answer to the first question?
Do only students hold misconceptions? Do teachers or families hold misconceptions that might impact their own understanding of mathematical concepts? If so, how might this impact student learning?

II. Working through the Articles

Reading the articles:
Use a jigsaw to have participants read the articles.

- Divide the teachers into groups of 4 and have them number themselves 1 – 4. All the #1 teachers will read the article: Balancing Act: The Truth Behind the Equals Sign; the #2 teachers will read the article: Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them; the #3 teachers will read the article: The Harmful Effects of Algorithms in Grades 1 – 4:; the #4 teachers will read the article: Mean and Median: Are They Really So Easy?.
- All the teachers who read the same article will discuss the article in terms of the following questions:
    - What misconceptions were addressed in the article?
    - How did these misconceptions impact student learning?
    - How did the teacher use this as an opportunity for learning?
- Have teachers return to their original grouping to share a brief summary of their article.

Each article provides some exemplars of ways teachers identify student misconceptions and use them as teaching moments. These articles also focus on a number sense view to teaching mathematics and how it enables students to achieve success when conventional instruction did not. As a group, discuss the following questions and record participants' answers on chart paper:

- How does a number sense view differ from conventional instruction?
- How did a good sense of number enable students to overcome misconceptions in each of the articles?
- What are the implications for teachers who are still relying on conventional wisdom to guide their teaching?
- How might you use the research from these articles to answer the questions, "What can I do to help my students succeed in learning mathematics?"
- How did the teachers in these articles use student work as a lens on number sense?

Summarize the Discourse:
Two possible ways to summarize the discourse follow:
Gallery Walk (see p. 12): Form new groups consisting of one participant from each of the three groups above. Groups physically move from chart to chart with the person whose article is on the chart taking 3 minutes to summarize the discussion around the article and then 2 minutes for the other members to respond and ask questions. Groups proceed through all three charts. Facilitator takes notes on important ideas that arise during discussions and closes with some whole group discussion.
Report Out: Each group takes 3 minutes to summarize the information on their chart to the whole group. Allow two minutes for the rest of the group to ask clarifying questions.

III. Keeping the Thinking Going

Have teachers collect examples of student work as exemplars of student misconceptions (also see article on student work samples on pp. 26-32). Ask participants to bring the student work samples to a second meeting to share with the group in terms of:

- What misconception did the student have?
- How did you (the teacher) use this as an opportunity to improve instruction?

Note: You may wish to do the sharing through a share point site rather than having a second meeting.