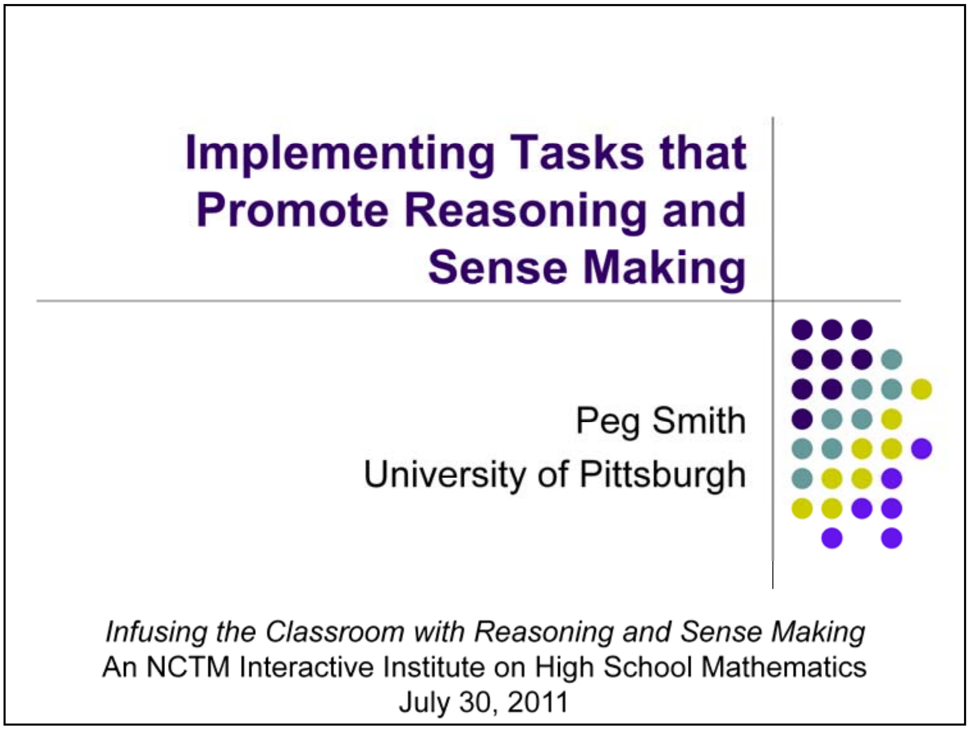


So for the last day and a half you have been been thinking about tasks that promote reasoning and sense making and how these are different in kind from the tasks you have been using in your textbooks.

What I want to talk with you about today is how to implement these tasks in a way that ensures that students really get a chance to engage in the habits of reasoning that we want them to learn.

The key to this to to plan for and facilitate a discussion that builds on students thinking and makes salient the mathematical ideas that are at the heart of the lesson.

The Five Practices (+)

1. **Anticipating** (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998)
2. **Monitoring** (e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)
3. **Selecting** (e.g., Lampert, 2001; Stigler & Hiebert, 1999)
4. **Sequencing** (e.g., Schoenfeld, 1998)
5. **Connecting** (e.g., Ball, 2001; Brendehur & Frykholm, 2000)

The five practices are

•**Anticipating** likely student responses to mathematical tasks

•**While students working on the tasks (in pairs or small groups), Monitoring** students' actual responses to the tasks

•**Selecting** particular students to present their mathematical responses during the whole class discussion

•Purposefully **sequencing when** these student responses are shared during the discussion

•Helping the class make mathematical **connections** between different students' responses

As you can see,

each of these has been discussed separately by various authors;

our contribution here is to integrate them into a single package.

The Five Practices (+)

0. Setting Goals and Selecting Tasks
1. **Anticipating** (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998)
2. **Monitoring** (e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)
3. **Selecting** (e.g., Lampert, 2001; Stigler & Hiebert, 1999)
4. **Sequencing** (e.g., Schoenfeld, 1998)
5. **Connecting** (e.g., Ball, 2001; Brendehur & Frykholm, 2000)

01. Setting Goals

- **It involves:**
  - Identifying what students are to know and understand about mathematics as a result of their engagement in a particular lesson
  - Being as specific as possible so as to establish a clear target for instruction that can guide the selection of instructional activities and the use of the five practices
- **It is supported by:**
  - Thinking about what students will come to know and understand rather than only on what they will do
  - Consulting resources that can help in unpacking big ideas in mathematics
  - Working in collaboration with other teachers

Hiebert, Morris, Berk, and Jansen (2007, p.51) argue that this level of specificity is critical to effective teaching:

Without explicit learning goals, it is difficult to know what counts as evidence of students' learning, how students' learning can be linked to particular instructional activities, and how to revise instruction to facilitate students' learning more effectively. Formulating clear, explicit learning goals sets the stage for everything else.

Nick Bannister could say – finding point of intersection of a linear function. But really doesn't get at the understanding he wants students to develop.

Nancy Edwards's Goals

She wants her students to:

1. Realize that examples are not enough to show that a claim is always true.
2. Recognize that there are many *different* ways to prove that a claim is true and that it is not the form that matters but rather the consideration of all cases and creation of a clear and logical argument.
3. Understand that there are reasons WHY mathematics works the way it does that can be explored and explained.

Page 2 of packet

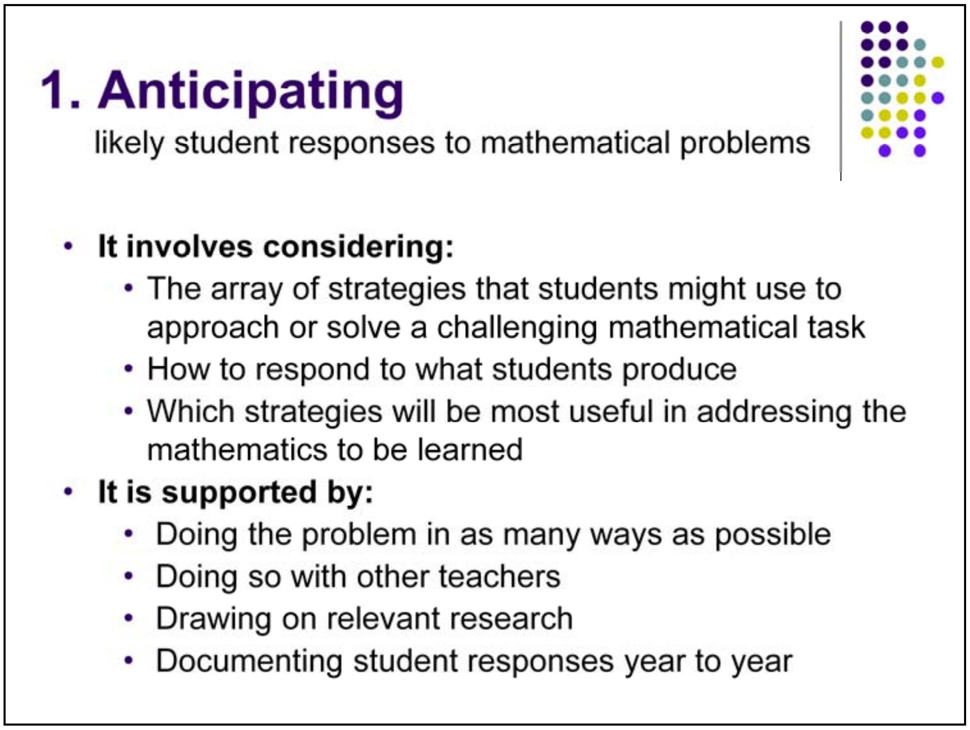
0_2. Selecting a Task

- **It involves:**
  - Identifying a mathematical task that is aligned with the lesson goals
  - Making sure the task is rich enough to support a discussion (i.e., a cognitively challenging mathematical task)
- **It is supported by:**
  - Setting a clear and explicit goal for learning
  - Working in collaboration with colleagues

Nancy Edward's Task

Prove that the sum of two odd numbers is always even.

I pick a simple task for us to consider, because I don't want us to get too tangled in the mathematics.



The first practice is for teachers to make an effort to actively envision how students might mathematically approach the instructional task (s) that they will be asked to work on. This involves much more than simply evaluating whether a task will be at the right level of difficulty or of sufficient interest to students, and it goes beyond considering whether or not they are getting the 'right answer.'

[Click to INVOLVES]

Anticipating students' responses involves developing considered expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn.

[

What Nancy Edward's Thinks Her Students Would Do

- Draw a picture
- Build a model
- Create a logical argument
- Use algebra
- Use examples and not know how to move beyond them
- Use the same variable in the representation of each odd number

2. Monitoring

students' actual responses during independent work

- **It involves:**
  - Circulating while students work on the problem and watching and listening
  - Recording interpretations, strategies, and points of confusion
  - Asking questions to get students back "on track" or to advance their understanding
- **It is supported by:**
  - anticipating student responses beforehand
  - Using recording tools

[Click INVOLVES]

Monitoring student responses involves paying close attention to the mathematical thinking that students actually use as they work on the problem. Commonly, this is done by circulating around the classroom while students work .

Lampert summarizes it – "If I watch and listen during small-group independent work, I am able to use my observations to decide what and who to make focal" during the discussion that follows

Monitoring Tool

List the different solution paths you anticipated

Strategy	Who and What	Order
Picture or model		
Logical argument		
Algebra		
Examples		
Confusing Variables		

Monitoring Tool

Strategy	Who and What	Order
Picture or model		
Logical argument		
Algebra		
Examples		
Confusing Variables		

Make note of which students produced which solutions and what you might want to highlight

The Case of Nancy Edwards

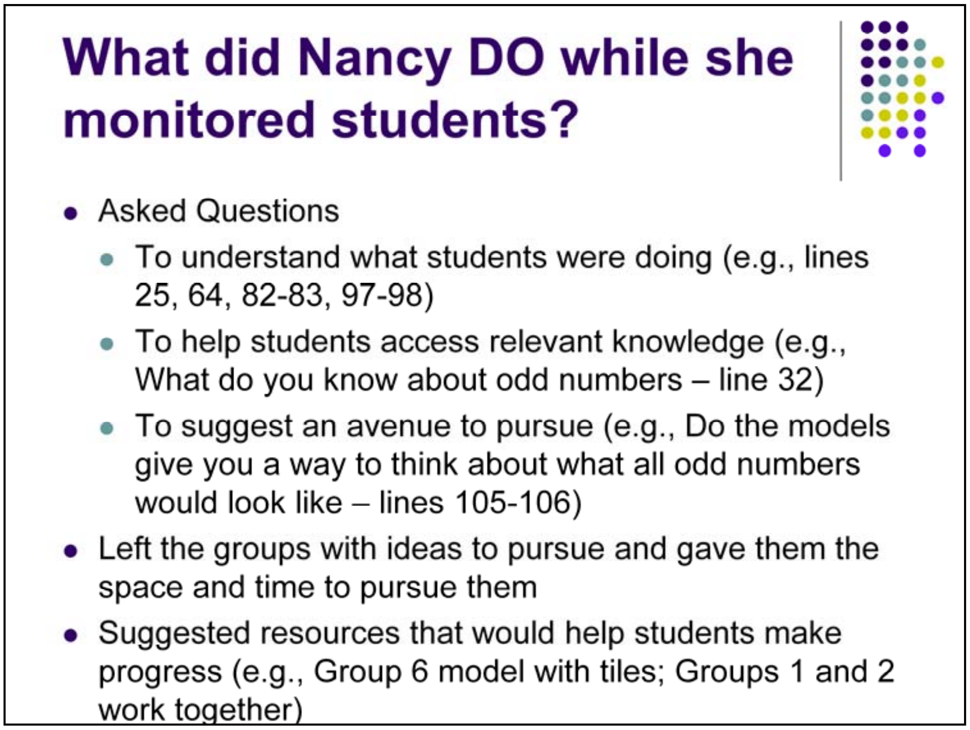
- Imagine that you are Nancy Edwards and that you are monitoring students as they work on the task.
- As you read the case (lines 1-115 only), use your monitoring sheet to makes notes on the data you would collect while students were working in their groups.
- Also note what Nancy does to support the development of her students' habits of reasoning while she is monitoring.

Monitoring Tool

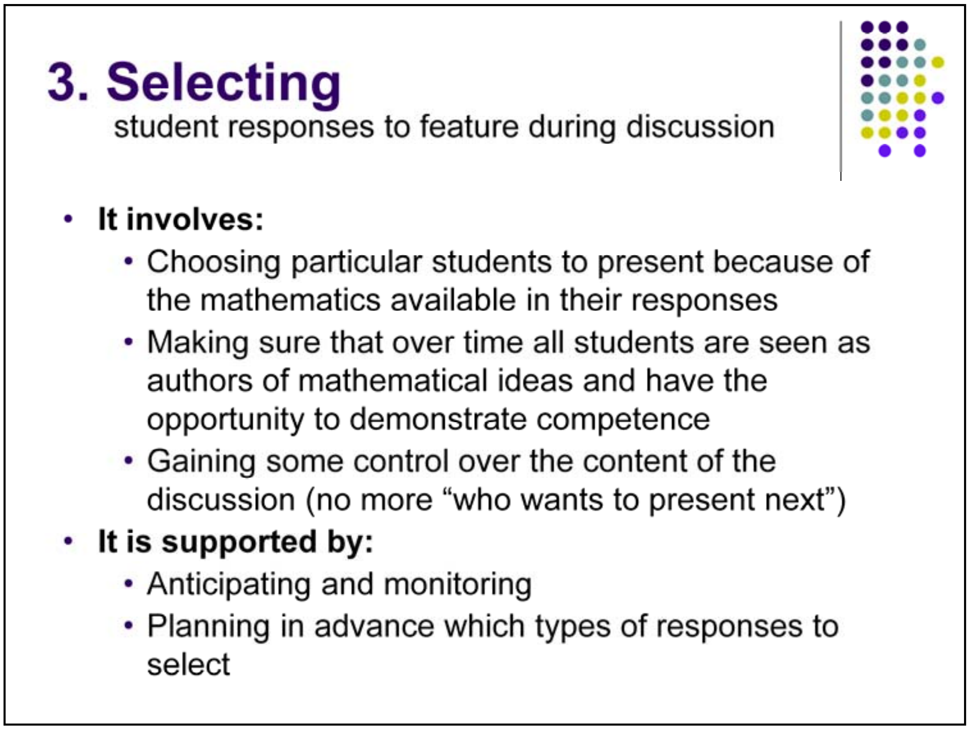
Strategy	Who and What	Order
Picture or model	• Group 1 – picture showing 1 example with 1 leftover • Group 6 – started with examples and went to model with tiles Neither group explained WHY it always works	
Logical argument	Group 2 made an argument based on each number having a loner number (the 1 that is left over when you divide a even # by 2)	
Algebra	• Group 3 used 2x+ 1 and 2a + 1 to represent two odd numbers • Group 5 used even and even + 1 as variables	
Examples	Group 6 started here and add a model but still has not identified the underlying structure of why this works	
Confusing Variables	Group 4 used the same variable twice – 2x + 1 and 2x + 1	

The chart serves as a record that can be used for a variety of purposes. First, the chart provides a record of who is doing what that can help a teacher keep track of the approaches that are available within the classroom and serve as a data source for making judgments about who will share what during the discussion. Second, the chart can help the teacher keep track of how students in the class are thinking about particular ideas and which students were selected to share their work with their peers on a particular day. Finally, the chart provides a historical record of what happened during the lesson that can aid the teacher in refining the lesson the next time is taught.

Data provide a vivid picture of where the class is.



Perhaps as important as what she did do is what she did not do. She did not tell students what to do or how to do it. She did not take over the thinking for them.



[Click INVOLVES]

Selecting is about determining what math students will have access to beyond what they were able to consider individually or in small groups.

It is about both WHO and WHAT to make focal.

Having monitored the available student strategies in the class, the teacher can then select particular students to share their work with the rest of the class in order to get "particular piece[s] of mathematics on the table," thus giving the teacher more control over the discussion as well as more time to plan

A typical way to do this is to call on specific students (or groups of students) to present their work as the discussion proceeds. Alternatively, the teacher may let students know in advance of the discussion that they will be presenting their work.

[If time] In a hybrid variety, a teacher might ask for volunteers but then select a particular student that he or she knows is one of several who has a particularly useful idea to share with the class. This is one way of balancing the tension between "keeping the discussion on track and allowing students to make spontaneous contributions that they consider…to be relevant."

[Click SUPPORTED and quickly say what's there]

4. Sequencing

student responses during the discussion

- **It involves:**
  - Purposefully ordering presentations so as to make the mathematics accessible to all students
  - Building a mathematically coherent story line
- **It is supported by:**
  - Anticipating, monitoring, and selecting
  - During anticipation work, considering how possible student responses are mathematically related

[Click INVOLVES & SUPPORTED BY]

Having selected particular students to present, the teacher can then make decisions about how to sequence the students' presentations. By making purposeful choices about the order in which students' work is shared, teachers can maximize the chances that their mathematical goals for the discussion will be achieved.

- For example, the teacher might want to have the strategy used by the majority of students presented before those that only a few students used in order to help validate the work that students did and make the beginning of the discussion accessible to as many students as possible.

- Or, if there is a common misconception that underlies a strategy that several students used, the teacher might want to have it addressed first so the class can clear up that misunderstanding in order to be able to work on developing more successful ways of tackling the problem.

- In addition, the teacher might want to have related or contrasting strategies be presented right after one another in order to make it easier for the class to mathematically compare them.

Monitoring Tool

Strategy	Who and What	Order
Picture or model	• Group 1 – picture showing 1 example with 1 leftover • Group 6 – started with examples and went to model with tiles Neither group explained WHY it always works	1st – explain what they did
Logical argument	Group 2 made an argument based on each number having a loner number (the 1 that is left over when you divide a even # by 2)	2nd – connect with Group 1
Algebra	• Group 3 used 2x+ 1 and 2a + 1 to represent two odd numbers • Group 5 used even and even + 1 as variables	4th- how does it relate to 1 &2 3rd – how does this relate to Group 1 and 2
Examples	Group 6 started here and add a model but still has not identified the underlying structure of why this works	HW-does this count as a proof? What could you add?
Confusing Variables	Group 4 used the same variable twice – 2x + 1 and 2x + 1	Do you need two different variables?

[Click INVOLVES]

Finally, teachers can help students draw connections between their Connecting may in fact be the most challenging of all of the five practices because it is here that the teacher must craft questions that will make the mathematics visible and understandable. Hence the questions must go beyond just clarifying and probing what individual students did and how; they must focus on mathematical meaning and relationships and make links between mathematical ideas and representations.

So, rather than having mathematical discussions consist of separate presentations of different ways to solve a particular problem, the goal here is to have student presentations build on each other to develop powerful mathematical ideas.

[Click SUPPORTED] Such connecting is supported by….

The Case of Nancy Edwards

Continue reading the case (lines 120 - 203) and consider:

- What connections got made?
- What did Nancy do to facilitate the discussion that goes beyond the five practices?

Connections

- All of the solutions were connected to each other and key ideas of number theory (e.g., an even number is divisible by 2, can be represented algebraically as 2n, pictorially as a 2 by n rectangle, or sets of two).
- Homework question 3 gets directly at what counts as a proof and the limitation of examples.

What Else Does Nancy DO?

- Encouraged students to question each other (lines 131)
- Asked students to relate their work to the work of other students in the class (lines 143-144)
- Created homework that require students to make sense of the arguments and reasoning of others (lines 180-185)
- Established a classroom climate in which students felt comfortable sharing their arguments and critiquing the arguments of others in a productive manner.

Page 11 of reasoning and sense making, we see that Nancy does many of the things that are listed as helping develop reasoning habits in the classroom.

A class room discussion is a key part of creating such a classroom and the five practices should help give you some leverage on that.

So What's the Point?

- Good tasks are a critical starting point for developing students capacity to reason, but tasks alone are not enough. The teacher needs to support students work on tasks *without taking over the thinking for them*.
- The 5 practices provide a model that is likely to help by providing a set of activities in which teachers can engage before and during the lesson that will help them use students thinking to advance their mathematical understandings.

By focusing on planning in advance the 5 practices leave less in the moment decision making.

Resources Related to the Five Practices

- Smith, M.S., Hughes, E.K., & Engle, R.A., & Stein, M.K. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School, 14* (9), 549-556.

- Stein, M.K., Engle, R.A., Smith, M.S., & Hughes, E.K. (2008).Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. *Mathematical Thinking and Learning, 10,* 313-340.

- Smith, M.S., & Stein, M.K. (2011). *Orchestrating Mathematical Discussions*. National Council of Teachers of Mathematics.

The book also has a chapter on questioning and a chapter on accountable talk that you might find help as you continue to think about how to support students and hold them accountable.

For additional information, you can contact me at

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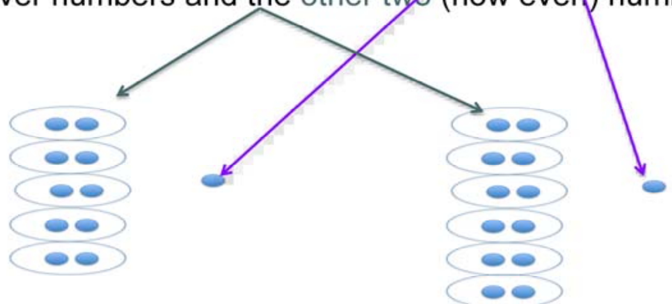
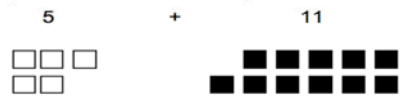
Draw a Picture

Every odd number (like 11 and 13) has one left over when you group by 2. Add the groups of two and you will get an even number (24). Now add all together the leftover numbers and the other two (now even) numbers.

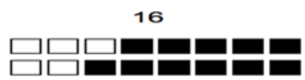


Build a Model

If I take the numbers 5 and 11 and organize the counters as shown, you can see the pattern.



You can see that when you put the sets together (add the numbers), the two extra blocks will form a pair and the answer is always even. This is because any odd number will have an extra block and the two extra blocks for any set of two odd numbers will always form a pair.



Logical Argument

An odd number = an even number + 1. e.g. 9 = 8 + 1

So when you add two odd numbers you are adding an even no. + an even no. + 1 + 1. So you get an even number. This is because it has already been proved that an even number + an even number = an even number.

Therefore as an odd number = an even number + 1, if you add two of them together, you get an even number + 2, which is still an even number.

Use Algebra

If a and b are odd integers, then a and b can be written $a = 2m + 1$ and $b = 2n + 1$, where m and n are other integers.

If $a = 2m + 1$ and $b = 2n + 1$, then $a + b = 2m + 2n + 2$.

If $a + b = 2m + 2n + 2$, then $a + b = 2(m + n + 1)$.

If $a + b = 2(m + n + 1)$, then $a + b$ is an even integer.