
An International Perspective on Mathematics Education:

Reflections on ICME-11

Preface

The travel grant to the National Council of Teachers of Mathematics from the National Science Foundation (Award ID 0714890) to support attendance at the Eleventh International Congress on Mathematical Education held in Monterrey, Mexico, was supported by the Mathematical Association of America (MAA), the American Mathematical Society (AMS), and the American Mathematical Association of Two-Year Colleges (AMATYC).

The Steering Committee consisted of Martha Siegel (MAA), William McCallum (AMS), Richelle (Rikki) Blair (AMATYC), Francis (Skip) Fennell (United States National Commission on Mathematics Instruction), Cindy Chapman (NCTM), and two members-at-large, Natalie Jakucyn and Patrick Scott. From the 413 applications, the Steering Committee selected 62 awardees, including 10 graduate students (from both mathematics and mathematics education), 5 community college instructors, 3 university mathematicians, 13 university teacher educators, 14 K–12 teachers (including 4 elementary school teachers), and 10 others, including curriculum specialists, coaches, and professional development providers. A list of the awardees appears in the introduction.

As part of the travel grant, John Dossey, assisted by Sharon McCrone and Katherine Halvorsen, prepared a report, *Mathematics Education in the United States, 2008: A Capsule Summary Fact Book Written for ICME-11* (2008), and 2500 copies were disseminated during the Congress. The report can be downloaded from <http://www.nctm.org/about/affiliates/content.aspx?id=16955>.

On behalf of the Steering Committee, thank you to the project organizer, Margaret Iding, Michigan State University, whose attention to detail and to the needs of the awardees was phenomenal, making the experience valuable and rewarding for everyone. Thank you to the awardees for their reports and cooperation in getting information to their group leaders and for the many presentations they gave to their peers and other interested leaders in mathematics education in the United States about the Congress and what they learned. I would also like to thank the project evaluator, Steven Ziebarth, associate professor of mathematics, Western Michigan University, for following up with the awardees and providing valuable advice for future travel grants. And finally, thank you to Ginny Lambert for her patient and thorough work in editing the chapters for this volume and to David Barnes from NCTM for facilitating the publication.

Gail Burrill
Principal Investigator
ICME-11 Travel Grant

Introduction

The International Congress of Mathematics Education (ICME) is held every four years under the auspices of the International Commission on Mathematical Instruction (ICMI). The Congress is a forum for mathematics educators from around the world to exchange ideas, information, and viewpoints and develop productive dialogue with their peers (www.icme11.org). In Monterrey, Mexico, July 6–13, 2008, more than 2000 educators from more than 90 countries attended ICME-11.

The National Council of Teachers of Mathematics (NCTM) obtained a grant from the National Science Foundation to help fund travel to the Congress for mathematics educators from the United States, including K–12 teachers, curriculum and district mathematics specialists, two- and four-year college and university mathematics educators, mathematics education researchers, graduate students, and mathematicians.

Teams of grantees were formed to focus on specific themes identified as areas of interest for mathematics education in the United States. The teams met during the conference, attended targeted sessions that dealt with their particular theme, and after the Congress, reported what they learned to their peers in the United States through talks at conferences and articles.

The findings from the theme groups are the basis for the first six chapters in this document. The last two chapters are articles written by members of two of the theme groups based on learning and discussions at the Congress.

The themes and group members were as follows:

Mathematics Education of Second-Language Learners

Group Leader: Patrick Scott, New Mexico Department of Education

Members:

Judy Ackerman, Montgomery College, MD

Evelyn Gordon, University of North Carolina

Gary Lewis, Illinois State University

Noemi Lopez, Harris County Department of Education, TX

Sailaja Suresh, Oakland International High School, Oakland, CA

Deby Valadez, Holiday Park School, Cartwright/Peoria, AZ

Alison Whittington, Chicago Public Schools

Craig Willey, University of Illinois at Chicago

Educating Students from Diverse Cultures

Group Leader: Cindy Chapman, Albuquerque Public Schools, NM

Members:

Rita Barger, University of Missouri—Kansas City

Edwin Dickey, University of South Carolina

Saul Duarte, Los Angeles Public Schools, CA

Guillermo Mendieta, Meaningful Learning, Fontana, CA

Jill Newton, Michigan State University

Hoa Nguyen, Florida State University

Jennifer Weisbart-Moreno, Claremont University, CA

Relationship between Research and Practice

Group Leader: Richelle (Rikki) Blair, Lakeland Community College

Members:

Heidi Berkenbosch, Prairie City Public Schools, IA

Hector Colon-Rosa, Inter American University of Puerto Rico

Susan Forman, Bronx Community College, NY

Crystal Hill, University of North Carolina

Kerry McKee, New Mexico State University

Chris Rasmussen, San Diego State University

Elizabeth (Fanny) Sosenke, Brooklyn Friends School, NY

Information and Communication Technology

Group Leader: Natalie Jakucyn, Glenbrook South High School, IL

Members:

Steven Blasberg, West Valley College, CA

William M. Carroll, Saint Ignatius College Prep, Chicago, IL

Jeffrey Choppin, Warner School of Education, University of Rochester, NY

John Mahoney, Banneker Academic High School, Washington, DC

Marilyn Mays, Dallas County Community College District, TX

Jodie Miller, Morristown-Beard School, NJ

Laura Watkins, Maricopa County Community College, AZ

Professional Development

Group Leader: Gail Burrill, Michigan State University

Members:

Jose Cedeno, Ranchito Avenue Elementary School, Simi Valley, CA

Carrie Chiappetta, Stamford Public Schools, CT

Cheryl Cleaves, State Technical Institute at Memphis, GA

Beth Herbel-Eisenmann, Michigan State University

Margie Hobbs, University of Mississippi

Linda Maples, Earle School District, AR

Li-Ming Yang, Education Development Center, Newton, MA

Rethinking the Curriculum in Primary and Secondary Mathematics Education

Group Leader: Martha Siegel, Towson University, MD

Members:

Susan Carter, University City Public Schools, MO
Mairead Green, Rockhurst University, MO
Cresenda Jones, Holmdel Township Public Schools, NJ
Glenda Lappan, Michigan State University
Aaron Large, Thurgood Marshall Middle School, Temple Hills, MD
Rheta Rubenstein, University of Michigan—Dearborn
Richard Seitz, Helena Public Schools, MT

The Role of Universities in Preparing Mathematics Teachers

Group Leader: William McCallum, University of Arizona

Members:

Efraim Armendariz, University of Texas
Jerry Becker, Southern Illinois University
Yvonne Lai, University of California at Davis
Angela Mentges, Sidney Middle School, OH
Annie Selden, New Mexico State University
Hortensia Soto-Johnson, University of Northern Colorado
Kazuko West, Keio Academy of New York, Purchase, NY
Steven Williams, Lock Haven University of Pennsylvania

Assessment

Group Leader: Francis (Skip) Fennell, McDaniel College

Members:

Mike Caraco, Burr and Burton Academy, North Bennington, VT
Beatriz D'Ambrosio, Miami University, OH
Karen Mirkovich, Prince William County Public Schools, VA
Felicia Phillips, Atlanta Public Schools, GA
Tom Reardon, Austintown Fitch High School, OH
Gail Yates, Christian County School District, Cadiz, KY

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Reflections on the Congress

Multilingual and Multicultural Education around the World

Gary Lewis

During the Eleventh International Congress on Mathematics Education (ICME-11), attendees had opportunities to learn about the challenges that nations around the globe face in educating students when a variety of cultures and languages exist in a single classroom. Many ICME-11 participants expressed concern about multilingual and multicultural education. The situations educators described varied dramatically across countries and even across different regions within each nation. Participants from many nations (e.g., Sweden) described how rapidly challenges associated with multilingual and multicultural education have grown over the past several decades as immigration from other nations has increased. Other countries, such as the United States (US), Mexico, Papua New Guinea, and Australia, have faced such challenges more regularly throughout their history because of their large indigenous populations, their nations' open immigration policies, or both.

Educators from nations such as Mexico described their situation as primarily involving a majority language and culture coexisting with numerous indigenous language groups. In other nations, characterizing the situation is very difficult because so many language and ethnic groups coexist. Papua New Guinea boasts over 800 indigenous, living languages and cultures, with the great majority of inhabitants living outside the few small urban centers. Similarly, the Fijian language comprises at least 300 *communalects*, which are created when people living in close proximity and using two dialects of a language merge them to form their own way of speech. Many of these communalects are so different from the languages of origin that they can be considered different languages.

This chapter will describe different approaches to timing the transition into a second language, contrast instructional and mathematical language, elaborate other challenges in multilingual and multicultural mathematics education, describe successful strategies supported by research, and give recommendations for the mathematics education of multilingual and multicultural students.

Different Transitions into the Majority Language

The range of approaches being used internationally to educate second-language learners can be encapsulated by a model presented by Maria Luisa Oliveras (2008). From her research on various countries' approaches to mathematics education in multilingual and multicultural environments, she found that countries' language and education policies generally fall within a spectrum of strategies, from cultural assimilation to cultural pluralism to intercultural approaches. Such a spectrum can exist within a country as well as between countries.

Perhaps one of the most progressive bilingual education systems described at ICME-11 was studied and presented by Eva Norén (2008) of Stockholm University.

Public attitudes toward the education of immigrants have shifted over the years, and Sweden has used three strategies at different points in history: a policy of education in students' native languages (1979–1991), Swedish-only education (1991–2004), and education in both native languages and Swedish (2004–present). In Sweden, as in other countries, researchers are aware that the mathematics performance of first-generation immigrants is weaker than that of native-born students.

In a cultural assimilation approach, the second-language learners are expected to adapt to the dominant culture and language. Some countries transition the students quickly or, in some cases, teach exclusively in the nation's official language or in English. In Mexico, a new generation of researchers is focusing on the education of students from indigenous groups such as the Yucatac, Mixtec, and Nahuatl, who live in remote, rural regions of the country with limited access to educational opportunities.

In the US, policies for educating students from minority language groups vary widely from state to state and are often geared toward cultural assimilation. In California, citizens voted to eliminate non-English instruction in public schools in 1998, forcing all teachers to use only English in the classroom. Similar legislation was also enacted in Arizona. Texas instructs students in their primary language for several years before transitioning to the majority language and lets such students take the state assessment in their primary language up until sixth grade.

In a cultural pluralism approach, second-language learners are expected to be competent in their mother tongue as well as that of their adopted culture. Countries such as Papua New Guinea, Botswana, Nigeria, Malawi, Tanzania, Brazil, China, and Korea educate native minority-language-speaking children in their first language for the first three to five years of schooling. They then transition the children into the primary language of the country or into English. Sometimes this immersion occurs even later in the primary grades. A group of researchers (Ní Ríordáin and O'Donoghue 2008) studied the Irish-English educational transition of adults in Ireland moving from mathematics instruction in Gaeilge (Irish) in secondary school to mathematics instruction in English in tertiary education.

Finally, in an intercultural approach, students can be educated in both languages in the same classroom. Schools espousing this approach hire native speakers to teach in the bilingual classrooms; often these teachers are immigrants from the same countries as their students. In Sweden, some bilingual teachers have taught in their home countries before living in Sweden long enough to become proficient in Swedish. In intercultural education, no one culture or language is considered dominant or superior to any other. This approach can transform society itself (Oliveras 2008).

Two Languages— Instructional and Mathematical

Kgomotso Garegae (2008), of the University of Botswana, noted that both first- and second-language learners can struggle with mathematics because of language. Garegae characterized language in mathematics as having two components – the language of instruction and the specialized language of mathematics, which consists of symbols and borrowed words. These symbols (e.g., $>$, \neq , \leq , \pm , $\frac{1}{4}$, \times , \div , Σ , χ , θ) and specialized language (e.g., *hypotenuse*, *triangle*, *simultaneous equations*) pose a problem because students' ability to interpret and conceptualize mathematical texts, especially word problems, varies. Thus, mathematical language becomes a second-language challenge for those whose language differs from the medium of instruction.

Garegae's findings showed that being incompetent in non-mathematical conversational English does not necessarily translate into incompetence in mathematical language; however, researchers noted that the use of mathematically specific vocabulary words often added to students' confusion about what they were expected to do mathematically.

Second-language learners may have difficulty understanding word problems because the second language is not as familiar to them. Researchers at the Center for the Mathematics Education of Latinos/as (CEMELA) interviewed Latino students who worked on problems from the National Assessment of Educational Progress (NAEP) about the thought processes they used while solving problems. The researchers found that many students had difficulty negotiating even simple word problems because they lacked language facility and problem-solving strategies (Anhalt, Fernandes, and Civil 2008). Unsuccessful students would rely more heavily on concrete or visual clues on the page, a strategy that often led to incorrect answers because drawings were not to scale or because images did not include all the information needed to answer the question correctly.

Many students learning English as a second language confuse related vocabulary words, such as *area* and *perimeter*, perhaps because these words are new to them. When interviewing students after a test and asking them about their thought processes, researchers found that while many of the students struggled with the exam questions, the tests did not accurately measure students' abilities in mathematics because of the language barriers. Unfamiliar phrasing in questions places heavy linguistic demands on the students, leading many who actually do understand the mathematics to misunderstand the question and provide a correct answer for the wrong problem. For example, many students understand the word *if* to mean a negation rather than a supposition.

The second reason students have trouble with language is that not all words are translatable; they either do not exist or have different meanings in the second language. What students learn in school can be hard to translate from one culture to another. For example, the Fijian economic system and way of life, which are based on sharing and redistribution, are not compatible with precision. In place of numerical classifiers, the people of Fiji use qualitative evaluations and gesture to indicate terms and phrases. Measurement is commonly determined using body parts. Hence, there is a lack of vocabulary for the standardized units discussed in mathematics classes and of concepts and words to use in discussing them.

For teachers to be able to successfully teach indigenous students, they need to understand which words in the relevant indigenous languages have different connotations and meanings. For example, different words are used for measurement and capacity and for geometric shapes and mathematical concepts. Adding to the challenge, many words used in mathematics do not even exist in some indigenous languages, so word-for-word translations are not always possible (de Bengoechea 2008).

Mercy Kazima, from the University of Malawi, spoke on "Mathematical Terminology in Teaching and Learning Mathematics in African Languages" (2008) and discussed the confusion caused by English terms that have one meaning in mathematics and another in everyday use. Teachers in Botswana code-switch by altering their language to signal a change in context rather than let the language hinder their students from understanding the mathematics. (A change in linguistic form

[language alternation] that signals a change in context [contextualization] may be described as code switching [Society for Linguistic Anthropology 2011]).

The problem of language is not limited to students but extends to practitioners and researchers. What is taught can also be hard to translate from one culture to another. In an activity in which pairs of students were given an object, the first student had to tell a fact that was true about the object, while the second had to mention something that was not true of the object. This kind of negation skill is not commonly taught in early elementary grades in the US, for example.

Related to these issues, one participant noted that even concepts as simple as *teaching* and *learning* can create misunderstandings across cultures. Several native speakers of languages other than English pointed out that their languages use a single word that includes both concepts; in other words, the roles of “teacher” and “student” are not easily discernible through vocabulary. Moreover, educational research and writings can be difficult to translate into such languages, so it may be challenging to communicate with educators who speak those languages.

Other Challenges in Multilingual and Multicultural Education

Several sessions at ICME-11 addressed additional challenges that students from minority groups face while attempting to learn mathematics in the majority language in a society. Students immersed in a new language have difficulties with the instructional and mathematical languages, as well as with the preservation of their culture. According to Natalia de Bengoechea (2008), one of the problems within indigenous education is that differences within the indigenous populations are often ignored because all indigenous groups are classified as one population with one set of needs. But in Mexico, the indigenous populations are incredibly linguistically, culturally, and geographically diverse.

In Ireland, researchers administered a mathematics word problem test to investigate language issues for language-minority students. They found a strong correlation between the native Gaeilge speakers' mathematics performance in English and language competency in English. When taught a minority language (Gaeilge/Irish), their mathematics scores increase, while the mathematics scores of students from the minority language (Gaeilge/Irish) drop after these students switch to the majority language (English). The researchers called such settings “subtractive bilingual environments” due to the impact that the dominance of the majority tongue and majority culture had on these minority-language students' learning. The challenge for Gaeilge speakers lies not in the relearning of mathematical concepts in the majority language but in transferring the mathematical skills from the mathematics register in Gaeilge to the new language of instruction.

An example of the US's struggle to educate Spanish-speaking students in mathematics was described by the researchers at CEMELA. The researchers found that when Latinos abandon the mores of their own cultural heritage for those of the majority group, their educational achievement gains do not last past the second generation.

Marta Civil (2008) discussed how students often talked past each other when trying to participate in class activities when another student was translating. Because

the student-translator has mathematical knowledge, any communications between the minority and majority group are filtered through the mathematical understandings of that student. This can make it difficult for students to participate fully and effectively in whole-class activities because the teacher can only translate for so many students, if he or she can even speak the second language, which is not always the case.

Another problem that arises when Spanish-speaking students in the US are fully immersed in English language instruction at the secondary level is that they may be placed in inappropriate courses because they lack fluency in the majority language. Even students with strong mathematical backgrounds may be placed in lower-track courses and end up bored.

According to Norén, researchers in Sweden found that students participate more in bilingual classes and get much better grades than they do in Swedish-only classrooms. Perhaps part of this success can be attributed to the diminished reliance on textbooks in these classrooms. Since the standard-issue textbooks are in Swedish, they often are used less in bilingual classrooms. In these classrooms, assessing students' understanding of mathematics is related less to the students' ability to understand the culturally specific problems used in the textbooks and more about their interaction with each other and the teacher.

One of the core challenges that exists within the bilingual system is the inherent power differential between the Swedish and immigrant teachers in the schools studied. While this was beyond the scope of the study, the researchers noted that the bilingual teachers in the study felt unaccepted in many situations at their schools and were concerned that they would not be able to continue teaching in their native languages. The researchers concluded that it is difficult to incorporate true bilingualism in schools because institutions are slow to respond to change. In spite of the positive effects of bilingual education on students' mathematics achievement, resistance to bilingual education is deeply seated in many countries' educational and political institutions.

Students in cross-lingual and cross-cultural contexts face unique challenges. Children encounter difficulties when moving from education or homes in one culture to education in another culture. Communication between students, parents, and teachers may be difficult. Immigrant parents sometimes perceive that instruction in the new culture is less strict than in their country of origin. It is confusing for parents to talk to teachers and at times to their children about mathematics in the second language. Similarly, parents often wish to help their students with their homework, but the students complain that it is difficult to translate mathematical questions for their parents when they are just beginning to understand those concepts themselves.

**Successful
Strategies
Supported by
Research**

Since 1992, the Department of Education of Papua New Guinea has developed educational and curriculum reform policies. The government wanted to promote and preserve the rich cultural and linguistic diversity of the over 800 indigenous languages of Papua New Guinea. These policies place heavy emphasis on community-based education through the use of vernacular languages and indigenous knowledge-based systems in teaching formal school subjects. A study of 125 children from four provinces indicated that children learning to read, write, and count in their own language

performed better than those taught early number knowledge in pidgin or English only (Matang 2008).

A collaborative research study between a group of indigenous Maori educators in New Zealand and immigrant educators in Denmark said that though their student populations were very different, they faced similar issues of low expectations and lack of support in the classroom and therefore needed similar strategies to help them succeed (Valero et al. 2008).

A study (Marshall, Musanti, and Celedón-Pattichis 2008) of New Mexico's bilingual classrooms found that mathematics teachers who encourage flexible thinking and language development achieve greater equity for their Spanish-speaking students. Teachers in the study used a Cognitively Guided Instruction (CGI) method, which focuses on visual representations and language development for all students, in their classrooms. Word problems were embedded in familiar situations, and students were expected to use different strategies when solving problems, knowing that they would have to explain their solutions each time. The researchers argued that by using students' native language in the classroom, teachers gave students greater access to challenging problems that helped them gain greater conceptual understanding of the mathematics.

The CEMELA researchers found that successful students are resourceful in using their mathematical knowledge and their language skills during a test, which suggests that instruction should help students develop this flexibility. Using a wide range of tools and problem-solving strategies to answer each question, these students are able to use previous knowledge to solve a problem and explain their reasoning. The researchers found that the most successful students develop a mathematical language that encompasses the new ideas and vocabulary they learn in class, which they are able to successfully apply to solving problems on a test.

A strategy for helping students who have difficulties with mathematical language is to create a mathematics register in the local language or with borrowed terms from other languages when they do not exist in the mother tongue. For example, some educators create terms that focus on the mathematical concept and not on the literal translation. Words for *set*, *base*, *sum*, *factor*, and *angle* are included in the register.

It was suggested that literal meanings of mathematical terms could be a source of confusion for the learner and that everyday meanings of words could interfere with mathematics learning. Kazima said that in Tanzania, educators are creating a mathematics register in the local language, whereas in Nigeria they use a mathematics register in six of the local languages. She mentioned that a study conducted in these nations in 2004 indicated that teaching mathematics using terms in local languages was effective.

Similarly, the Fijian Mathematics Vocabulary project (Bakalevu 2008) is an attempt to develop a Fijian mathematics vocabulary. In Malawi, Kazima reported that educators have overcome this obstacle by using borrowed English terminology for their mathematics register. Students learn the English words for mathematical concepts in primary school, but the language of instruction used to teach the concepts is their native language. This approach allows for a greater level of precision in the mathematical terms used in the classroom, because these terms have only one meaning. The challenge is that students are learning words in a new language in addition

to learning the mathematics, often making it difficult for students to remember the new words and their meanings.

Kenneth Wolff and Mika Munakata (2008) discussed strategies to improve the science and mathematics achievement of recent immigrants in the US state of New Jersey by using field trips and hands-on activities to engage students in the use of mathematics in the real world. Accommodations for second-language learners, such as introducing key vocabulary words, using diagrams and visual aids, and having students work in groups are embedded into everyday instruction for all students. Even the class structure is set up for second-language learners to succeed; bilingual aides help students in the classroom, and there is time for student translations within lessons. The researchers have found these strategies effective in engaging students in the mathematics classroom. Students are excited about learning math because they know that math is used in interesting ways in the real world.

One of the few sessions in which participants discussed a teacher's action research was a Shared Experience Group led by Sue Thomson (2008) from the Australian Council for Educational Research. Thomson is a mathematics teacher of second-language learners who are recently arrived immigrants from diverse linguistic and cultural backgrounds and who are reasonably well prepared in mathematics. At her school, the language policy states that all students must learn English in order to pass their matriculation exams to gain entrance to university. Second-language learners can learn mathematics in English by taking an extra mathematics class, like the one Thomson teaches, where she uses activities in which students use spoken and written English to complete mathematical tasks in multilingual, heterogeneous groups. Thomson piloted several of these activities with her English-speaking students, only to find out that they too had a very limited understanding of the mathematical vocabulary words that were regularly used in the classroom.

In one activity, students work together to match picture cards to word cards. In another, students draw pictures based on another student's description of a hidden diagram without using the actual word that defines it (e.g., a student describes how to draw a parallelogram without using the word for the object, but using words that describe any of its properties or characteristics). The students complete crossword puzzles using mathematical definitions and words in English and describe visual proofs of the triangle sum theorem or the Pythagorean theorem. Students learn conversational and mathematical English as they negotiate these group tasks. Thomson also encourages students to practice making sounds in English that are difficult for them to articulate. Conference participants agreed that it would be beneficial to employ some of the activities Thomson used with her English-language students to help them develop mathematical registers in their own language.

Future Recommendations

Further research related to the use of multiple languages, vocabulary in indigenous languages, mathematics registers, and code-switching in teaching and learning mathematics is needed. Teachers need to do and share action research, and further education of teachers with respect to multilingual and multicultural issues is important if all students are to learn mathematics.

During a panel discussion on equal access to quality mathematics, panelist

Olimpia Figueras (2008) from Mexico described the tension that languages of indigenous peoples contribute to the struggle for mathematical equity. Researchers need to perform an analysis of mathematical vocabulary words in indigenous languages to see how quantitative ideas are used and how those may or may not translate into the majority language.

Because majority- (English) language speakers have trouble with specialized language, Garegae suggested that teachers teach English language in mathematics classes and focus on dual/multiple meanings of English words.

The debate over language use in schools is politically charged, and some people in Malawi believe that they should develop a register with terms in the home language. Kazima reported that teachers seem to be happy using the borrowed math terms; however, there is no research on the effectiveness of the Malawi strategy on teaching and learning. Because there are so many indigenous languages and because language is so important in mathematics, a great deal of research still needs to be done in this area.

Both sides in the debate speak to the need to address the use of multiple languages within teacher preparation programs. Programs need to ensure that teachers are familiar with the languages and cultures of the students they are expected to teach and should address the issues involved in teaching in multilingual classrooms. De Bengoechea supported Kazima's point of view and suggested that in the short term, Mexico needs to develop a certificate program for teaching mathematics in indigenous contexts.

In her presentation, "When the Home Language Is Different from the School Language: Implications for Equity in Mathematics Education," Civil emphasized the need to build the "listening" skills of teachers. She suggested that teachers be engaged in action research as they learn about immigrant students. Civil reported that much of teachers' action research doesn't get published and is just presented at conferences, so there is a limited amount of shared knowledge about how to help second-language learners in the mathematics classroom.

Civil claimed that educators need to focus on more than just language in instruction. Key constructs in helping students from other cultures and/or language groups are power, identity, and participation. CEMELA recognizes this and is trying to help students and their communities view their heritage as an asset and work toward retaining that heritage by integrating it into students' educational experiences in an effort to retain gains that the first and second generations make in their educational endeavors.

The challenge that exists for researchers and educators is developing all second-language learners' proficiency in mathematics by creating a mathematical language that includes a range of problem-solving strategies, so that students are not hindered by the use of their second (majority) language.

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Educating Children of Diverse Cultures

Cindy Chapman

In addition to the focus on the multilingual education (Lewis, in this volume), another aspect of multiculturalism related to education is the belief in the importance of respect and dignity for all students and the conviction that while diversity may be challenging, the richness it offers more than makes up for its difficulties.

In a plenary panel, “Equal Access to Quality Mathematics Instruction” (2008), Bill Atweh of Australia spoke about the difficulty of achieving both quality and equal access simultaneously. Quality mathematics without attention to equity leads to elitism, whereas equitable education without quality mathematics leads to a watered-down curriculum. Thus, “equity in mediocrity” is easy to achieve, while “equity with quality” is quite difficult.

Ubiratan D’Ambrosio (2008) spoke about the fact that quality doesn’t simply mean doing what is being done now but better. To achieve quality, the mathematics community needs to consider ideas of social justice in the teaching of mathematics. Respect, solidarity, and cooperation are essential, and educators must be sure that students’ cultural roots are honored. D’Ambrosio also spoke about the importance of recalling the contributions of the common person toward the evolution of mathematical ideas.

Ethnomathematics

In 1985, D’Ambrosio defined *ethnomathematics* as the math cultural groups—such as national-tribal societies, different employment/professional groups, or different age groups—practice. In the Plenary 6 panel at ICME-11, he discussed the potential contribution of ethnomathematics to achieving social justice and peace with dignity for all through promoting dialogues that endure and allow communication between educational institutions and local cultural communities. Examples of this came from the Topic Study Group 33, on “Mathematics Education in a Multilingual and Multicultural Environment,” chaired by Natalia de Bengoechea and Maria Luisa Oliveras. Participants learned about a textbook development project in China that focused on the food, architecture, and farm work of a minority population in the Xinjiang region. A project involving two schools in Turkey and one school in Rhode Island used Turkish rugs to study fractions, patterns, and geometry concepts.

Written and Unwritten Language

Respecting and understanding diversity includes recognizing and addressing the difficulties that language can create for children of diverse cultures. In addition to the language issues described by Gary Lewis (this volume), the role of gesture as complementary rather than merely supportive to speech in communication was considered significant with respect to cultural diversity. Gestures from different cultures may send different signals about learning, and teachers of students of diverse cultures need to be mindful not only of language differences but of gestures as well.

Student Attitudes

Student attitudes toward learning mathematics often seem to depend on the culture of the country. Michèle Artigue (France) discussed the differences between eastern and western cultures, and these were emphasized in China's national presentation, a Japanese lesson study session, and a Topic Study Group on Primary Education presentation on problem solving in Japanese primary schools.

Student attitudes and effort in other countries often seem different from what educators in the United States experience in their own classrooms. In particular, the work ethic of Asian and American students differs. Asian students' approach may be traceable to Confucius, who believed the only way one could understand a subject was through long and careful study (Lin n.d.). Because everyone has the same potential, all can achieve mastery through hard work. Harold Stevenson (1992) has conjectured that the differences between Asian students and American students are attributable to these philosophical beliefs, parents' efforts and expectations, and societal attitudes. Asian students work hard to pass tests in order to move to the next level of education. To avoid being shamed by the failure of their children, parents are heavily involved: they make sure that students complete their homework, attend class when their children are absent so that they can take notes, and observe their children's behavior in the classroom.

At ICME-11, Chinese educators talked about the work ethic of Chinese students and teachers, citing a Chinese saying that illustrates it: "Unpolished jade will never shine. To teach without severity is a dereliction of duty." For the Chinese, extensive and continued practice is a critical part of learning, although the country's educational leaders are very concerned about the narrow goal of exam-focused education.

In the US, students tend to believe that innate talent and abilities that are beyond their control determine their success in school. They spend less time in school but about the same amount of time on subject matter. At the high school level, American students spend more time on subjects other than mathematics, science, and history than do students in Japan; Japanese elementary students spend three times as much time on mathematics as Americans (Sykes 1995).

According to Stevenson, American students have a higher opinion of their own abilities but do not see hard work as the way to achieve their goals (1992). Compared with Asian parents, American parents are not as involved with the education of their children and expect teachers and the system to lead their children to success (Van Schaak n.d.).

To students in Japan, life centers on school. They spend more hours there than do American students, but the extra time is devoted to recesses, which allow for rest and assimilation time between subjects. Stevenson explains that knowledge is not forced in Asian elementary schools. The students are encouraged by their teachers to construct their own ways to do mathematical problems. At ICME-11, Japanese educators explained that students enjoy and expect to engage in challenging problems, even ones that might be beyond their skill level. In American schools, by contrast, teachers are considered the source of knowledge and often are involved in confrontation and conflict with students.

Rosetta Zan (Italy) presented a longitudinal study that investigated a multidimensional way of looking at student attitudes. Students in first through thirteenth grades

narrated their own “stories” regarding their feelings about mathematics. These stories tended to revolve around students’ emotions (I like/don’t like math), self-efficacy (I can/can’t do math), and vision of math (math is a collection of skills/abilities versus math is problem solving/creative). One finding was that positive attitudes toward math (I like math and/or can do math) were more often included in stories in which students’ vision of mathematics was that mathematics is problem solving and creative.

Summary

Educators must respect and incorporate the culture of their students in their programs of study. They need to be knowledgeable about the different meanings of words and gestures in different cultures and must realize that students have different attitudes toward learning and education in general and toward mathematics in particular. Without such cultural awareness, equity with quality cannot be achieved for all students.

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Research and Practice: An International View

Richelle (Rikki) Blair

Research related to learning and teaching mathematics was a central theme in many of the sessions of the Eleventh International Congress on Mathematical Education (ICME-11). Researchers and practitioners in many countries face similar issues and questions about mathematics education. The following chapter describes the perspectives of researchers and practitioners, communication issues, research that should be connected to practice, strategies for connecting research to practice, strategies for involving teachers in the research, and recommendations discussed at ICME-11.

Perspectives of Researchers and Practitioners

In Plenary Session 1, “What Do We Know? And How Do We Know It?” Michèle Artigue and Jeremy Kilpatrick (2008) looked at mathematics education as a social science, not as a scientific field. The session considered mathematics education as evolving from a consolidation of sociocultural and anthropological approaches to a new focus on teachers’ knowledge, beliefs, and practices.

Teachers and researchers often have different perceptions about research and its role in the task of educating children. Researchers study a problem or situation with a goal of having practitioners implement changes in their classroom based on the research. However, often the results of research are not implemented because the practitioners perceive the research as not applicable, not providing resolutions to problems they face, or only theoretical.

Researchers do not always agree on what approach to use. Opinions expressed during discussions in the ICME sessions varied, with some participants noting the advantages of different theoretical perspectives and others stating that there is a need for a single theoretical perspective that could be used when researching any level of mathematics education. Ed Dubinsky (Dubinsky et al. 2008) spoke about the goal of using a single theoretical approach based on the ideas of Piaget, the constructivist APOS theory of learning (Action, Process, Object, and Schema), which can be applied to all mathematical concepts to study how students learn. According to Dubinsky, “This theory posits the use of certain mental mechanisms to build mental structures that an individual might use to deal with a mathematical situation... Our pedagogy focuses on helping students build the structures, after which even difficult concepts can become assessable.”

Practitioners look to research for answers to questions on theoretical content and pedagogical practices for particular situations they encounter. In the plenary panel debate “What Do We Need to Know? Does Research in Mathematics Education Address the Concerns of Practitioners and Policy Makers?” Paul Cobb and colleagues (2008) responded to questions related to this topic sent to them by North American teachers in advance of the Congress. The questions reflected a view that educational research provides a definitive yes-or-no answer to a research question. Teachers

expect that research in mathematics education will provide prescriptions to solve teaching problems, similar to research used in the medical profession.

Angel Gutiérrez (2008) stated that the implementation of research was limited by teachers' beliefs that available research was not practical enough to be useful. Researchers were seen as being disconnected from the realities of the everyday classroom. Cobb argued that research in mathematics education cannot answer questions with simple yes-or-no answers because classrooms are part of complex instructional systems. When researchers conduct studies, they investigate a particular question with a particular group and study the findings specific to the situation of the group. In most cases, extrapolating the results of the research to another setting or classroom is problematic.

Disagreeing with Cobb, Shiqi Li asserted that educational research has the power to address the complexities of teaching and learning, arguing that case studies are a powerful approach for connecting research and practice. Researchers could generalize case studies in order to apply the research to other situations. However, practitioners cannot apply the research in a general way without analyzing the particular settings of the mathematics classrooms, both their own and the classroom represented in the case study. Case studies show the research in a situational perspective—why answering “yes” to a question or a practice may be right for one situation and not for another.

Although some problems may be identified and investigated through research, a second belief teachers hold is that research is not usable because researchers do not investigate what teachers can do to resolve problems. Rosetta Zan (2008) provided practitioners with theoretical clarity regarding students' negative attitudes toward mathematics. On the basis of 1265 essays written by students, teachers diagnosed students as having a negative attitude toward mathematics, as indicated by (1) an emotional disposition (I like or do not like mathematics), (2) self-efficacy (I can or I cannot do mathematics), or (3) a vision of mathematics (mathematics is or is not an exact science). Factors that influenced how students' attitudes changed over time included the introduction of different mathematics topics, moving from one school to another school, and specific events that affected the student. Regardless, according to the analysis of students' essays, the teacher was the most important factor in attitude change.

The United States, Spain, China, Brazil, and the Netherlands have similar problems with teaching fundamental algebraic concepts. Topic Study Group 11, “Research and Development in Teaching and Learning of Algebra,” led by Rosamund Sutherland and David Carraher (Sutherland et al. 2008) addressed key issues and topics about the use of language in algebra, the concepts of equality and equivalence, and the development of an understanding of variable. For example, students still encounter obstacles and make common mistakes when working with variables: for example, they believe that $2n - n = 2$ or $3n = n \cdot n \cdot n$. One US study reported by TERC focused on middle school students and concluded that younger students use variables in problem solving. These studies did not provide examples of promising, replicable activities because they did not give guidance for teachers on how to change their practice.

A third reason many teachers may not use research might be because the teachers perceive the applications as theoretical. For example, in her presentation, “Linking

Research and Practice: What are the Research Needs of U.S. Practitioners?” (2008), Judith Reed discussed a case study by Groth and Bergner where, after watching a video of a classroom lesson and reading the transcript, in-service participants worked individually to evaluate the lesson and then met in small groups to compare interpretations. Interestingly, every person took a different approach to the activity. While participants reached some common conclusions, each individual’s approach highlighted different aspects of the lesson, and it was difficult to engage in a productive discourse. Participants generally agreed that analyzing lessons using different approaches was a valuable activity. However, as the group discussed different theoretical approaches for viewing mathematics education research, the need for a common approach when sharing research became obvious. This suggested that research shared with practitioners should be carefully constructed if the messages are to lead to a real discussion about the implications for teaching.

Another reason teachers may not incorporate research into practice may be the differences in attitudes between teachers and policy makers. For example, more experienced teachers may ask why they should implement change when, based on past experience, they would have to undo the change at a later date. Also, teachers often feel they do not have time or money to locate and read research in addition to attending in-service programs. On a larger scale, some policy makers think the changes recommended by research are worthwhile but too costly. Also, while case studies can suggest promising approaches to teaching or learning, few large-scale studies are able to implement these approaches because of the prohibitive expense of doing random controlled trials within the education system.

Communication Issues in Relating Research to Practice

To make research available to practitioners, it must be published in printed or electronic form that is made available to teachers and must be integrated into teacher education programs, either for certification or in post-certification in-service programs. In the United States, teachers at the middle school level often have an elementary certification with little background in mathematics or in mathematical pedagogy. Thus, the opportunities for educating teachers about the role of research and how it can impact their practice can be limited.

In a plenary session, “The Impact of Research Findings in Mathematics Education on Students’ Learning of Mathematics,” Gutiérrez (2008) emphasized the importance of research being available to teachers while recognizing that the lack of a pathway for communicating the research to teachers limits implementing the research. Education professionals from Finland, Sweden, Puerto Rico, the United States, Brazil, Germany, and Portugal posed different questions regarding, and brainstormed solutions to, the lack of communication between researchers and practitioners. In countries such as Taiwan, Sweden, and Finland, where the public sees teaching as a profession, better communication exists between researchers and practitioners. In the United States, Judith Reed and the National Council of Teachers of Mathematics (NCTM) are actively trying to help teachers access research through a special section of the NCTM website.

Another reason research may not be available to practitioners could be the way articles and studies are written. Researchers often write reports using terms

unfamiliar to teachers or in a foreign language that may be mistranslated. Teachers may not understand the work or connect the research to practice.

One way to ensure communication among stakeholders is to design research studies to be more mindful of different kinds of stakeholders. A study involving four urban middle schools in the United States (Cobb 2008) is investigating practices and structures that have potential to support the improvement of teaching. These include teacher access to colleagues who use high-quality instructional practices, designated time for teacher collaboration with peers, placing school-based mathematics instructional leaders to support teacher learning, and professional development for mathematics leaders to support their effectiveness in that role. The research suggests that these practices need to be grounded in a shared vision of mathematics education held by school and district units, including personnel in leadership, curriculum and instruction, research and evaluation, education of English language learners, and special education. Implementation requires funding and support for the time needed to enact the professional development components. In addition, stakeholders should recognize that they are mutually accountable for fulfillment of the vision. The study was in its beginning phase at ICME-11, and results will not be available until later, but the findings might inform ways in which research can be integrated into practice.

What Educators Need to Know

In “How Do You Teach Nursery Children Mathematics? In Search of a Mathematics Pedagogy for the Early Years,” Sue Gifford (2008) shared useful information about how children learn mathematics and how teachers can support children’s construction of mathematical knowledge and understanding. Adult-led mathematics-focused group activities have been found to be effective in increasing children’s mathematical knowledge and understanding. Teachers should be given specific instruction and shown examples of methods that improve children’s mathematical abilities, such as the need to create cognitive conflicts for children and to help children reflect on their own thinking. As teachers become more aware of the mathematical thinking of young children, including typical errors and misconceptions, they can use this knowledge to plan instruction.

Knowledge of children’s learning trajectories, an under-researched area at present, could fill a gap in teachers’ pedagogical knowledge. Douglas Clements (Clements and Sarama 2008) defined a learning trajectory as a description of children’s thinking and learning and a related conjectured route through a set of instructional tasks. This route includes methods of supporting children’s learning through curriculum and teaching strategies. When teachers use trajectories, they learn about mathematics and become more aware of how children think while doing mathematics.

Jere Confrey and her colleagues (2008) presented a database of articles on rational number reasoning and an analytical synthesis of the articles. Some key findings included: (1) rational number reasoning is complex; (2) internationally, students lack sufficient experience moving within multiplication and division spaces; (3) partitioning develops independently and parallel to counting; (4) there are three dominant meanings of a/b in the rational number literature; and (5) the sequence of concepts can make a fundamental difference in how students are prepared. Confrey argued that a/b should be introduced prior to the addition of rational numbers. The findings

of this research synthesis could have an impact on how and when rational number concepts are introduced to students.

Darcy Hallett (2008) reported on what makes studying fractions challenging and gave some suggestions for helping students with this issue. According to Hallett, the study of fractions is challenging for three reasons: (1) fractions represent a relationship, not just a number; (2) the algorithms are more complicated; and (3) the nature of the relationship between denominator and numerator changes across different contexts. Hallett presented four ways to look at fractions—part/whole, quotient, operator, and intensive quantity. His research showed that children understand quotient situations as a model of fractions better than the other models. Teaching the quotient model first helped students understand other models. He also discussed whether conceptual understanding or algorithms should be taught first. He concluded from his research that it depends on the child. For some students, a solid mastery of procedures helped with conceptual understanding, but for others, without conceptual understanding, there was no mastery of procedures.

Clements (Clements and Sarama 2008) discussed an intervention called TRIAD (technology-enhanced, research-based, instruction, assessment, and professional development), which is currently being tested with low-income preschoolers. Learning strategies in the program include the use of visual tools such as 5- and 10-frames and methods to promote children's verbal skills with the goal of increasing student vocabulary as well as mathematical understanding. Professional development, assessment, and instruction are closely linked via technological tools and include distance education courses, interactive professional development with support for teaching based on learning trajectories, and a software management and reporting system for student progress.

Historically, early childhood education has been provided through an exploration-based model. Gifford (2008) stated that spontaneous free play alone is insufficient for children to learn as much mathematics as they are capable of learning. Key mathematical learning opportunities do not necessarily arise during free play. Fenna Van Nes and Jan de Lange (2008) examined children's use of mathematical representations, such as finger patterns and domino-dot configurations, specifically as spatial structures. The study explored the relationship between children's use of these spatial representations and children's numerical abilities while examining children's development of counting strategies beyond unitary counting. A goal of the research was to assist children who are having difficulty in counting by helping them see the spatial structure of numbers. This study pointed to the power of 10-frames as a tool for building children's number sense and further developing their visual memory. Another consideration was the child's choice of finger patterns during number work. It was helpful for the teacher to attend to how children were naturally using their fingers and to see if there were certain ways of doing things that are useful for mathematical understanding for most children or for particular children in various cultures.

L.S. López (2008) studied the processes younger students use during problem solving (exploration, acquiring new information, local monitoring) and how a child's use of processes and strategies contributes to predicting accuracy in problem solving. Specific questions were used to help young children think about and verbalize the processes they use to solve problems. This information is useful for teachers because

they often expect children to choose a strategy or give answers promptly. Using Lopez's questions about processes helps both children and teachers become aware of what happens during problem solving beyond strategy selection and use.

Some researchers combined content and pedagogy. In Topic Study Group 19, "Research and Development in Problem Solving in Mathematics Education," Richard Lesh and Thomas Fennewald (2008) presented developments in mathematical problem solving, student strategies, assessment, and computational tools for all levels. They gave definitions for *problem solving*, *modeling*, and *problem posing*. Some of the major connections to practice from this session were: (1) a current focus on showing how concepts develop through problem solving; (2) the metacognitive and cognitive processes teachers use when solving problems; and (3) the importance of understanding as well as reaching a correct solution. The emphasis was on mathematical modeling and the usefulness and effectiveness of technology to assist in the development of mathematical thinking through problem solving.

Dani Ben-Zvi (2008) identified the following research activities as crucial to developing statistical literacy and reasoning skills:

1. Developing and testing empirically a theoretical framework for informal inferential reasoning and the role of argumentation in the inferential process
2. Mapping statistical reasoning as it occurs in the social context of the classroom
3. Focusing on learning difficulties in order to advance the understanding of the challenges in learning and teaching statistics and to inspire improved instructional methods and materials, enhanced technology, and alternative assessment methods.

Strategies for Connecting Research to Practice

Combining sociocultural factors and mathematical content and connecting them to teaching practices was the focus for Ubiratan D'Ambrosio (2008). He said that mathematics, in particular ethnomathematics, should help to generate responsible creativity and ethical world citizenship. In the context of the term *ethnomathematics*, *ethno-* means "of the natural, social, and cultural environment"; *mathema-* means "doing/ knowing," that is, learning, explaining, or coping with and solving problems; *-tics* are ways, styles, and techniques. In order to meet students' material and spiritual needs, one should use ethno+mathema+tics—ethnomathematics. D'Ambrosio also discussed the notion of students doing ethnographic research as an effective and powerful way to connect research to practice. This session brought together ideas of culture, mathematical content, and teaching practices presented in other lectures.

In her lecture, Merrilyn Goos (2008) focused on sociocultural factors that influence the learning and development of mathematics teachers, teacher educators, and researchers. She used Valsiner's zone theory (VZT) to explain the development of teachers and teacher educators. VZT takes into account the zone of proximal development, zone of free movement, and the zone of promoted actions.

In Topic Study Group 32, "Gender and Mathematics Education," the focus was on understanding sociocultural factors. In Gilah Leder's paper (2008), "Equity: The Case for and against Gender," she reported that females performed better on open-ended questions in Australia, Iceland, and Iran, all other factors being equal. Females from rural areas in Iceland performed better than those from urban areas. Leder also reported that females in same-sex classes performed the same as those in mixed-sex classes.

When looking at gender issues, it is necessary to determine whether inadequate educational opportunities are given to girls and to examine teaching styles, societal values, ways females and males are socialized, and the power structure of society. Jillian Knowles (2008) focused on the relationships between culture, mathematical content, and teaching practices by analyzing the role of affective interactions between female and male students with female teachers. She discussed a model to help teachers look at the relationship between female students' having low mathematics self-esteem and the role that these students' relationship with a female teacher plays in helping them. These affective problems interact with cognitive functioning. Knowles said that if teachers understand themselves as gendered people, they might be able to free themselves from gendered restrictions in their mathematics endeavors.

Strategies for Involving Teachers in the Research

Teachers can either be involved directly in research by conducting it themselves or can be made aware of research and be instructed in how to apply it. A strategy to give classroom teachers a more comprehensive understanding of the process and findings of research is to involve them in action research using models such as Barbara Jaworski's research/inquiry model, "in which reflective practices, teachers' engagement in research, and partnership between teachers and academics/educators" are involved (2008). Instead of conducting research for and about the teacher, research should be done *with* the teacher (de Andrade 2008). Collaborative research involving both researchers and teachers increased teacher knowledge and consequently improved students' learning, according to Ferdinando Arzarello and colleagues (2008). In Arzarello's work, as teachers' thinking changed over the course of the project, they began to think about what children were learning, what was happening in a child's head, and how they connected with and extended students' understanding, rather than responding only to the answers students gave.

In Japan, teachers continue learning to teach through a systematic plan of lesson study in which teachers collaboratively design and implement a research lesson, then reflect on what students learned from the lesson and revise the lesson accordingly.

According to China's national presentation, teaching is viewed in China as one of the top three most respected professions. Teachers are given the time and support to study and improve their own classrooms. Chinese teachers are viewed as producers of research, not just as consumers of research. They also have many outlets for publishing their classroom case studies, in contrast to the situation in the United States, resulting in more professional journals oriented to teachers than to researchers.

In Finland and Sweden, all secondary school mathematics teachers complete a master's degree that includes a thesis for which the prospective teacher conducts a piece of publishable research on the teaching and learning of mathematics.

Recommendations

- Better communication is needed between researchers, educators of teachers, teachers, and the community. Artigue (Artigue and Kilpatrick 2008) said the analysis of international studies such as TIMSS (Trends in International Mathematics and Science Study) showed that changes need to go beyond the surface. Policy makers need to be aware of these international lessons. Teachers and

school personnel need to participate in the development of an action plan in order to implement the research in practice. Ways to communicate about research to teachers, educators, and policy makers need to be analyzed. Printed and electronic publications, conferences and in-services, and organizations such as NCTM are possibilities. Research material needs to be translated and available in a variety of languages in order for more to benefit.

- Educators and researchers need to compare the preservice and in-service training and certification of teachers internationally and how this training relates to research and is put into practice. Across the globe, educators and researchers need to incorporate research into teacher training programs, perhaps using the models of Japan and China.
- Researchers should develop strategies for helping teachers recognize the impact on student learning of what they do in and out of the classroom. Culture, gender, attitude, home and classroom environment, and learning materials all affect the education of students. Teachers need to study their materials, incorporate research findings, and anticipate student responses. They need to be aware of the path that children typically take in learning mathematical concepts and procedures so that they can use that pathway as a lesson-planning guide to ensure children gain crucial learning.
- Teaching skills need to be constantly updated and furthered through in-service sessions and conferences, and teachers must read both written and electronic publications. Teachers need to be active researchers and solicit research that addresses problems or questions that arise.
- A.L. Semenov (2008) stated that the mathematics taught should relate to students' environment. He discussed the major challenges to content and methods of primary and secondary education. School mathematics should provide students with "models of reality" and "patterns of reasoning," the ability to transfer mathematical knowledge from real life to models and interpret simulation results, and experience in searching for new solutions to unusual problem situations. These areas of focus could be used to inform the development of curriculum, textbooks, district and state curriculum guides, and teachers' individual lesson plans.
- Researchers should be more aware of their audience. The language needs to be clear and understandable to the teachers and educators using the results of the research. Publications must be properly translated, and they should be applicable to large groups other than just the study group and to groups with different backgrounds and needs.
- Further research needs to be done in many areas. Little empirical evidence is available that would help educators improve the mathematics education of their students (Evans et al. 2008a). The research that is available is often presented from the perspective of the teacher educator, not the teacher. Some research exists on the effects of professional development on the teacher, but there is very little on the process of educating teachers. Research is needed on how to prepare teacher educators to provide in-service education for teachers and on how teacher educators can deliver in-service education in a systematic way. Research that

examines how implementation of effective educational practices in the situated environment of a school and school district can be supported is scarce.

- Researchers should be willing and available to actively engage teachers in identifying questions for research and for conducting research at both preservice and in-service levels. Active research should be a part of teacher preparation programs and, as in Japan, part of the ongoing process of teaching.
- Research should also be brought to bear in designing the training of teachers. Bill Atweh (2008) commented on the fact that quality teaching improves achievement for all students, but the least advantaged students benefit the most. The need to connect research to practice is of particular importance for those who are the least advantaged. Every child has the right to equitable mathematics education, but a great number of children do not have access.
- Quality mathematics education may look different in different countries because the qualifications needed to become a teacher in different countries vary. Research is needed to help the community understand the roles subject matter knowledge and pedagogical content knowledge play in the development of teachers.
- In order for teachers and educators to put research into practice, it must be available, applicable, and communicated in a way that is accessible and understandable.

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The Role of Technology in Teaching and Learning Mathematics: An International Perspective

Natalie Jakucyn and William M. Carroll

Increasing availability of new technologies provides challenging opportunities for educators to reconsider how learning and teaching might change in mathematics classrooms. Just as scientific and graphing calculators transformed the curriculum in earlier decades, computer algebra systems (CAS), dynamic geometry systems (DGS), networked electronic notebooks, easy access to the Internet, and other advances should make mathematics educators rethink the topics that should be addressed in mathematics classrooms as well as how students and teachers might interact with each other and with the curriculum. Few would want teachers to return to extracting square roots or computing regression equations by pencil-and-paper methods. Similarly, researchers and educators must ask how can CAS, computers, and other technological advances improve mathematics education—what will be lost and what will be gained? Most importantly, how can they deepen students’ skills and conceptual understanding with technologies that actively allow them to explore mathematics using interactive multiple representations? Beyond the classroom, facility with technology will prepare students for the workplace of the 21st century and their participation in a global community.

International trends in mathematics education can help researchers and practitioners examine the impact of new technologies on instruction and learning. The Eleventh International Congress on Mathematical Education (ICME-11) provided a forum for investigating how educational systems across the world incorporate technology in their curricula. Summarizing what was presented about technology at ICME-11, this chapter is divided into five sections: international progress and problems in implementing mathematics education technology, research about the effectiveness of technology, innovations in technology for mathematics teaching and learning, technology in mathematics education in the United States, and recommendations for incorporating technology into mathematics instruction. The discussion can inform mathematics educators about international trends in mathematics education and ways in which technology can support teaching and learning.

International Progress and Problems in Implementing Mathematics Education Technology

The initial plenary lecture at the Congress, “What Do We Know? And How Do We Know It?” (Artigue and Kilpatrick 2008) and the plenary lecture-panel debate, “What Do We Need to Know? Does Research in Mathematics Education Address the Concerns of Practitioners and Policy Makers?” (Cobb et al. 2008) made clear that teachers around the world share many of the same questions and concerns about the use of technology. Paul Cobb, from the US, discussed survey questions submitted by ICME-11 participants that included “How do calculators impact students’ skills?” and “Is ‘traditional’ or ‘modern’ teaching more effective?” Thirty years after hand-held calculators were brought into mathematics classrooms, some teachers and policy

makers, along with part of the general public, still wonder whether calculators and other technology are useful tools or crutches that diminish students' skills and understanding. In his plenary lecture, Jeremy Kilpatrick addressed this situation in two observations: "It is easier to change what is taught than to change what people expect to be taught," and "Some issues in mathematics education are like vampires that rise from the dead every decade or so."

Realistically, the amount of technology a teacher can incorporate effectively is limited not only by her or his own knowledge but by time and the demands of the mathematics curriculum and assessments as well as the culture of the school. Lack of basic educational technology and needed infrastructure is problematic in many parts of the world. Teachers sometimes struggle with the cost and availability of mathematical technology. When asked about the use of technology in Indian schools, Ajit Kumar (2008) noted that in large areas of India, electricity is not always reliable, and even when it is, few schools have projectors or classroom computers for demonstrations. Similarly, mathematics educators from Uganda reported that there was no use of calculators or computers in Ugandan public schools because of lack of funding. In contrast, since 1997 the Mexican Ministry of Education has been sponsoring the EMAT Project, which aims to test and implement a model that incorporates a variety of technology environments (including CAS, spreadsheets, and dynamic graphing software) into the teaching of mathematics in public secondary schools. Research was done with nearly 90 teachers and 10,000 students at secondary schools, including very poor schools with dirt floors. The project was groundbreaking in changing the role of the teacher and the attitude of children and opened the door for richer ways of incorporating technology in schools (Garcia-Campos and Rojano 2008).

While "scaling up" the implementation of new technology can be an issue nationwide, it can be problematic even within a single school. For example, Kumar reported on the use of CAS in teaching calculus. A goal of the authors' investigation was to help students move from perceiving themselves as passive recipients of knowledge (algorithms and rule) to exploring concepts and connections with a computer-based system. While the CAS-based class was successful in furthering students' understanding, interest, and achievement, the lecturer reported that none of his colleagues were willing to shift from traditional teaching of calculus.

Some teachers are resistant to using CAS-based calculators because they cannot be used on all high-stakes assessments. Even when they are allowed, tests are often written in such a way that there is little or no advantage to having an advanced calculator, resulting in some students and teachers seeing little advantage to using more advanced technology.

Soon Yee-Ping (2008), from Singapore, presented a workshop on enhancing elementary students' learning by engaging them in open-ended problem-solving activities on the scientific calculator. Rather than focusing on skills, the activities were investigative and open-ended, focusing on mathematical connections and conceptual thinking. Yee-Ping related that Singaporean students' high achievement on international tests might be partly attributable to the fact that calculators are required on the national examinations. In contrast, a Taiwanese educator reported that Taiwanese secondary school students never use calculators in their schools because they are not allowed on the examinations. Finland, another high-achieving nation, has no national

examinations, and individual schools are allowed to decide what technology they want to use or whether to use it at all (Kupari 2008).

The use of calculators in these three nations illustrates how high-stakes assessments can determine not only what mathematics is taught but how it is taught. Use and non-use of calculators on high-stakes tests, or the lack of any such exam, seems to greatly impact how much class time teachers are willing to invest in technology. Instructional time is needed for students to learn the syntax and gain fluency in any technology, decreasing time available for mathematics instruction. When current curriculum does not support or require the use of advanced technology, teachers need to weigh the cost and benefits of using technology (Hoyles 2008). The contrast between the use of calculators in Singapore and Taiwan relative to whether they are allowed on the end-of-school examination highlights how assessment plays a role in whether technology is included in the classroom.

Research about the Effectiveness of Technology

In her plenary lecture, “Technology and Mathematics Education,” Celia Hoyles, from the University of London, provided an interesting perspective on how to implement new technology and learning. Her view was that implementation of technology and new curricula requires intensive and sustained training as well as ongoing support to succeed. While it might be expected that advances would come more quickly in education, she has found that a successful implementation might mean that approximately 10 percent of teachers have changed their teaching practices initially and continued interventions were needed to increase this proportion. Her study of incorporating technology in a factory provided a framework for thinking of the analogous situation in education. Workers who received helpful feedback about their use of technology were more positive about the technology and more apt to use it to enhance their work performance. In a similar fashion, students and teachers might be more apt to use technology in mathematics if the gains in understanding and performance are clear.

Hoyles and other speakers noted that the primary goal should not be the mere inclusion of technology. While calculators are now common in many classrooms, several speakers noted that calculators are often used in three modes: teacher demonstration of a procedure, guided practice where students follow the teacher, and student practice. Little open-ended problem solving is observed. Even though the mathematics that can be done on a calculator may be more sophisticated, in many classrooms the calculator has become an “electronic pencil” rather than a problem-solving, reasoning, and investigative tool. Along with facilitating learning by reducing cognitive load (Karadag 2008), more important is the manner in which technology allows students to interact with mathematics to deepen their learning and comprehension. What is needed, Hoyles and others such as Dick (2008) noted, is not more technology but “well designed micro-worlds where the things that matter are the things that you have commands to change.”

Carolyn Kieran, from Canada, discussed the role of using CAS to improve both conceptual and procedural knowledge for learning at all levels. Since 2002, Kieran has been conducting a large comparison CAS study of academically weak tenth-grade students. At ICME-11, she released data from a study that analyzed the improvements of two classes of these algebra students in both *technique* (being able to

do) and *theory* (being able to explain why and to note some structural aspects), in the context of tasks that invited technical and theoretical development. At the outset, both the CAS class and the non-CAS class scored at the same levels in a pretest that included technical and theoretical components. However, the CAS class improved more than the non-CAS class on both components but especially on the theoretical component. Because students were able to analyze correct answers from their CAS work, they were able to see patterns. Kieran stated, “This kind of assurance, which led the CAS students to theorize, was found to be lacking in the uniquely paper-and-pencil environment where students made few theoretical observations” (2008). In addition, she said that the theoretical observations, in turn, helped improve students’ technical ability.

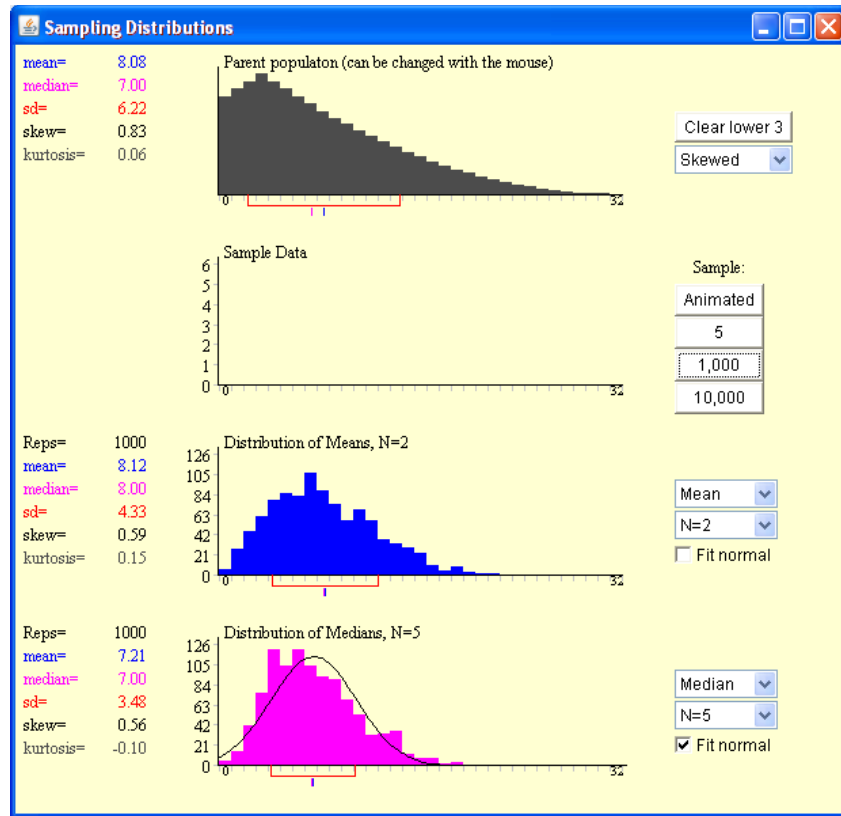
From one extensive review of the literature from Sweden, Per-Eskil Persson (2008) provided evidence for positive gains in understanding made when graphing and CAS-based calculators were used in the classroom. Results from four large meta-analyses (each based on 43 to 79 papers) and nine experiments had similar conclusions: conceptual understanding and problem solving improve, traditional skills are not lost, and lower-achieving students benefit from graphing and CAS calculators. Research on the use of CAS in university mathematics departments in the UK, US, and Hungary was presented by Zsolt Lavicza (2008), who focused on the voluntary results of 1103 questionnaires sent to 4500 university mathematicians. Percentages using CAS for teaching and research were similar across the three countries, with 67 percent using it in research and 55 percent using it in teaching. These results also indicated that university mathematicians use CAS for teaching more extensively than elementary and high school teachers.

Innovations in Technology for Mathematics Teaching and Learning

Many free web-based mathematical applications exist. Software packets with applets are now often included with texts and are also available at no cost to anyone with access to the Internet, allowing teachers and students to investigate what happens to mathematical objects as parameters are changed. For example, applets allow students to observe how the spread and shape of a sampling distribution changes as the sample size increases (see fig. 1) or how the secant line approaches the tangent to a curve as $\Delta x \rightarrow 0$.

GeoGebra includes a dynamic geometry system as well as a spreadsheet and workspace with CAS capabilities. Its developers’ goal is to “provide free and ready-to-use materials for immediate classroom use” (Preiner and Hohenwarter 2008). Examples presented at J. Preiner and Markus Hohenwarter’s workshop resembled the micro-worlds postulated by Hoyles, where students could operate on mathematical objects in multiple representations, change parameters, and examine the results of their actions.

Currently, activities created for particular dynamic graphing software (DGS) programs such as Cabri II Plus, Geometer’s Sketchpad, or GeoGebra must be rewritten to be used in another program, making it difficult for teachers to share lessons and activities across platforms. The aim of Intergeo, a web-based project, is to create a repository of activities that can be readily used across any of these DGS programs, reducing teacher search and preparation time. Created through a collaboration of ten



**Fig. 1. Interactive statistics applet
(Rice Unix Facility 2008)**

European nations, its goals are to “offer content in a searchable and metadata-tagged portal,” to write it in a “common file format based on open standards” in order for the user to access any program of choice, and to test the content in the classroom. “All stakeholders, software teams, resource authors, teachers and learners will be involved, in order to promote quality enhancement cycles” (i2geo n.d.).

The Le@rning Federation (Education Services Australia 2008) is another example of a large-scale project creating free online curriculum materials for teachers. This Australian and New Zealand project creates learning objects such as audio, graphics, texts, and animation that present mathematics (and other educational) topics on a variety of levels from procedural/skill through reasoning and meta-cognitive. For example, the Le@rning Federation’s Wishball Tournament (see fig. 2) is an activity in which students use and develop their place-value, operation, and estimation skills as they strive to reach a target number in a minimal number of turns.

An interesting study that showed promise for teacher education and support was a distance learning program involving teachers in rural areas who would otherwise have difficulty interacting with peers outside of their own school (Evans et al. 2008b). Through interactions between instructors and teachers at a distant site, researchers found the teachers learned how to incorporate more problem solving, active learning, and discussion in the classroom through their experiences in these types of activities. Three additional ICME-11 sessions by Maria Andrade-Archiga and colleagues, Man-

uel Juarez and others, and A. Reyes (2008) discussed the advantages of distance learning for teachers and students using video or computers both in school and at home.

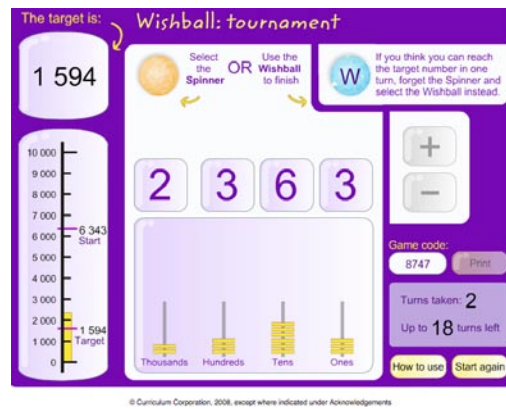


Fig. 2. Wishball Tournament

Counter to the current testing and accountability trend, several presenters at the conference emphasized the need to develop new tasks that cannot be reduced to simple pencil-and-paper interactions (Artigue and Kilpatrick 2008) and to collect examples of new classroom scenarios that illustrate the effective integration of technology and learning (Kyngios and Laborde 2008). These mathematical tasks would move instruction from teacher-guided demonstrations to student-based explorations. Gail Burrill (2008) showed examples in which students change a mathematical object and reflect on the mathematical consequences of that change. For instance, to investigate the properties of a parallelogram that are essential in computing area, figures 3 and 4, from TI-Nspire (Texas Instruments Educational Technology handheld and software), illustrate where changing a point drives a change in the shape.

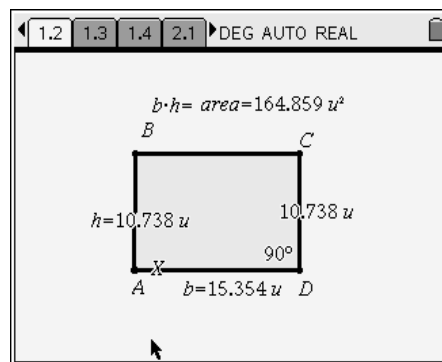


Fig. 3. Area of square

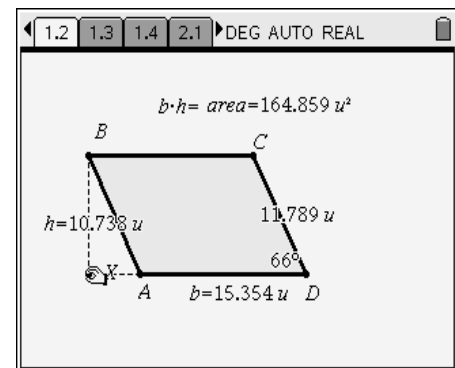


Fig. 4. Area of parallelogram

Anthony Or (2008), of Hong Kong, demonstrated dynamic tasks using Cabri 3D to investigate sections of a cube or to rotate and investigate three-dimensional figures using a three-dimensional DGS (see figs. 5 and 6).

Drag the yellow, green and red points to obtain different sections.
 Drag the blue point to see the section. Click the figure and right-button drag to change view angle.

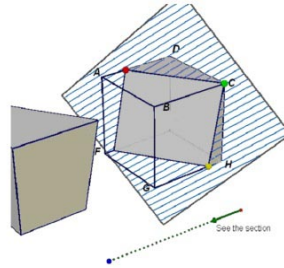


Fig. 5. Three-dimensional shapes

Drag the yellow, green and red points to obtain different sections.
 Drag the blue point to see the section. Click the figure and right-button drag to change view angle.

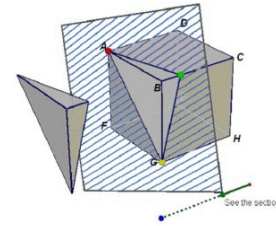


Fig. 6. Sections of a cube

Technology in Mathematics Education in the United States

The United States is a microcosm of the world, with some school districts investing heavily in technology and training of teachers, while other poorer districts are not able to afford even calculators.

Despite the technology recommendations in NCTM's *Principles and Standards for School Mathematics* (2000), Francis (Skip) Fennell (2008) reported that the recent recommendations from the US National Mathematics Advisory Panel (US Department of Education 2008) diminish the role of calculators in teaching. A review of 11 studies that met the panel's rigorous criteria (only one study was less than 20 years old) indicated a limited or lack of impact of calculators on calculation skills, problem solving, or conceptual development over periods of up to one year. The panel cautioned that use of calculators slowed automatic fluency in computation but did recommend computer use for drill and practice (US Department of Education 2008, 50).

Mathematics Education in the United States 2008: A Capsule Summary Fact Book (Dossey, Halvorsen, and McCrone 2008), a report prepared for ICME-11 by NCTM and the United States National Commission on Mathematics Instruction and describing the current state of mathematics education, shows that US students have made steady gains over the years in which calculators and other technologies have found greater use in classrooms. For example, John Dossey reports that ACT math scores have increased (see fig. 7) at the same time calculator use has increased. In 1996, only 11 percent of eighth graders reported using graphing calculators in class; by 2007, this proportion had increased to 50 percent.

The National Assessment of Educational Progress (NAEP) achievement scores for US fourth and eighth graders have shown a steady increase, with their 2004 scores significantly higher than in all previous years (see fig. 8). Scores rose as more technology was being used in classrooms, but the relationship is only correlational and not causal. The claim is not that calculators and other technology increased scores. A number of other factors changed classroom mathematics over the past three decades, such as the development and implementation of a new version of the NCTM Standards and more students taking advanced mathematics. These data simply presented evidence that students were learning more mathematics at the same time calculators and other technology attained greater use in the classroom.

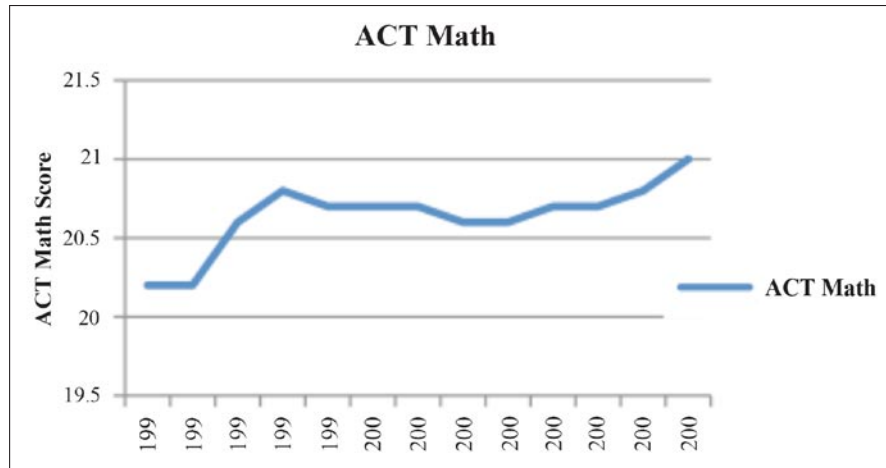


Fig. 7. ACT math scores

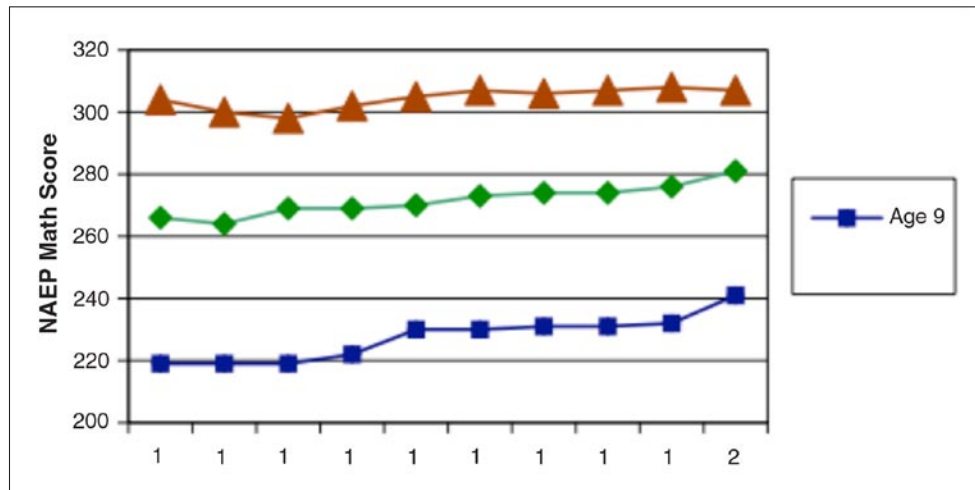


Fig. 8. Longitudinal NAEP scores

The Advanced Placement (AP) Statistics curriculum presents a model for how curriculum and testing can encourage, support, or require the use of technology. Students in AP Statistics are required to learn to use a graphing calculator as part of the curriculum. Printouts from software programs like Minitab are also included in the AP Statistics texts and on the exam, motivating instructors and students to use and explore these statistical programs.

Recommendations for Incorporating Technology in the Mathematics Curriculum

Given the amount and variety of technology available to teachers and the potential of these technologies to transform learning and instruction, how does the mathematics community go forward? Five related recommendations are discussed here: more research on the effectiveness and use of educational technology; making evidence of effective use of technology available to key stakeholders; integrating new technologies in curricula, texts, and assessments; developing repositories of excellent classroom-ready technologies; and addressing global equity issues.

First, in order to evaluate the effectiveness and use of technology in the classroom, more research is needed. Persson points out two issues that need to be addressed—how students’ understanding of mathematical symbols and syntax is impacted by calculator use and teachers’ beliefs and knowledge about use of the calculator in the mathematics classroom. Clearly, educators need evidence of the benefits of new technology in deepening students’ mathematical understanding. This is especially important given the claim of the National Mathematics Advisory Panel that it could only identify 11 studies that investigated the impact of calculators on learning algebra (US Department of Education 2008, 50). Results from research should clarify what happens to “traditional skills” in this era of new technologies and should make clear what students who use technology gain. The results should be understandable and readily available to practitioners and policy makers and not just for researchers, as in the case of the Swedish study. Furthermore, researchers need to be aware that it is at the school level that instructional decisions are made.

Research is needed on why teachers do not use more technology. In conversations at ICME-11 with mathematics educators from Australia, Great Britain, and other nations, teachers and policy makers indicated that while technology was available, teachers used it much less than expected, which is in accord with Hoyles’s report that only 10 percent of teachers welcome instructional technologies and use them effectively. A sustained effort needs to be made to reach more teachers at the elementary, secondary, and university levels. What does a sustained effort entail? What works to help teachers to begin use of and to continue to use the technology?

Research is also needed on how technology can be used in new ways to help students think about mathematics. Tasks that truly use the potential of these technologies, such as student-based explorations on mathematical objects, need to be further developed.

Second, teachers, policy makers, and the general public should be aware of the results of the use of technology in advanced mathematics, research, and other applications. Published research and scenarios provide evidence of use of technology in promoting mathematical thinking. Results of controlled classroom studies or meta-analyses of existing studies should be disseminated to the educational community and wider public if they are to understand the impact calculators (and other technology) can have on basic mathematical skills and conceptual knowledge. Soliciting articles that report the effects of technology for publication in NCTM’s *Mathematics Teacher* and similar journals would be a good first step. Teacher education programs can be a primary source of learning about technology in the classroom. The behavior of new teachers is greatly shaped by the culture of the school, the beliefs of the mathematics department, and the attitude of the veteran teachers with whom they work. NCTM and other organizations can play an important role in making and sustaining transformation at the school level.

Third, change at the classroom level will succeed if technology is integrated in the curriculum, texts, and assessments. Any whole-scale adoption in the US will require rethinking an already overcrowded curriculum that focuses on acquiring skills and algorithms and too often on test preparation. Results from research may help the mathematics education community identify the technology that is viewed as integral to their curriculum. Is it time to move to CAS in the secondary curriculum? Should

DGS become a major factor in the secondary geometry curriculum? If so, in what portions of the curriculum, who will decide, and what will it replace?

It was projected that in 2010, Victoria, Australia, would have a CAS-active secondary mathematics curriculum with accompanying commercial materials, including tests and other assessments. Currently, few US texts incorporate tasks that use CAS or dynamic geometry systems, and too often these activities are enrichment activities skipped by teachers who believe they have an already overwhelming curriculum to complete. Given the breadth of the US mathematics curriculum, it is unlikely that most teachers will devote instructional time to learning and using technology, including calculator and computer screen representations, unless it is integrated more seamlessly into the curriculum and assessments. As technology-rich curricula are written, marketing such materials to US teachers at large will be a challenge. The US curriculum, as well as those in other countries, will have to be reconsidered both to allow time for learning how to use the technology and for designing the deeper investigative explorations that can be supported by technology. One means of scaling up the use of technology successfully is to mandate it on high-stakes testing, such as in Singapore. If national, state, and local school mathematics departments require use of technology on assessments, teachers may begin to see the value of spending class time learning how to use it.

Fourth, in developing repositories of excellent classroom-ready technologies, online sources can be helpful to teachers who wish to use technology in an exploratory manner. Currently, teachers must search the Internet or other resources for instructional aids, without knowing about the usefulness or integrity of the applications they access. For example, while many secondary school teachers use interactive applets to support teaching statistics, calculus, and other classes, a centralized location on the Internet with the best applets as well as free CAS programs and other tools could be helpful in facilitating their use in the classroom. The Math Forum, the National Science Digital Library, the National Library of Virtual Manipulatives, and NCTM's Illuminations have each in their own way presented opportunities to find and use technology in teaching, but what is needed is one convenient and free website with links to high-quality technology experiences and more support for teachers looking for these resources, such as the one proposed by i2geo.

Finally, equity is both a national and international issue that mathematics organizations need to address. As researchers, university educators, teachers, and policy makers work to improve mathematical learning and instruction, they must work toward ensuring that all students, both throughout the United States and around the world, have access to high-quality mathematics, including technology that helps all to become active learners and participants in the global community. International mathematics organizations, like the International Commission on Mathematical Instruction (ICMI), as well as organizations in industrialized countries, like NCTM, need to play a role in helping all students have access to excellent instruction, curricula, and technology. This lack of equity is a reality that mathematics educators must acknowledge and address if all students are to be able to effectively take part in the global economy.

A wealth of technology with great potential for improving instruction is available to teachers. Teachers can transform how mathematics is presented in the classroom. More importantly, technologies can allow students to represent concepts and

interact with mathematical representations more fluidly than teacher or textbook presentations allow. ICME-11 provided participants the opportunity to look across international educational systems and to consider how technology, new curricula, and excellent teaching could transform and deepen the mathematics learned by students moving into the 21st century, a century where technology will transform schools and society in ways that cannot yet be imagined.

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Professional Development across the World

Carrie Chiappetta and Gail Burrill

Sessions at the Eleventh International Congress on Mathematical Education affirmed the fact that professional development of both preservice and in-service teachers is central to mathematics education across the world. Despite differences in approaches, resources, and cultures, many of the models, concerns, and issues related to mathematics education seemed to be universal and not country specific.

This chapter focuses on four questions and how these were addressed during the Congress:

1. What are some professional development models in place in other countries?
2. What common characteristics and issues related to professional development exist across countries?
3. What is the role of technology as a way to deliver professional development?
4. What recommendations emerged from the discussions for providing effective professional development?

Professional Development Models across Countries

In the lecture “Societal Challenges to Math Education in Different Countries,” Masami Isoda (2008) stated that society has two challenges. The first is how to get teachers to change, and the second is how to get students to change. Many nations have instituted changes in their curriculum requirements to reach these goals, but the teacher is the mediator of the curriculum. A common hypothesis seems to be that professional development is a means to get teachers to change which, in turn, will help students change.

According to the lecture entitled “Educating Practicing Math Teachers: What’s Missing in the Literature,” given by Ruhama Even (2008) of the Weizmann Institute of Science, a consistent professional development system for teachers around the world does not exist. Professional development programs can be short-term or long-term, focusing on either content, pedagogy, or both, with or without financial and academic support. Some programs charge individual fees, some are free, and some pay the individual directly or indirectly by granting them eligibility for pay raises through increased graduate credit. The amount, degree, and nature of teacher preparation and how teachers engage in continuing their professional growth, whether it is individual or collaborative, varies from country to country and even within one country.

One international trend now serving as a model for a variety of countries is lesson study, which started in Japan in the 1800s. Lesson study is a cyclical process in which teachers set goals and collaboratively design, conduct, and evaluate research lessons. In addition to the growing popularity of lesson study in the United States, two major international projects involve lesson study. The Asia-Pacific Economic

Cooperation (APEC) Lesson Study Project involves 21 countries including Chile, China, Indonesia, Korea, Mexico, Thailand, and Vietnam. The Japan International Cooperation Agency–Center for Research on International Cooperation in Educational Development (JICA–CRICED) Joint Project is a project between Japan and three regions: Central and South America, Bosnia and Herzegovina, and Afghanistan.

Lesson study is also the model used in a program in Singapore that focuses on professional development for both teachers and school leaders. The four Singapore Centers for Excellence for Professional Development provide professional development for teachers, while the Educational Development Leadership Center provides professional development for school leaders. Each school has a School Staff Developer in charge of professional development.

In 2001, the Mathematics Curriculum Standards of Full-time Compulsory Education were established by the Ministry of Education of the People’s Republic of China. Some districts or individual schools in provinces in China have mandated study groups that meet once a week for a half day to determine daily lesson plans using one centralized standard curriculum. These groups are led by a master teacher and are observed by a training administrator who has a background in mathematics or science education.

Similar to the United States and China, Australia has different laws and regulations governing education in each state. New South Wales implemented new teaching accreditation standards in 2004 requiring all teachers to meet professional teaching standards in professional knowledge, professional practice, and professional commitment, as well as requiring new teachers to go through a mentoring program.

Common Characteristics and Issues

Isoda looked at the APEC Lesson Study Project and listed what teachers said about professional development. She found that a third of those surveyed did not participate in professional development. One-fifth recognized it existed; fewer than half (40 percent) of the teachers indicated that the professional development was in place and working, with only 7 percent saying that it was working well. Perhaps one of the reasons for these divergent responses was that “teacher ‘tradition’ is stronger than teacher education,” according to Christer Bergsten (2008) of Sweden. Many teachers gained knowledge and instructional strategies from the professional development sessions they attended but resorted to their “traditional” ways when they were back in their classrooms.

Characteristics of successful professional development

The common characteristics of successful, sustainable professional development are positive teacher attitude, collaboration, reflection, and both financial and academic support, according to Konrad Krainer (2008). In his lecture, Krainer stated one key to professional development is that it needs to be sustained by being accessible, applicable to the teachers’ needs, and easy to understand and use.

Positive teacher attitude towards change is not always easy to achieve. For example, in New Zealand, secondary math teachers were critical of the Numeracy Project. They resisted putting students into collaborative groups, as they still believed that lecturing was the best teaching strategy for their students.

Even though changes in curriculum in countries such as China had teachers relying less on lecture and working more collaboratively, conference participants from different countries felt that a need existed for professional development to help secondary mathematics teachers move away from lecturing. In her lecture, “Collaborative Learning for Mathematical Level Raising: What Does It Take?” Rijkje Dekke (2008) suggested that there are two types of teachers: the “process teacher,” who stimulates key and regulating activities, and the “product teacher,” who gives content help and is seen as a “traditional” teacher. In one study Dekke cited, the main finding was that students who had a “process teacher” achieved at higher levels than those with a “product teacher” who was oriented to using lecture to deliver content.

Conference speakers from many countries discussed issues involved in helping teachers become more comfortable with collaborating and collectively learning to change their instructional practices. The culture of mathematics teachers seems to be different from the culture among teachers of other academic subjects. An Austrian professional development program focused on having mathematics teachers work together in groups on a project and then disseminate the information to other mathematics teachers. The researchers discovered that mathematics teachers had a more difficult time meeting as a group and seeing the value of such meetings than, for example, science teachers.

Not only do secondary mathematics teachers have difficulty working together, they have trouble working with teachers from other academic disciplines. The group from Austria agreed that working in isolation hinders teachers’ growth in becoming effective teachers and that mathematics teachers have almost a “fear” of working with teachers from other disciplines. The researchers speculated that in work with other disciplines, mathematics is seen as a tool to analyze information and not necessarily as its own entity; moreover, mathematics teachers are seen as the “experts” and therefore need to know all of the answers, which makes some teachers uncomfortable.

Collaboration is an integral part of professional development, according to Krainer. Lesson study is about collaboration; teachers set goals and work together to design research lessons. The focus is not on developing a single lesson—it is about the shared lesson planning process, which is intended to improve teachers’ understanding of mathematics teaching and learning. The planning process includes anticipating possible student answers, identifying common student misconceptions, and creating key questions to ask to advance, clarify, and assess student thinking.

In Sweden, teachers are involved in professional development groups based on their common interests, such as cooperative learning. Thus, the teachers take ownership, and the research is meaningful to them. Teacher study groups in Sweden meet biweekly to read and discuss pertinent literature, apply it to their practice, and then discuss the results of the application. In Brazil, a professional development opportunity was offered to teachers that involved working in small groups on interdisciplinary projects. Such programs, where teachers learn from one another, seem to fit in with the Professional Learning Communities (PLC) movement in the United States.

A third characteristic of effective professional development that goes hand-in-hand with collaboration is reflection. In her lecture “Transforming Math Practices of Learning and Teachers through Digital Technology,” Celia Hoyles (2008) stated that the key for continuing professional development is opportunity and time for teachers

to work together in small communities and networks to reflect on their practice, with the support of a specialist or expert when required. For the reflection to be successful, teachers need to think not only about the content but about the way content is being delivered in their classrooms. Reflection should focus on what students are learning and what works best for the teacher and the student. The personal nature of reflection can also be motivating to teachers.

In one subgroup of Discussion Group 7, Maitree Inprasitha (2008) from Thailand shared that teachers have taken on more of a lesson-study, open-ended approach at the primary level. As a result, teachers are beginning to plan collaboratively and are taking the opportunity to reflect on their practices, and students are beginning to construct knowledge for themselves.

Krainer's fourth characteristic of successful professional development is sustainability through both financial and academic support. Professional development models from various countries around the world are funded in different ways. In some models, teachers pay for the professional development themselves, while in other models, professional development is free. The financial source of teacher reimbursement also varies. One professional development program in England is funded through charitable foundations; therefore, the question arose about the sustainability of such funding.

Because teachers' salaries are so low in some countries, many teachers hold several teaching jobs simultaneously. This makes it difficult for them to attend professional development sessions and receive training beyond their teaching degree. In addition to having limited time for professional development due to their workload, materials and resources also are limited. This scarcity makes it difficult to implement any type of professional development in countries like Colombia.

Teachers in the United States often take part in professional development for extra credits, for pay raises, or to fulfill teacher-certification requirements. In different countries and in certain US states, teachers can be supported by study loans, reimbursement of course fees, new study grants, or full sponsorship for postgraduate studies. Teachers in Israel are required to participate in 56 hours of professional development each year. These professional development sessions can be taken for credit toward an increase in salary. This can be a way to motivate teachers to take the professional development courses.

In addition to financial support, teachers need professional support. The National Center for Elementary School Teachers of Mathematics at the University of Haifa in Israel provided a general framework for professional connections between various educational institutions dealing with mathematics education at the elementary levels. In Brazil, a professional development opportunity was offered to teachers that involved working in small groups on interdisciplinary projects. Faculty from the University of Sao Paulo provided the professional development for teachers who volunteered to be a part of the program, which has continued for the past six years.

Content vs. pedagogy

According to Bergsten, the goal of professional development is to have a "knowledgeable teacher." Being "knowledgeable" means that the teacher should know the content and the pedagogy in order to act in the moment using as many instructional strate-

gies as necessary. Speakers from each country, with the exception of China, said that mathematics content training was needed in professional development. Some professional development programs, such as those described by the study group from Austria, found teachers needed to be more than just “exposed” to the mathematics content. Support, such as classroom observations and debriefing, co-teaching and/or modeling lessons, and discussions about the types of questions that probe for deeper student understanding, was necessary to help teachers better enable their students to learn the mathematics.

During the plenary lecture, “What Do We Know? And How Do We Know It?” (Artigue and Kilpatrick 2008) Jeremy Kilpatrick from the United States shared the viewpoints of those who have influenced mathematics education. Kilpatrick said that Felix Klein highlighted a discontinuity between high school mathematics and university mathematics and suggested updating the school curriculum, taking into account university instruction and the needs of high school teachers. Kilpatrick elaborated on the distinction, raised by Klein, between understanding mathematics and understanding how to teach mathematics. He said that teachers of mathematics need to know the content and how to go more in depth with the content but also have to understand common student misconceptions.

Strategies for providing teachers with professional development for both content and pedagogy varied. During the plenary “Current Trends in Mathematics,” Jose Antonio de la Peña (2008) suggested the worst-case scenario for training middle school teachers is one in which the middle school mathematics teacher has both middle school mathematics content courses and middle school pedagogical training. He argued that elementary pedagogical training and secondary mathematics courses were most beneficial for middle school teachers.

In Portugal, primary teachers volunteered for a two-year program in mathematical content. Classroom observations focused on student learning, not on the teacher. A professional development opportunity in England allowed teachers to take courses at a university with the goal of learning both mathematical content and pedagogy. Each class consisted of a content piece and a pedagogical piece. During the content piece, teachers were taught as if they were the age of the students in their classes; this part of the program lasted 16 months. During the second part of the class, which lasted 22 months, teachers discussed the pedagogy used in lessons so that they could implement these strategies in their classrooms.

Research on effective professional development

In her topic study lecture, “Describing Effective Teaching of Numeracy: Links between Principles of Practice and Teaching Actions,” Tracey Muir (2008) from Australia described how teachers’ beliefs impact the way they reacted to what students do with their understanding of numeracy. In her research, Muir observed teachers’ instruction, tasks assigned, questioning methods, and use of representations and modeling. She also interviewed teachers after their lessons and had them view videos of their teaching. In her conclusions, Muir noted several implications from the study relevant to the delivery of effective professional development. Teachers’ beliefs, knowledge, and achievement of principles of practice influenced their actions, which then reflected the quality of the mathematics instruction. By using open-ended

questions, teachers scaffolded concepts to help students develop understanding and probed further into the thinking of their students. Viewing videos helped teachers see what they could have done to improve the lesson (such as the questions posed). Teachers' expectations, along with the culture teachers have developed in their classrooms, impacted the way students learned mathematics.

Technology as a Way to Deliver Professional Development

Technology can play a part in professional development for teachers. Portals on websites, such as on the UK's National Centre for Excellence in the Teaching of Mathematics site, can connect teachers with online communities, which widen their professional circles. Teachers can work face-to-face via the Internet, doing mathematics together as learners. Technology can also be used to deliver instruction to teachers. They can learn the mathematics and pedagogy online, but personal support within the classroom on how to teach the content to students seems to be important.

Recommendations

Research is needed to answer each of the first three questions in this chapter. Information is missing about the curriculum expectations and the differences in professional development between and within countries. What are the related pedagogies, structures, theories, concepts, and frameworks? Questions arose about the format and structure of the models. Because reports on teacher learning are primarily available in scholarly publications mainly in English-speaking countries, those who do not read scholarly journals or those who do not read English are at a disadvantage.

The second question dealt with common characteristics and issues related to professional development. A major issue, according to Even (2008), is the lack of information about the characteristics of those who are successful in educating and supporting practicing teachers. A question arose about what kind of training is given to those who provide professional development. The people who deliver professional development to mathematics teachers are called by various names, including *professional developers*, *teacher developers*, *teachers of teachers*, *teacher leaders*, *coaches* (in the US), and *teacher educators*. There needs to be research and information about what these titles really mean and if can they be used interchangeably. The lack of consistency in the terms used to describe those people who do the training speaks to the confusion about the training of the trainers themselves.

How does professional development help teachers become better teachers? Research about which forms of professional development are successful and how to develop positive teacher attitudes, promote and sustain meaningful collaboration among teachers, and develop reflective practitioners, as well as what constitutes effective support, is needed.

Strategies for attracting teachers to meaningful professional development must be studied. Almost all of the professional development discussed involved volunteer teachers. In the plenary session, "What Do We Need to Know? Does Research in Mathematics Education Address the Concerns of Practitioners and Policy Makers?" Teresa Rojano (2008) noted more research needs to be done about why mathematics teachers resist collaboration and how to get them to change. In addition, although the value of teacher reflection about their practice was not questioned, the quality of the

reflective activity was. Finally, research on financial and academic support needs to be done to determine the kinds of support that are the most successful and the most sustainable.

The discussion of professional development at ICME-11 suggested that many countries have a similar focus for the professional development of their mathematics teachers and similar concerns about their teachers and the professional development being given. These common foci can help create collaboration among countries. Although duplicating successful professional development models across countries is not practical or sensible, given the different cultures, expectations, and roles of education across the world, learning and adapting from others can help create successful learning experiences for teachers in the United States.

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Rethinking the Primary and Secondary Curriculum in Mathematics: An International Perspective

Martha J. Siegel

Educators from all countries represented at ICME-11 were concerned with the challenge inherent in rethinking the primary and secondary curriculum in mathematics. Educators are grappling with the questions and decisions surrounding the complex issues of the mathematics curriculum for the 21st century. If mathematics is an international language and if everyone is part of a global economy, why isn't there an international mathematics curriculum on which all can generally agree? Are the children, economies, and cultures so very different that educators cannot agree on certain standards? How does a mathematics curriculum foster procedural skills, problem-solving skills, and communication, while providing an atmosphere that leads to positive attitudes for continuing the study of mathematics for those who are capable? By adjusting teaching strategies to best suit the learning style of each student, can the international community agree on some basic learning goals? What practices make a curriculum teachable? Can we assess students on these standards?

From the start of ICME-11, in the plenary session, "What Do We Know? And How Do We Know It?" by Michèle Artigue and Jeremy Kilpatrick (2008), parts of the curriculum were under scrutiny by the mathematics community, while other parts seemed to be the subject of little research activity. Many topic study groups, discussion groups, and lectures touched on these challenges: to design a curriculum, to implement a curriculum, to assess the curriculum, to provide textbooks that reflect the intended (planned) curriculum, and to educate and support teachers as they implement the curriculum. Several speakers at ICME-11 agreed that there is no ideal curriculum. This chapter will discuss what is meant by curriculum, sociopolitical influences on curriculum, curriculum in the United States, and recommendations and reflections.

What Is Meant by Curriculum?

In her presentation, "Mathematics Curriculum: A Vehicle for School Improvement," Barbara J. Reys (2008) defined *curriculum* as "the broad construct of both what society values and expects in terms of mathematics content to be learned in the K–12 school system as well as the materials used by teachers to deliver mathematics instruction to students." Reys stated that this broad definition of what students learn in mathematics includes four forms of curriculum: intended, textbook, implemented, and assessed. Basically, the curriculum includes what the teachers have access to and use and what the students learn. The intended curriculum contains what the curriculum developers (educators, researchers, and the general society) think students should know upon completion of mathematics study at a certain level. This content is called *objectives*, *expectations*, or *standards* and is used for classroom instruction, textbook writing, and assessment (Reys 2008).

In order to determine what a student needs to learn, Jere Confrey (2008), in

“Student and Teacher Reasoning on Rational Numbers, Multiplicative Structures, and Related Topics,” discussed the massive work involved in synthesizing more than 500 research articles on the subject. The synthesis facilitated drawing a detailed concept map that showed learning trajectories for this material. Such research is of enormous benefit to those who design curricula. For example, the research revealed many important links in the development of multiplicative reasoning. One of these, according to Anna Sfard (Sfard and Ben-Zvi 2008), was using “scaffolding” as a technique that helps students at each stage to form the conceptual structures they will need to proceed further in the subject. The research is vital for curriculum developers.

In the schools, mathematics stands alone as an independent subject or is integrated with other curriculum areas. Integrated curricula more commonly exist in the primary grades. Teachers have a renewed interest in integration because increased curriculum demands have limited their instructional time. Although number and algebra are sometimes integrated, more advanced mathematics often is not because it “is an organized sequential body of knowledge that does not lend itself to being easily integrated with other curriculum areas and therefore must be taught in isolation” (Perger and Thomson 2008). Curriculum developers may work in different ways depending on the level.

The second type of curriculum identified by Reys is the textbook curriculum. This includes the textbooks, workbooks, computer technology, and any other material that is prepared to equip the teacher to teach mathematics. These tools specifically enable the students to meet the expectations of the intended curriculum. Denisse Thompson and Sharon Senk (2008) emphasized, “As noted by Begle (1973) and Valverde et al. (2002), curriculum materials have a powerful influence on what students learn.” In some countries textbook adoption is national, and in others, textbook decisions are made independently by cities, districts, or teachers. Thus, the intended curriculum must be effectively incorporated into the textbook curriculum.

The third type of curriculum, implemented curriculum, relates to when the intended curriculum is taught, what is taught, and how it is taught (Vale 2008). Reys noted that the implemented curriculum is “often a function of the district-adopted textbook and/or the individual teachers’ preferences.” Teachers vary in their use of textbooks by choosing certain problems or topics, by switching the order of subjects, by deleting certain topics or reviews, or by using supplementary material, “resulting in an implemented curriculum that looks very different from the intended curriculum” (Reys 2008; from Ball [1996]; Cohen [1990]; Lloyd [1999]; Remillard [2000]). Kilpatrick explained, “Two classrooms in which the same curriculum is supposedly being ‘implemented’ may look very different; the activities of teacher and students in each room may be quite dissimilar, with different learning opportunities available, different mathematical ideas under consideration, and different outcomes achieved” (Artigue and Kilpatrick 2008).

To accommodate the diversity in classrooms, schools may group the students by mathematics abilities that are often determined by a single test. This type of homogeneous grouping, or “streaming,” often widens the achievement gap. The result is that stated expectations may not be met because some students require more time for processing. The opposite approach is heterogeneous grouping. A unique way of teaching heterogeneous classes is a “tiered curriculum” approach, such as that used in

Melbourne, Australia, where some teachers use a differentiated curriculum that “has multiple learning programs and different approaches and activities for the students to meet their particular needs, interests, and preferred ways of learning,” making it “appropriate for the diversity of student need and hierarchical in terms of level of complexity” (Vale 2008). This program allows the student to choose activities based on the recommendation of the teacher, increasing student interest. However, Colleen Vale stated in her presentation that the program is not appropriate for all students because some do not benefit from independent individualized learning and need whole-class lessons. Clearly, coverage may not match stated expectations. In other words, the intended curriculum may not be implemented as intended.

The fourth type of curriculum, according to Reys, is the assessed curriculum, which “refers to the content focus of accountability assessments designed to monitor student learning in relation to the intended curriculum” (2008). International tests such as Programme for International Student Assessment (PISA) or Trends in International Mathematics and Science Study (TIMSS) affect the curriculum in many countries. Curricula may be designed around those of the countries’ students who are most successful on these tests, or they may be tailored to the curricula of other countries that present high test scores. Commonly, curricula may be designed around high-stakes tests.

Educators use different strategies in testing. These strategies also affect curricula. One technique is to test skills and concepts separately. Some representatives argued for prohibiting calculators on one part of an examination and encouraging calculators on another part. Determining whether students can apply what they have learned is foremost in some testing strategies. The assessed curriculum reflects what portion of the intended expectations has been learned from the teacher and textbook. Teachers may have different perceptions of what students have learned when they look at the same results on these tests.

Sociopolitical Influences on the Curriculum

The study of curriculum in each country is both a social and a political science. The goals of the society, the government, and the people in the current environment determine, in large part, what is considered important and essential for children to learn. “Educational objectives and content naturally cannot help but being influenced by cultural and social factors, because education is a vehicle to enculturate the following generation into values of the society” (Iwasaki and Baba 2008). Because each group decides what is important to teach, the curriculum is going to vary between groups. Zalman Usiskin (2008) reinforced the role of culture in teaching mathematics by saying that three criteria were “necessary when making curriculum decisions: (1) importance in understanding or coping in society, (2) importance for future work in mathematics or applied mathematics, and (3) importance in understanding what mathematics is about.”

In “Comparison of Objectives between School Mathematics in the Western Countries and That in the Eastern Countries—Focusing on Upper Secondary Education,” H. Iwasaki and T. Baba (2008) noted that in the US and most European nations (which they called the Western Countries), there was decentralization of mathematical objectives, and in Taiwan, Korea, and Japan (which they called the Eastern

Countries) there was centralism, which reflects the East Asian educational philosophy of “collectivism.” In the Eastern countries, the general formative objectives are concentrated on mathematical thinking, problem solving, and creativity as applied to individual interests and needs without mention of social practicability. In the Western countries, the behavioral objectives are described in more detail, and social objectives focus on forming a civil society and hence stress the application and utilization of mathematics in society.

Cochair of the discussion group “Reconceptualizing the Mathematics Curriculum” Yeping Li, formerly of China and now on the faculty of Texas A&M University, talked about the cultural nature of the mathematics curriculum and the role of the culture in forming and transforming school curricula (Schielack and Li 2008). Through an examination of a common topic in mathematics curriculum materials from the US, Hong Kong, mainland China, and Singapore, Li provided contextual explanations of cross-system similarities and differences in the teaching of algebra using nine eighth-grade textbooks.

The influences on and dimensions of curriculum reconceptualization as discussed at the Congress might be summarized in table 1. The US is influenced by state and national responsibilities and by cultural differences within the country. Portugal, like the Eastern countries of Singapore and China, has a centralized system/standards of education. In Singapore and Portugal only nationally approved textbooks are used. Singapore, which is influenced by international comparisons, has a newly designed curriculum focusing on problem solving, and its students are frequently high scorers on TIMSS.

Table 1. Curriculum comparisons by country

Country	Influences on curriculum reconceptualization	Dimensions of curriculum reconceptualization
USA	State vs. national responsibilities Cultural differences	
Singapore	Centralized system Only approved textbooks International comparisons	Focus on problem solving
China	Centralized standards Parents focus on skills	Traditionally a heavy emphasis on skills Reforms focusing on problem solving
The Netherlands	Both national and school-based summative assessment Emphasis on math for all	Three different strands (vocational, polytechnical, and university) in secondary school Emphasis on conceptual understanding No methodological constraints
Portugal	Centralized system Only approved textbooks Both national and school-based summative assessment	Two dimensions: content and capacities Methodological guidelines given
Denmark		Traditionally subject-matter driven Reform focusing on mathematical competencies

Chinese mathematics educators do not see situational learning and abstract learning as a dichotomy. In writing reform textbooks for all of China, developers stated a goal to avoid such a dichotomy. Although it focuses on problem solving, China's mathematics curriculum traditionally has a heavy emphasis on skills, with the expectation that parents will ensure that students master these skills.

Henrique Manuel Guimaraes, a member of Jane Schielack and Yeping Li's discussion group, "Reconceptualizing the Mathematics Curriculum," was involved with the design of Portugal's national curriculum. He discussed the interaction of the modern syllabus with methodology and the tension between content standards and process standards (2008). He mentioned knowledge versus capacities, understanding versus memorization, conceptual versus procedural, intuition versus rigor, autonomy versus control (for administrators, teachers, and students), and national versus local goals as important issues in the design of curriculum.

Portugal and the Netherlands are influenced by both national and school-based summative assessment and give methodological guidelines in their curricula. The difference between the two countries is that Portugal has two dimensions in its curriculum—content and capacities—and the Netherlands, which emphasizes conceptual understanding, has three different strands for application of mathematics principles at the secondary school level: vocational, polytechnical, and university. Denmark's focus is completely different. Curriculum is traditionally driven by subject matter, and reform is focused on mathematical competencies, not applications.

Curriculum in the United States

As mentioned above, in the past the United States has allowed local control for deciding the intended curriculum. However, two major factors influenced the desire for change. First, American students were not achieving as well as students from other countries on international tests, causing concern that Americans would not be able to compete in the global economy as well as people from other countries (Reys 2008). In addition, "as the population became more mobile and as communication across states around the education of students increased, the great differences in what students experienced in schools in different states became more apparent" (Lappan 2008).

In the United States, sociopolitical influences are evident in the four forms of curriculum that Reys (2008) defined. A push for national standards started in the mid-1980s, and the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards for School Mathematics* in 1989. In 2000 NCTM released *Principles and Standards for School Mathematics*, which called for a coherent and concise curriculum across all grades. In 2002, the US federal government passed the No Child Left Behind Act, requiring every state to create grade-level learning expectations for kindergarten through grade 8. The high school frameworks were articulated as either single-subject or integrated courses. To support the need for curricular coherence, in 2006 NCTM published *Curriculum Focal Points for Prekindergarten through Grade 8: A Quest for Coherence*.

The Center for the Study of Mathematics Curriculum (CSMC), funded by the National Science Foundation and led by Reys, conducted a careful analysis of the grade-level mathematics learning expectations (GLEs) for 42 states. This analysis of the

GLEs led to another set of questions related to the examination of mathematics curriculum standards and materials. This work was meant to challenge those who have responsibility for setting curriculum standards to produce standards that have the quality and clarity to stimulate the development of excellent mathematics programs in schools. Because of differences in the level of specificity, the language used, and the grade placement for the different standards, Reys stated, “While some argue that the US has a national curriculum, namely the NCTM Standards, others believe that interpretation at the state level has introduced variation and therefore continued and advanced the historical ‘local-control’ mentality of curriculum articulation in the US.”

The discussion group “Reconceptualizing the Mathematics Curriculum” chaired by Schielack and Li examined the NCTM Focal Points. According to Schielack (2008), the concept perspective of the Focal Points highlighted the most important mathematical ideas for each grade and presented them as interconnected concepts, skills, and procedures. At a particular grade level these concepts were either in the foreground or background, depending on whether they are being introduced or emphasized or are connecting to other concepts. Schielack said, “The paragraphs explaining the Focal Points highlight the depth of the ideas and give a sense of the range of knowledge, skills, contexts, and problems that might accompany the development of the focal point. The mathematics teachers and instructional leaders can design effective instruction based on focused content supported by appropriate connections, contexts, and tools and addressed through the development and application of mathematical processes.” She felt that the advantage of such an organizational structure is that everyone involved in teaching—teachers, materials developers, test developers, administrators, and the public—can understand what is occurring.

(*Link to http://www.nctm.org/uploadedFiles/About_NCTM/Corresponding_Societies/MathEdUS2008-Dossey.pdf) *Mathematics Education in the United States 2008: A Capsule Summary Book*, written for ICME-11 by John Dossey, Katherine Halvorsen, and Sharon McCrone under the auspices of NCTM and the US National Commission on Mathematics Instruction, has an extensive bibliography. The list gives references for many curriculum documents, starting with one presented as far back as 1894.

Since US teachers have autonomy in their classrooms, the implemented curriculum varies from the intended curriculum. In a study on two field-trial versions of materials developed by the University of Chicago School Mathematics Project (UC-SMP), teachers had to complete so-called Opportunity-to-Learn (OTL) forms, noting whether they taught the content the students would need to answer particular questions. Thompson and Senk (2008) reported that this exercise helped the curriculum developers understand “the conditions under which their materials were field-tested, how differences in achievement among classes might be understood [and how they] related to differences in opportunity to learn, and what revisions are needed prior to commercial publication.”

Even when the intended curriculum is the same, Thompson and Senk identified reasons why implemented curriculum differs: the teachers do not necessarily teach the same content, do not assign comparable problems, and provide different opportunities for students to learn. Also, teachers may cover the content at different times

in the students' educational experiences. Thompson and Senk said, "At the end, assessment just reporting achievement without some indication of teachers' perception of the extent to which the assessment items reflect the implemented curriculum may lead to biased reporting or inappropriate interpretation of any achievement differences." Their insight was gained from teachers using the same textbooks, and they suggested the possibility of more variability if teachers use different textbooks for the same curriculum.

Recommendations and Reflections

The general feeling from interacting with international colleagues at ICME-11 reinforced the commonality of concerns about curriculum and the international desire to use every opportunity to truly engage students where they are and to help them to reach their maximum capability in mathematics.

But Tony Gardner (2008) of the UK asserted that curriculum reform was not necessary. Mathematics teaching is hard, and perhaps educators need to improve rather than reconceptualize. It is like marriage, he claimed. New wives don't make marriage easier; one has to work hard to make it work, no matter what! He and several others warned against the lure of mathematical literacy as a substitute for mathematics and raised the concern that numeracy and quantitative literacy are attractive notions to politicians. According to Gardner, mathematics educators should not allow these to replace school mathematics.

More research is needed to understand the role of curricula in learning. According to Reys (2008), "Progress on understanding the complex role that curricula play in the learning of both teachers and students will help advance the development of future generations of mathematics curricula, of professional development strategies for supporting teacher learning around curricula, and of mathematics teacher education practices."

Teachers' knowledge of the scope of the entire curriculum is a necessary element in their ability to teach at every level. Isolation of teachers from one another and the lack of ongoing communication and collaboration can hamper the implementation of a curriculum. The role of administrators in the implementation process is another factor in what constitutes the implemented curriculum.

Across countries, educators are questioning whether assessments are testing what students should learn. Can we agree on the importance of certain topics, concepts, attitudes, and skills? The writing and selection of test items and the analysis of results play increasingly important roles in making policy decisions concerning school mathematics. It is critical, therefore, that increased attention be given to the emphases in the curriculum. Research activity also requires the creation of tools and methodologies for evaluating the impact of curriculum on teacher behavior and student learning. Researchers need to monitor the influence of high-stakes assessments on decisions teachers make regarding the implemented curriculum.

Research is needed when assessments are being compared. Said Senk, "Given this variability in implementation of curriculum, it is essential that curriculum developers and researchers document the variability and find some way to report implementation as part of research reports about curriculum and achievement. Without such documentation, it is difficult to interpret results or to make comparisons

between classes.” What is the role of all the supporting materials, including textbooks and in-service teacher education, in making it possible for the intended curriculum to be implemented in every school, by every teacher, for every student? When results on assessments differ, is the difference explained by a difference in intended curricula, textbook curricula, implemented curricula, or assessed curricula?

At the foundation of decisions on curriculum is the question, “What is considered ‘educated’ in a culture?” Is high-quality mathematics education a human right? Can people agree as a society that knowing some reasonable level of mathematics opens opportunity for all citizens and provides, like reasonable skill in language arts, gateways to an educated electorate and a skilled workforce? Are all children to be educated to be able to do the same mathematics? Are educators preparing a workforce with specific needs? Is there sufficient support for the most gifted? Are there common goals in mathematics curricula across the globe?

Perhaps it is possible to design an international scope and sequence for what all students should know and be able to do. The value of international meetings such as ICME is that the research community can begin to tighten research standards and define paradigms for further evaluation of global curricula.

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Two General Issues in US Mathematics Education

The following section contains papers written about two broad issues related to mathematics education in the United States, preparing and supporting teachers and assessment, informed by discussions and experiences from ICME-11.

The Role of Universities in Preparing Mathematics Teachers: Insights from the International Congress on Mathematical Education

Yvonne Lai, William McCallum, Hortensia Soto-Johnson¹

What mathematical and pedagogical content do preservice teachers master? What factors in the preparation of teachers predict their future students' achievements?

The first purpose of this report is to compare a selection of Asian and European teacher preparation programs with US teacher preparation programs. Then efforts related to mathematical knowledge needed for teaching will be described. Among other goals, this recent strand of research seeks to identify the mathematical content mastery of teachers that predicts their students' success. The components, measuring, and the application of Mathematical Knowledge for Teaching for secondary mathematics teachers will be described. Finally, conclusions with brief recommendations for universities to improve the preparation of future mathematics teachers will be made.

For the purposes of this article, mathematicians are members of college or university faculty who primarily teach mathematics courses or actively participate in mathematics research. Mathematics education researchers are college or university faculty who may teach mathematics courses but whose primary responsibilities involve conducting mathematics education research and supervising future teachers or teaching mathematics education courses. Teachers are those who teach classes in kindergarten through grade 12. Although these communities may overlap, it is useful to distinguish between them in order to address how they can interact and contribute to the preparation of future mathematics teachers.

Teacher Preparation Programs and Professional Development

In *Understanding Others, Educating Ourselves*, a report by the National Research Council board on international comparative education studies, the authors argue that cross-national comparisons may sharpen and expand the practices of US education-policy makers and researchers. International comparative studies can help “define what is achievable,” “observe and characterize consequences of different practices and policies for different groups, under different circumstances,” and “identify and question beliefs and assumptions that are taken for granted” (Chabbott and Elliott 2003, 8–9). Moreover, the methods, models, and goals of the US education system have been influenced by those of England, France, Germany, Italy, and Japan (Chabbott and Elliott 7).

Teacher preparation programs from abroad

In the past two decades, US policy makers and the academic community have begun searching for factors in teacher preparation that predict student success. The data sets

for such studies are varied and include international comparisons. In these international studies, data sets range from large-scale surveys to detailed analysis of a few classrooms. Phenomena concerning student achievement emerging from international studies are necessarily confounded with sociocultural, educational, and environmental effects. However, the information may shed insight on the design of future revisions and studies of US teacher preparation.

The Trends in International Mathematics and Science Study (TIMSS) is perhaps one of the most recognized international comparisons. TIMSS, administered by the International Association for the Evaluation of Educational Achievement (IEA), evaluated fourth- and eighth-grade students in 2003 and 2007, and eighth-grade students in 1999 (2008). Among all countries participating in the TIMSS, five significantly outperformed the US in all three years: Chinese Taipei, Hong Kong SAR, Japan, Korea, and Singapore. Some potential differences in the philosophy and implementation of teacher training programs in Chinese Taipei, Korea, and Japan are briefly highlighted below.

In Chinese Taipei, teacher preparation has been reformed to include a yearlong induction program (Darling-Hammond 2007). The Taiwanese and Korean teacher preparation programs are compared to that of the US in the Mathematics for the 21st Century study (MT21), which William Schmidt presented at ICME-11 (Schmidt et al. 2008). In “Lesson Study in North America: Progress and Challenges,” Catherine Lewis notes teacher educators in the US have shown a considerable interest in Japanese lesson study (2006, 2-4). Japanese lesson study is a research cycle and professional development practice for elementary school teachers.

In recent years, Finland has become a high achiever on another international evaluation, the Programme for International Student Assessment (PISA). Finnish teachers are included in the discussion because they are also required to master educational research methodology.

In comparing the teacher preparation of Taiwanese, Japanese, Korean, Finnish, and American teachers, four areas in which differences may occur are ongoing professional development, teacher background in educational research and pedagogical studies, teacher mathematical background, and the presence of a professional, collegial community for teachers.

In reports a decade ago, the achievement gap between American students and those from Taiwan and Korea was suspected to come from flawed US math curricula (Schmidt et al. 2008, 6). However, the difference in achievement may not only stem from a curricular gap for the students but also from a “preparation gap” in preservice teacher programs. The authors of the MT21 report provide an argument for the existence of such a “preparation gap” through a study and analysis of future middle school teachers from six countries,² including their pedagogical and mathematical coursework. In that report, the term *pedagogical coursework* means coursework that addresses “mathematics that has a direct relationship to the teaching of school mathematics,” that is, courses that aimed for “deeper understanding” of topics in the typical middle school curriculum (Schmidt et al. 2008, 31–32).

The MT21 findings indicated that Taiwanese and Korean preservice teachers expressed satisfaction with pedagogical coursework, whereas “US future teachers did not feel that they had had adequate opportunities” to learn the mathematics needed

for teaching (Schmidt et al. 2008, 32). The sample used for the MT21 study was not statistically representative of all middle school teachers in the countries surveyed. However, efforts were made to include a “reasonably representative sample of the whole country including some of the variation found across all teacher preparation institutions in the country” (18).

The “preparation gap,” in terms of teachers’ satisfaction with the pedagogical training opportunities available to them, is further supported by the data gathered on German students by both MT21 and TIMSS. Fourth-grade students from Germany and the US earned comparable scores on the 2007 TIMSS. The German and US scores fall into the intermediate category of mathematical understanding, where “students can apply mathematical knowledge in straightforward situations.” In contrast, countries with top averages on the TIMSS fell into the category of high understanding, where “students can apply their knowledge and understanding to solve problems” (Institute of Education Sciences 2008). The German teachers surveyed by the MT21, like the US teachers, did not express satisfaction with the opportunities afforded to them for pedagogical training (Schmidt et al. 2008, 31);³ both the German and US future middle school teachers were significantly less satisfied than the Taiwanese and Korean future middle school teachers. These data suggest further investigation into the cultural or curricular factors contributing to the difference in satisfaction, so as to characterize a more precise relationship between the difference in satisfaction of teachers and the relative performances of German, US, Taiwanese, and Japanese middle school students.

Among those in the MT21 sample, Taiwanese and Korean preservice teachers possessed on average the highest mathematical knowledge in two ways. First, the Taiwanese and Korean preservice teachers attained on average the highest scores on exams administered in algebra, functions, number sense, geometry, and statistics. Secondly, Korean and Taiwanese preservice teachers completed on average the most advanced coursework: abstract algebra, calculus, multivariable calculus, differential equations, functional analysis, complex functions, differential geometry, and topology. Typical future teachers in these two countries studied between 79 and 86 percent of these topics, whereas preservice teachers in the US usually completed coursework in less than 50 percent of these topics (Schmidt et al. 2008, 28). It is worth noting that some research suggests that there is not a statistically significant correlation between teachers completing units of coursework beyond calculus and student achievement in the US (Begle 1979). Thus, although there is a striking contrast between the level of advanced mathematics taken by US teachers in comparison to teachers from Taiwan and Korea, whatever impact teacher knowledge of advanced mathematics has on student achievement may arise from factors more subtle than the sheer number of required courses.

As in Taiwan and Korea, Japanese preservice middle school teachers are required to complete more mathematical coursework than US preservice teachers. Furthermore, Japanese middle school teachers continue their professional development with lesson study—a four-phase cycle centered on the analysis of live classroom lessons. The following description of lesson study comes from Catherine Lewis (Lewis, Perry, and Murata 2006) and Tad Watanabe (personal communication, 2008).

In the first phase, a group of teachers examines curricula and devises goals

related to the immediate and long-term mathematical development of their students. In the second phase, the teachers select, revise, or write a lesson plan that addresses these goals, as well as “anticipated student thinking, [a] data collection plan, models of learning trajectory, [and a] rationale for chosen approach.” In the third phase, one of the teachers teaches the lesson while the others collect data. In the fourth phase, the teachers participate with others in a “formal lesson colloquium. . . to illuminate student learning, disciplinary content, lesson and unit design, and broader issues in teaching-learning” (Lewis, Perry, and Murata 2006, 4).

In Japan, the practice of lesson study ensures that in-service teachers do not create or deliver lessons in isolation; they conduct continuing research on lessons and are able to create lessons based on students’ comprehension and preconceptions of mathematics. The practice of lesson study demonstrates a cultural commitment to the preparation and continued development of teachers’ professional knowledge, as well as to the practice of educational research in mathematics.

Another high-achieving country is Finland, whose teacher preparation programs in both lower and upper secondary levels also emphasize research. Fifteen-year-old students in Finland attained the highest overall score and the second highest average score (after Hong Kong SAR) on the 2003 Programme for International Student Assessment (PISA), which emphasized mathematics that year (OECD 2004). Additionally, Finnish students have placed consistently in the top few score rankings in all categories tested by the PISA since its inception.⁴ The case of Finland is particularly interesting, as the country has historically been a low achiever.

In Finland, all teachers attend two to three years of government-funded graduate training; commonly encompassed in this training are a year of training in a school partnered with a university and a master’s thesis entailing genuine research methods and analysis. Moreover, teachers in Finland learn to “develop and evaluate local performance assessments”; they are “sophisticated diagnosticians, and they work together collegially to design instruction that meets the demand of the subject matter as well as the needs of their students” (Darling-Hammond 2007, 68–69). Finnish teachers are given the background to view their classrooms with the perspective of a researcher.

Finnish teacher preparation programs require graduates to be fluent in education research methodology; they complete courses on research techniques which they apply to a master’s thesis, mandatory for all prospective teachers, whether they will go on to become elementary or upper secondary teachers (Kansanen 1999). The master’s theses produced show great command of conducting research: teachers write theses of typically 80–120 pages in length, and these documents include a literature review, a report on data collection methods, and data analysis. Once in the workforce, Finnish teachers are entrusted with “a considerable degree of decision-making authority as concerns school policy and management,” including “almost exclusive responsibility for the choice of textbook” (Kansanen 1999, 44).

At ICME-11, discussion among participants revealed that teacher programs are extremely competitive in Finland. Such a high degree of independence may be especially effective in Finland because of the competitiveness and depth of its teacher preparation programs. Indeed, lower secondary teachers complete a program to which only 10 percent of all applicants across all universities are accepted (Valijarvi et al. 2002, 42); only 15 percent of all applicants to any preparation program, upper

or lower secondary, are accepted (Kansanen 2003, 3). This selectiveness may only be one part of the Finnish success; the quality of the teachers may also be related to teachers' background in educational research methodology, the responsibilities of the teachers once they are in the workforce, and the type of support for teaching the government provides.

The government has backed this philosophy; the Finnish national LUMA program has fueled "great efforts" to use technology in schools, "enhancing teacher training in both subject and pedagogical studies; and increasing experimentation" (Valijarvi et al. 2002, 23). As Valijarvi argues, the combination of government support for scientific inquiry into educational practices, excellent teachers, and high teacher autonomy may be strong factors in the high Finnish performance on the PISA. Given the success of both Finnish and Japanese students, and the emphasis on teacher-as-researcher in both nations, the ability of teachers to observe and reflect upon their classroom with a scientific eye may also contribute to the success, and help reinforce the other potential strengths, of the Finnish system.

Comparison with US teacher preparation

In comparing US teacher preparation programs with those profiled in the previous section, the following areas will be discussed: the Conference Board for the Mathematical Sciences (CBMS) recommendations for teacher preparation, induction and mentorship of prospective and new teachers, and potential obstacles to forming local professional communities of teachers.

The CBMS recommends that preservice elementary teachers in the US complete two to three mathematics courses and preservice secondary teachers complete 40 hours of mathematics courses (2001).⁵ Along with mathematics courses, all preservice teachers enroll in education courses and complete field work. Education courses include educational psychology, technology in the classroom, classroom management, special education, and integrating reading into the content area. Student teaching comes at the end of the certification program, usually during the last semester of the senior year. Thus, the bulk of pedagogical coursework occurs before the preservice teachers enter the workforce. As a result, the pedagogical ideas learned in a preparation program may be theoretical rather than practical, and there is no widespread, continued professional development available while teaching.

In the US, the supervision of student teaching varies throughout the country and even within the same department. A mathematics educator, a faculty member from the education department, a graduate student, or a practitioner may serve as the university field supervisor, which may include three to five visits during the semester. In some cases, the formal mentorship provided during student teaching may not continue. As cited in Glazerman and colleagues (2008, A-2), a 2003–2004 US Department of Education report found that only 35.2 percent of K–6 teachers have an induction period that included "a mentor in the same field, supportive communication, common planning time or regularly scheduled collaboration with other teachers in their subject area, and participation in a seminar for beginning teachers." Another 27.6 percent of teachers had "mentorship and supportive communication" without all the other components listed above, and the remaining 35.2 percent lacked both a mentor and supportive communication with an administrator or department chair (as cited in Glazer-

man et al.[2008], A-2)⁶. As for the subject area of mathematics, the National Science Board found that 66 percent of middle school and high school mathematics teachers participated in a teacher induction program (National Science Board 2004, 31). Through surveys of teachers and district representatives, Hall found that only 33 states require mentoring programs; of those, 22 states both require and finance mentoring programs for new teachers, and 23 mandate mentor training (Hall 2005, 213–216).

The US Department of Education's 2003–2004 Schools and Staffing Survey reported that only 48.1 percent of all public school teachers in the US have completed a master's degree or higher, and experience in educational research is not required of US teachers. "In the United States, researchers and teachers are separate groups" (Kessel and Ma 2001, 473). US professional development programs lack widespread government support, and new US teachers in general lack the support of a professional community of colleagues.

Dedicated time for collegial discussions about teaching may be difficult to schedule for many US teachers because of their supervisory duties. Indeed, LeTendre and others (2001) found that "US teachers reported a work week that was significantly influenced by planning for (and adjusting to changes in) non-teaching work." In contrast, "few German teachers reported supervisory duties," and Japanese middle school teachers reported flexible supervisory duties (LeTendre et al. 2001, 8) as well as reduced responsibilities in their first year of teaching (Darling-Hammond 1999, 341).

The MT21 study found that there may be a pedagogical preparation gap in US teacher programs compared with Taiwanese and Korean programs. This preparation gap included deeper understanding of mathematics needed for teaching. The practice of lesson study and Japanese students' high achievements on TIMSS suggest that teacher research on knowledge relevant to teaching, including students' ways of understanding, may be critical to successful teaching. Finnish teachers are entrusted with a high degree of autonomy in their schools, including the choice of textbook, a decision that likely requires understanding of students and the curriculum.

Two important features of teacher preparation programs may be the teaching of deep understanding of school mathematics as well as of student knowledge. One of the most promising areas of research related to these ideas is Mathematical Knowledge for Teaching, a collaboration between mathematicians and mathematics education researchers that will be discussed in the next section.

Teachers' Mathematical Knowledge

A central issue discussed at ICME-11 and related to the role of universities in preparing future teachers was the topic of Mathematical Knowledge for Teaching (MKT), "the mathematical knowledge, skills, habits of mind, and sensibilities that are entailed by the actual work of teaching" mathematics (Bass 2005). MKT includes the pedagogical preparation that the MT21 study considered, namely, "deeper understanding" of "mathematics that has a direct relationship to the teaching of school mathematics." Explicating exactly what MKT constitutes in the US, including mathematical content, has been the subject of recent research.

By its nature, the identification of MKT requires the collaboration of mathematicians, mathematics education researchers, and teachers. The description of the features

of MKT can benefit from the expertise and personal experiences of all three communities. In fact, the notion of MKT was pushed to the forefront of current research via collaborations between Deborah Ball (Ball et al. 2008), a mathematics educator, and the mathematician Hyman Bass. Their work is heavily drawn from Ball's experiences as a practitioner and the experiences of members of her research team, Mark Thames and Geoffrey Phelps, who based their work on US elementary school mathematics. Additionally, a number of teams have examined MKT for secondary school mathematics, and some of their efforts and examples are described below.

Components of Mathematical Knowledge for Teaching

Over the past decade, the working definition of MKT has been refined. A hierarchy of the elements of MKT, as proposed by Hill, Ball, and Schilling, is summarized in figure 1. The purpose of such categorizations is to help design instruments that will identify factors in teachers' mathematical knowledge that contribute to their students' achievement.

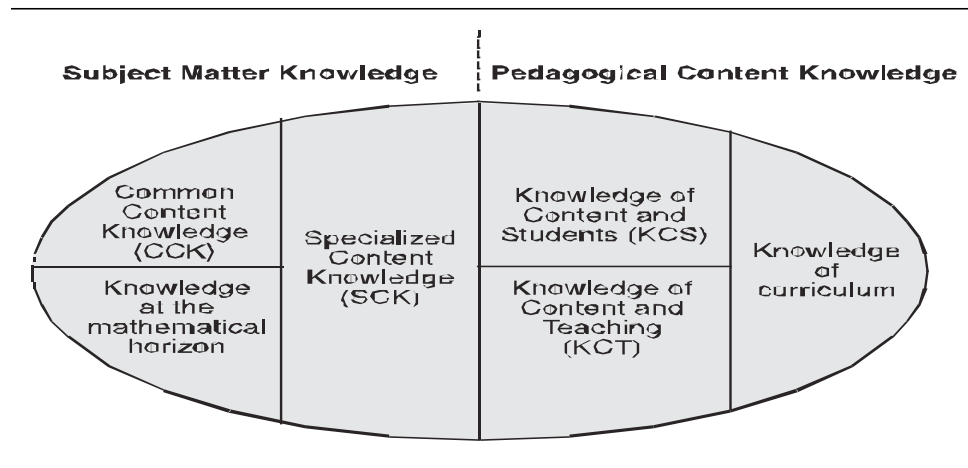


Fig. 1. Domain Map for Mathematical Knowledge for Teaching, as defined by Hill, Ball, and Schilling (2008)

The “deeper understandings” of mathematical topics that arise in teaching, considered by the MT21 study, correspond to the Subject Matter Knowledge of MKT. The constructs comprising Subject Matter Knowledge are Common Content Knowledge, Specialized Content Knowledge, and Horizon Knowledge. As Kessel and Ma (2001) discuss, the specific mathematics associated with these constructs may vary by country; here, we are interested in Ball's work, which concerns the US.

Common Content Knowledge is “the mathematical knowledge and skill used in settings other than teaching.” In instruments used to measure MKT, questions assessing Common Content Knowledge may include, “What is a number that lies between 1.1 and 1.11?” Answers to questions on Common Content Knowledge may “require knowing that a square is a rectangle, that $0/7$ is 0, and that the diagonals of a parallelogram are not necessarily perpendicular. These are not specialized understandings but are questions that would typically be answerable by others who know mathematics” (Ball, Thames, and Phelps 2008, 399).

Specialized Content Knowledge is mathematics that “is not needed or used in settings other than mathematics teaching. This uniqueness is what makes this content knowledge special” (Ball, Thames, and Phelps 2008, 396). One of the many components of Specialized Content Knowledge is error analysis, for example, in a potentially unexpected error such as $307 - 168 = 169$. In this case, the student has “‘borrow[ed]’ 1 from the hundreds column, ‘carr[ied] the 1’ to the ones place, and subtract[ed] 8 from 17, yielding 9. The process continues by ‘bringing down’ 6 and calculating $2 - 1 = 1$.” Subtraction is a procedure that many students struggle to learn. To help their students, teachers must perform error analyses swiftly, as students in a real-time classroom cannot wait for a teacher to ponder the motivation for errors. “Analyses such as this are characteristic of the distinctive work teachers do, and they require a kind of mathematical reasoning that most adults do not need on a regular basis” (Ball, Thames, and Phelps 2008, 397).

Creating examples such as the above serves several purposes. First, the work initiates conversations between mathematicians and mathematics education researchers about designing instruments that measure MKT in preservice and in-service teachers. These examples are also teaching tools that mathematics education researchers could use with preservice and in-service teachers in coursework or professional development activities. Mathematics and the teaching of mathematics can be used as common ground for meaningful conversations between mathematicians, practitioners, and mathematics education researchers.

The last category of MKT, Horizon Knowledge, is a provisional category seeking to describe “an awareness of how mathematical topics are related over the span of the mathematics included in the curriculum. First grade teachers, for example, may need to know how the mathematics they teach is related to the mathematics students will learn in third grade to be able to set the mathematical foundation for what will come later” (Ball, Thames, and Phelps 2008, 403).

Measuring MKT

Since the early 2000s, Ball and Hill have led efforts to develop measures for MKT, now tested in samples of over 2000 US students and their teachers (Hill, Ball, and Schilling 2008; Hill, Rowan, and Ball 2005; Hill, Schilling, and Ball 2004). The results indicate a relationship between MKT in professional development and modified classroom practices and improved student learning, which gives a potential explanation for the TIMSS and MT21 findings regarding US students and teachers discussed in the first section. Past studies have mostly shown inconclusive correlations. For example, Begle’s study (1979) did not indicate statistically significant correlations between units of courses taken beyond calculus and student gains. Monk (1994) suggests that while teachers’ taking four courses beyond calculus might be associated with improvement in US high school student performance, there was not much impact from the fifth course and beyond. Hawkins, Stancavage, and Dossey (1998) found US fourth-grade students’ performance was higher for those whose teachers majored in mathematics education rather than in mathematics.

In light of previous trends, the findings of Ball, Hill, and their colleagues are especially promising. They have developed a construct—namely, mathematical knowledge for teaching—where expertise correlates positively with student gains. The

assessment and conception of this construct is the product of approximately 15 years of development work, and the power of the construct in part comes from the careful consideration of components forming MKT and their potential for assessment.

As reported by Sharon Senk (Senk, Thompson, and Johnson 2008) at ICME-11, the first rigorous large-scale international investigation of teachers' background and knowledge, including their MKT, is underway. The Teacher Education and Development Study in Mathematics (TEDS-M) is the product of collaborative efforts by members of Michigan State University and the Australian Council for Educational Research; it will profile teachers in 19 countries.⁷ TEDS-M builds upon the efforts of MT21, with many of the TEDS-M instruments based upon those of MT21. According to Senk (personal communication, 2008), the TEDS-M data, which have already been collected, include probability samples of approximately 400 elementary and middle school teachers from approximately 50 institutions in the largest participating nations. For nations that produced fewer than 400 elementary and middle school teachers per year, sampling plans were tailored to each country, with the consultation of a specialist from Statistics Canada and a representative of the Data Processing and Research Center in Hamburg, Germany, of the International Association for the Evaluation of Educational Achievement. Initial international reports were released in 2010.

MKT for secondary mathematics

Research on MKT related to secondary mathematics has been investigated by several teams, including the Situations Project with principal investigators from Penn State and the University of Georgia, the Center for Mathematics Education associated with the Education Development Center (EDC), Knowledge of Algebra for Teaching (KAT) with principal investigators at Michigan State University, and KnotSS (Knowledge for Teaching Secondary School) with principal investigators at the University of Arizona Institute for Mathematics and Education.

In MKT for secondary mathematics, we also find collaboration between mathematicians, teachers, and math education researchers. The work of Al Cuoco and colleagues (1997) at EDC has included fostering "habits of mind" needed for secondary mathematics; one of his projects is Focus on Math, a partnership between mathematicians, math education faculty, and teachers.

KAT proposes a framework of "core concepts/procedures, representations, applications, and reasoning/proof" for algebra. Kathleen Heid (2008, 9) of the Situations Project reported at ICME-11 that the current working definition for MKT for secondary teaching includes "mathematical processes of symbolizing/representing, proving/justifying, defining, and generalizing." Below is a hypothetical but representative preconception that students possess about symbolic representation (from McCallum [2007], 2), who is a principal investigator for KnotSS:

$$\begin{aligned} \int \frac{1}{2x^2 + 4x + 4} dx &= \int \frac{1}{x^2 + 2x + 2} dx \\ &= \int \frac{1}{(x+1)^2 + 1} dx = \arctan(x+1) + C \end{aligned}$$

The student has factored a 2 from the denominator and then lost it. McCallum

observes, “(T)here is a situation in which it is quite permissible to lose the factor of 2, namely in solving the equation $2x^2 + 4x + 4 = 0$. For a student with a weak grasp of the difference between equations and expressions, the superficial procedural similarity between solving equations and transforming expressions is a snare.”

McCallum suggests that the failed integration may arise from a conceptual understanding of the sort of symbolic representation covered in a typical high school algebra curriculum, rather than a careless mistake. From this discussion, McCallum supports the position asserting that proficiency should be evaluated on more than simple procedural fluency; assessment should consider conceptual understanding, strategic competence, adaptive reasoning, and productive disposition as well (Kilpatrick, Swafford, and Findell 2001). Since rich assessment of algebra is often delayed until functions are introduced, it is possible that these errors won't be diagnosed until after students complete their course on algebra. Furthermore, student preconceptions may depend on their prior experiences, including instruction and curriculum.

One motivation for investigating MKT for secondary school mathematics is the Conference Board of the Mathematical Sciences' recommendation for “developing deep understanding” (Conference Board of the Mathematical Sciences 2001, 13) in prospective teachers. A more nuanced notion of student preconceptions may help develop measures of MKT for secondary school concepts, as well as inform pedagogical methods taught to preservice secondary school teachers. One goal of describing MKT is to find measurable forms of knowledge that impact student achievement and that can be used as a basis for teacher preparation programs.

Conclusions

One question broached at ICME-11 was: As components of MKT are identified, are they teachable to future teachers? If MKT is teachable, then teaching it would require collaboration between mathematicians, mathematics education researchers, and teachers. These three communities are valuable contributors to the strands comprising MKT for elementary, middle, and secondary mathematics. Teachers can at the very least provide guidance regarding curriculum, knowledge about students, and knowledge about teaching. Mathematics education researchers can serve as experts in the knowledge about students and teaching constructs. Mathematicians can work with mathematics education researchers to identify productive habits of mind and processes of mathematical reasoning, as well as intellectually honest treatments of mathematics. These three communities together can make concerted efforts to improve our teacher education programs. This may involve coteaching courses, coauthoring curriculum materials, or conducting research together.

Efforts to incorporate such strategies have begun but are not standard practices. For example, Grassl and Mingus (2007) detail the benefits of coteaching an undergraduate abstract algebra course through the eyes of a mathematics educator and mathematician. A number of mathematics texts and curriculum materials showcase the contributions of mathematicians and mathematics education researchers, but these tend to be mathematics texts for preservice elementary teachers. Few such mathematics texts designed for preservice secondary teachers exist. On the other hand, mathematicians and mathematics education researchers have made great strides in collaborating on mathematics education research; Ball and Bass are a prime example.

Mathematicians are challenged to embrace communication with teachers and mathematics education researchers through coauthoring textbooks designed for prospective teachers, in coteaching courses where prospective teachers are the primary audience, and in ongoing professional development for practicing teachers.

Teaching and learning are areas in which it is difficult to isolate factors with statistically significant impact. Mathematical Knowledge for Teaching is a promising strand of research, but it is only one avenue of research. The construct of MKT was first studied by Ball and Bass, and the validity and reliability of their assessment instruments were determined in collaboration with those fluent in policy work and statistics. To determine other potential lines of research, it may be fruitful to examine features of teacher preparation in high-achieving countries and hypothesize culturally appropriate analogues to study in the US. The process of such research will require an amalgamation of a vast array of professional knowledge and skills. Interdisciplinary efforts that include the expertise of teachers, mathematicians, mathematics education researchers, cognitive scientists, psychologists, and other social scientists to investigate features of teacher training programs that positively impact student performance are needed.

Notes

1. The first author is partially supported by NSF-RTG DMS-0602191. The second author is partially supported by NSF DUE-0525009.
2. Bulgaria, Germany, Korea, Mexico, Taiwan, and the United States.
3. German students averaged 525, and US students averaged 529; the top average was 607, achieved by Hong Kong SAR, and the lowest score was obtained by Yemen, at 224. The mean score across all countries for fourth graders was 500. We cannot compare German and US achievement for eighth grade, the other grade tested by TIMSS, as German eighth graders did not participate in TIMSS.
4. The Programme for International Student Assessment (PISA) tests 15-year-olds from principal industrialized countries (including the US) on skills and knowledge in mathematics, science, and literacy. According to the PISA website, every three years, PISA assesses “how far students near the end of compulsory education have acquired some of the knowledge and skills essential for full participation in society.” The focus of the PISA rotates through mathematics, science, and literacy; the design of the 2003 PISA emphasized mathematics.
5. As described in the CBMS requirements for teachers (6).
6. The information cited here is taken from the US Department of Education 2003–2004 Schools and Staffing Survey, as reported by Glazerman et al. (2008, A–2)
7. Botswana, Canada, Chile, Georgia, Germany, Italy, Malaysia, Mexico, Norway, Oman, Philippines, Poland, Russia, Singapore, Spain, Switzerland, Taiwan, Thailand, and the United States.

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What About the Assessment Gap? We Need to Address It—Now!

Francis (Skip) Fennell, Mike Caraco, Beatriz S. D'Ambrosio, Felicia Phillips, Karen Mirkovich, Tom Reardon, and Gail Yates

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Commentary on Critical Issues in Mathematics Education: *What About the Assessment Gap? We Need to Address it—Now!*

Francis (Skip) Fennell, *McDaniel College, MD*

Mike Caraco, *Burr and Burton Academy, VT*

Beatriz S. D'Ambrosio, *Miami University of Ohio*

Felicia Phillips, *Atlanta Public Schools, GA*

Karen Mirkovich, *Prince William County Schools, VA*

Tom Reardon, *Fitch High School and Youngstown State University, OH*

Gail Yates, *Christian County Schools, KY*

The following thoughts have been influenced by sessions related to assessment presented at the 11th International Congress on Mathematics Education (ICME) held in Monterrey, Mexico in July, 2008.

The No Child Left Behind (NCLB) Act (2001) has forced schools and school districts to not only account for the mathematics performance of all students, including student subgroups, but to also to publically report performance data and identify plans to strengthen that mathematics performance.

But it has also spawned assessment and teaching practices that overemphasize state assessments and created a “teach to the test” mentality as an effort to ensure that more and more students reach a school’s Adequate Yearly Progress (AYP) benchmark.

To reach AYP levels and demonstrate proficiency, many state assessments skim the surface as they attempt to measure state curriculum standards that often contain far too many expectations, address concepts and skills that are less than important, overemphasize skills, and have far less emphasis on complex content, problem solving, and rich mathematical problems that require students to show their work. (There are some notable exceptions to this situation discussed in recent NAEP reports.)

Against this reality backdrop, Secretary of Education Arne Duncan recently (June 14, 2009, at the 2009 Governors Education Symposium) announced that to help measure the soon-to-be-released set of common core curricular standards that states agree upon—no small feat—the U.S. Department of Education will provide \$350,000,000 to states and state consortiums to create rigorous assessments linked to the new common core standards. (See the New England Common Assessment Program [NECAP] as an exemplar.) This funding will come from the Race to the Top funds available from the American Recovery and Reinvestment Act (ARRA). Secretary Duncan noted that “once new standards are set and adopted you need to create new tests that measure whether students are meeting these standards.” He continued, “We need tests that go beyond multiple choice—and we know that these kinds of tests are expensive to develop. It will cost way too much if each state is doing this on its own. Collaboration makes it possible for this to happen quickly and affordably.” A reauthorized NCLB based on common curriculum standards would present a leaner, more streamlined set of curricular expectations and perhaps more flexible guidelines for reaching NCLB.

As common, more focused, and coherent curriculum guidelines are considered along with more flexible guidelines for defining AYP, it is worthwhile to consider assessment issues generally and internationally. While NCLB and the National Assessment of Educational Progress

(NAEP) highlight achievement gaps within many states, school districts, and schools, yet another gap exists that is problematic. Yes, we have an assessment gap and it needs to be addressed—now.

Ruiz-Primo, Shavelson, Hamilton, and Klein (2002) identify a continuum of assessment distance as a model for articulating the distance between assessment events and their potential instructional impact. Class-based or *immediate* assessments include informal observations, classroom discussion, and artifacts from a lesson. *Close* assessments are those that teachers embed within their lessons or use to monitor progress. Together, *immediate* and *close* assessments define the typical formative assessments used in this country and internationally. *Proximal* assessments are also teacher or classroom driven, but are the formal unit tests or end-of-chapter exams included in curriculum materials and perhaps mandated by the school district, and are more summative in nature. *Distal* and *remote* assessments include the state NCLB-required assessments, standardized achievement tests, ACT and SAT tests, and other such assessments and are also summative. These distal and remote assessments serve a purpose and are important. They provide assessment “snapshots” that indicates how, generally, students are doing but the results have little meaning diagnostically, other than to examine particular types of items along with student errors and successes—classic item analysis issues regarding item difficulty and discrimination.

Continuum of Assessment Distance

- **Immediate:** informal observation or artifacts from a lesson;
- **Close:** embedded assessments and semi-formal quizzes following several activities or lessons;
- **Proximal:** formal classroom exams provided by particular curriculum materials and perhaps required by the district;
- **Distal:** criterion-referenced achievement tests such as those required by NCLB; and
- **Remote:** broad outcomes measured over time using norm-referenced tests.

FORMATIVE AND SUMMATIVE ASSESSMENTS:

While formative and summative assessments are often used as complementary approaches to assessment, all too often, formative and summative assessments are viewed by

teachers and students as distinctly different from each other. The point here is that linking formative and summative assessments together can help close the assessment distance in the continuum discussed earlier. Now, perhaps more than ever, the mathematics education community has the opportunity suggest very strongly that the full continuum of assessments are important opportunities for all students to more fully show what they know, and that any reauthorization of No Child Left Behind must consider varied assessments from immediate to remote or formative and summative.

EQUITY AND ASSESSMENT: Many countries successfully meet the needs of their multilingual students in classrooms where the language of instruction may differ from the languages used by students at home. Policies regarding the language of instruction vary tremendously throughout the world and even within countries. The decision in some areas of the United States to prohibit teachers and children from using languages other than English during instruction and on assessments may need serious reexamination, given the experiences of so many countries that embrace the multilingual nature of the members of their classroom communities. These policies have a significant impact on issue of access to educational opportunities, both as a result of opportunities to learn and as a result of their performance on standardized assessments, and need to be addressed.

CLOSING THE ASSESSMENT GAP: There is a gap, an assessment gap, and it certainly needs to be closed—now. We are over-assessing far too many of our students and the assessments are many, varied, and, far too often, not connected to teaching and learning. It is time, right now, to blur the assessment continuum. Teachers need to use assessment to help inform their teaching, to assist them in determining student needs and interventions, and to compare student progress across instructional units and grade levels. From immediate to distal, from formative to summative, the assessments need to be part of a plan—a well-articulated plan that focuses on using assessment to truly assist in the teaching and learning process. To do this right, the mathematics education community in the United States must focus more carefully on issues of equity as it relates to assessment. If we seek evidence of mathematical understandings with varied forms of assessment, we could paint a different picture of the mathematics achievement of many of our students. The conversation about achievement has to become much more robust, and

to do this, our assessment gap must close. It must blend formative and summative assessments if we are to honor and celebrate the knowledge of our students and the knowledge base of the communities to which they belong. This is especially important as we consider the potential of common curriculum standards and assessments.

In closing, we submit the following questions, just as an initial step in considering your own assessment plan, and linking assessment to important issues of teaching and learning:

- What is your state or school district's assessment plan?
- How do you use formative and summative assessments to determine student needs and interventions?
- How does your assessment plan accommodate the needs of mathematics learners whose primary language is not English?
- Is the use of formative assessment a regular component of every teacher's mathematics lessons? How do you know?
- Do students have opportunities to demonstrate what they know via assessments that are not test-like?
- How do you implement and use the data gathered from a full range of assessment opportunities—from immediate to remote and both formative and summative?
- How will your formative and summative assessments change as common curriculum standards become a reality?

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